Economics 311 Midterm Exam

John Kennan, December 19, 2014

Time allowed: 2 hours

Do **FOUR** questions (all questions have equal weight).

IMPORTANT: Explain your answers carefully. A good diagram is often more effective than a lot of words (but you must explain what the diagram means). You get no credit for unsupported assertions or guesses. Write as if you are trying to convince an intelligent person who does not already know the answers. If your answers would not convince such a person, it will be assumed that you do not fully understand the answers.

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1. The Phoenix Moons, a professional football team, has a stadium which seats 30,000 people. All seats are identical. The optimal ticket price is \$5, yet this results in an average attendance of only 20,000.

   (a) Explain how it can be profitable to leave 10,000 seats empty.

   Solution
   In order to sell more seats it is necessary to reduce the price charged for all of the seats. If $MR = MC$ with some seats empty, then selling more seats would mean that cost increases more than revenue (e.g. if $MC = 0$, selling more seats would mean that revenue falls).

   (b) Next week the Moons play the Tucson Turkeys, who have offered to buy an unlimited number of tickets at \$4 each to be resold only in Tucson. How many tickets should be sold to Tucson to maximize profits? (i) 30,000, (ii) more than 10,000, (iii) 10,000, (iv) less than 10,000, (v) none. Explain your answer.

   Solution
   (ii) more than 10,000, assuming that marginal cost is less than \$4 (if demand is highly elastic, marginal revenue could be above \$4 with $p = 5$; in that case nothing changes; but it is more plausible to assume that the marginal cost of selling seats is negligible). There is now an opportunity cost of selling seats in Phoenix, so the profit maximizing choice is where $MR = 4$.

   (c) Given your answer to part (b) above, what price should the Moons charge their own fans, to maximize profit? (i) \$5, (ii) more than \$5, (iii) between \$4 and \$5, (iv) \$4, (v) less than \$4. Explain your answer.

   Solution
   (ii) more than \$5, assuming that marginal cost is less than \$4. Since the quantity sold is reduced, a higher price can be charged.

2. Design a contract to maximize the expected profits received by a risk-neutral principal who will hire a risk-averse agent. The agent's utility function is

$$u(c, e) = \log(c) - e$$

where e is effort (high or low), and c is consumption, which is equal to the wage payment specified in the contract. The principal can observe gross revenue, but cannot observe the agent's effort. The agent has an outside option that is a sure thing with utility level $-\frac{1}{2}$. The low effort level is zero, and the high effort level is $\frac{1}{2}$.
Gross revenue depends on the agent's effort level. If effort is high, revenue R is 20 with probability $\frac{1}{5}$, and 25 with probability $\frac{4}{5}$. If effort is low, R is 20 with probability $\frac{3}{5}$, and 25 with probability $\frac{2}{5}$.

Solution
The contract specifies wages for each realization of output. Expected profit given high effort is

$$\pi = \sum_{i=1}^{n} p_i (R_i - w_i)$$

Choose $\{w_i\}$ to maximize this, subject to an incentive constraint

$$\sum_{i=1}^{n} p_i \log(w_i) - e \geq \sum_{i=1}^{n} p_i^0 \log(w_i)$$

and a participation constraint

$$\sum_{i=1}^{n} p_i \log(w_i) - e \geq u^0$$

High effort means R is 20 with probability $\frac{1}{5}$, and R is 25 with probability $\frac{4}{5}$, so expected revenue is 24. Low effort means R is 20 wp 3/5 and R is 25 wp 2/5, so expected revenue is 22. After paying for the agent's effort, the expected net payoff from high effort is $23\frac{1}{2}$, so this is what the principal wants to implement, unless it costs too much to provide the incentive.

The individual rationality (aka participation) constraint is:

$$\frac{1}{5}\log(W_{20}) + \frac{4}{5}\log(W_{25}) = 0$$

so

$$w_{20} = -4w_{25}$$

where w is the wage in logs.

The incentive compatibility constraint is then:

$$\frac{3}{5}w_{20} + \frac{2}{5}w_{25} = -\frac{1}{2}$$

so

$$-12w_{25_{2}} + 2w_{25} = -\frac{5}{2}$$

This gives

$$\begin{aligned} w_{25} &= \frac{1}{4} \\ w_{20} &= -1 \end{aligned}$$

profit is $24 - \frac{1}{5} \times \exp(-1) - \frac{4}{5} \times \exp\left(\frac{1}{4}\right) = 22.899$

If the principal chooses to implement a constract such that the agent supplies low effort, then profit is $22 - \exp\left(-\frac{1}{2}\right) = 21.393$.So the high-effort constract is more profitable.

3. A consumer has an income of $\$2,000$ per month, which is used to rent an apartment, and to buy food. Apartments can be rented at a monthly rate of $\$1$ per square foot. The consumer's utility function is

$$u(f, x) = \log(f) - \frac{1000}{x}$$

where f is food, measured in pounds, and x is apartment size, measured in square feet.

   (a) If the price of food is $\$4$ per pound, what is the optimal consumption plan?

   Solution
   Marginal utility of each good is infinite when the quantity is zero, so corner solutions can't be optimal.
   Equating marginal utility per dollar gives

$$\frac{1000}{x^2} = \frac{1}{f p_f}$$

   so

$$p_f f = \frac{x^2}{1000}$$

   Using the budget constraint

$$I - \frac{x^2}{1000} = p_x x$$

   and since $I = 2,000$ and $p_x = 1$ this implies

$$\begin{aligned} 0 &= x^2 + 1000x - (1000)(2000) \\ &= (x + 2000)(x - 1000) \end{aligned}$$

   So $x = 1000$ and then $f = \frac{1000}{4} = 250$.

   (b) If the price of food rises to $\$8$ per pound, does the consumer rent a smaller apartment?

   Solution
   No. The solution for x above was obtained without using any information about the price of food.

4. There are 3 people who live next to a lake. They all care about the quality of the water in the lake, but some care more than others. They have quasilinear preferences represented by utility functions

$$\begin{aligned} u_1(y,x) &= y + \log(x) \\ u_2(y,x) &= y + 3\log(x) \\ u_3(y,x) &= y + 5\log(x) \end{aligned}$$

where x is the water quality and y is a (private) consumption good. Each person has an endowment of 2 units of the y good.
Each person can make the water cleaner, at a cost:

$$c(q) = \frac{1}{2}q^2$$

where q represents the quantity of the private good used to clean the water, meaning that consumption of the private good is $y = 2 - \frac{1}{2}q^2$.
The water quality is determined by the equation

$$x = q_1 + q_2 + q_3$$

where q_i is the choice made by person i, taking the others' choices as given.

(a) Find a Nash equilibrium.

Solution
Write the utility functions as

$$u_i(y,x) = y + a_i \log(x)$$

Taking q_2 and q_3 as given, person 1 chooses q to maximize

$$u_1(y,x) = 2 - \frac{1}{2}q^2 + a_1 \log(q + q_2 + q_3)$$

The first-order condition is

$$\begin{aligned} q &= \frac{a_1}{q + q_2 + q_3} \\ &= \frac{a_1}{x} \end{aligned}$$

So $q_1 = \frac{a_1}{x}$, and similarly

$$q_i = \frac{a_i}{x}$$

Adding these equations gives

$$x = \frac{1+3+5}{x}$$

so $x^2 = 9$, and $x = 3$, and

$$\begin{aligned} q_1 &= \frac{1}{3} \\ q_2 &= 1 \\ q_3 &= \frac{5}{3} \end{aligned}$$

(b) Is the equilibrium outcome efficient?

Solution

No (because there is a nonpecuniary externality). Cleaner water benefits all of the consumers, but each consumer only takes the individual benefit into account when deciding how much of the y good is allocated to water-cleaning. For example, setting $q_1 = \frac{4}{3}$ would yield $x = 4$, and if person 3 then gives 1 unit of the y good to person 1, then person 3 is better off (because the marginal utility of the clean water is 1 for this person when $x = 5$, and above 1 when x is lower). This change also makers person 2 better off.

Efficiency requires that the marginal cost must be the same for each person (as long as everyone has positive consumption of the y good). Otherwise the person with the highest marginal cost could pay someone else to spend a little more on water-cleaning.

One efficient solution maximizes the sum of utilities. Then the sum of the marginal utilities is equal to the marginal cost, and the marginal cost.

$$\frac{x}{3} = \frac{9}{x}$$

so $x = 3\sqrt{3} \approx 5.196$.

The optimal solution for person 1 is to have both of the other people use all of their endowment to clean the water, and then person 1 adds $q_1 = \sqrt{5} - 2 \approx 0.236$

5. A monopolist faces the demand curve

$$P = 60 - \frac{2}{5}Q$$

where Q is the annual quantity sold, and P is the price, in dollars. Labor is the only input, and the labor supply curve is perfectly elastic at a wage of \$2 per hour. The production function is

$$Q = \sqrt{10L - 5000}$$

where L is hours worked.

(a) Find the profit-maximizing price and quantity.

Solution

Marginal Revenue is

$$MR = 60 - \frac{4}{5}Q$$

Cost is

$$C = wL$$

with

$$Q^2 = 10L - 5000$$

so

$$C = w\left(\frac{Q^2}{10} + 500\right)$$

and marginal cost is

$$MC = \frac{wQ}{5}$$

Since $w = 2$, equating MC and MR gives

$$60 - \frac{4}{5}Q = \frac{2Q}{5}$$

so the optimal quantity is $Q = 50$, and the price is $p = 40$.

(b) Suppose a price ceiling of \$35 is imposed. What is the new profit maximizing plan, and how much profit is made?

Solution

Now marginal revenue is \$35 up to the point where the demand price falls below the ceiling; this point solves the equation

$$35 = 60 - \frac{2}{5}Q$$

so $Q = 62.5$. At this point marginal cost is 25, so it is optimal to produce up to this point. Selling more would mean that marginal revenue falls below marginal cost, so the profit-maximizing point is at $Q = 62.5$.