A. The point of this problem is to give you practice solving a bargaining game by backward induction. The wording of the question left open for interpretation some of the details of who pays what to whom when. Different readings of the question gave rise to different sizes of the pie to be split between the employer and union, and different offers made by either side. For this reason, I accepted any reasonable interpretation of the structure of the bargaining game, and awarded most of the points based on whether you used a correct backward induction argument to solve the game.

Suppose we reach the beginning of the 50th week, there has been no agreement, the size of the pie is $P_{50} (= $500), and it is the employer’s turn to make an offer. Since the union has an outside offer of $240, the employer cannot offer less than this since the union will reject it. But the union has no reason to reject an offer of exactly $240, and the employer has no reason to offer more than that. So the employer offers the union $240, takes the remaining $50 - $240, and pays the negotiator $200. Going back a period, the size of the pie is $P_{49}$ and it is the union’s turn to make an offer. The union will make an offer that is as greedy as possible, but still acceptable to the employer. To offer the employer less than $P_{50} - $240 - $200 would be a mistake, since the employer would reject the offer, realizing that by holding out for one more period he will get $P_{50} - $240 - $200. But the employer has no reason to reject an offer of exactly $P_{50} - $240 - $200, and the union has no reason to offer more than that. So the union offers the employer $P_{50} - $240 - $200 and takes the remaining $P_{49} - (P_{50} - $240 - $200)$. Then we go back a period, to the 48th week, when the size of the pie of $P_{48}$ and it is the employer’s turn to make an offer. In this case, to make sure that the offer is acceptable to the union, the employer needs to make sure that the union does not (i) decide to leave the negotiations and work elsewhere at $240 a week; (ii) take the unemployment benefit that period and wait one period and collect $P_{49} - (P_{50} - $240 - $200)$. It is easy to see that we only have to be concerned about (ii) (that is, if the employer avoids (ii), she avoids (i) as well). Thus, the employer will offer the union $P_{49} - (P_{50} - $240 - $200) + $130, take the remaining $P_{48} - (P_{50} - $240 - $200)$, and pay the negotiator $200. And so on and so on, working backwards until we reach the first week. This initial split is what the two parties (assuming they are both fully rational, have complete information about the structure of the game, and that the union wins the toss and gets to make the first offer) will agree to immediately, without incurring the pie-shrinking costs of ongoing negotiations. But this is only half of the solution, since it was derived assuming that the union makes the first move so that it is the employer’s turn in the 50th week. We must now go back to the end of the game and see what happens if it is the union’s turn to make an offer. The backward induction argument will have exactly the same flavor as the one just described. The expected outcome of the bargaining game then, before the coin toss, is simply 1/2 times the split found when the union moves first plus 1/2 times the split found when the employer moves first.
One final word about solving the problem by backward induction: after following the argument for a few periods you should see a pattern. This should allow you to jot down a simple recursive equation and jump to the beginning of the game right away. Alternatively, you can always use brute force, with or without (preferably with) the help of a computer program such as Excel, painstakingly going through each of the fifty periods.

B. First note that the employer can observe whether a worker is of type a or b. What the employer cannot observe is whether a given a- or b-worker is qualified or not. Therefore, the employer can set different standards for each type of worker.

1. Let \( N \) be the number of workers of each type (i.e. there are \( N \) a-workers, and \( N \) b-workers), and \( t_a \) and \( t_b \) be the test scores required to assign an a- or a b-worker to a good job respectively. Then, the expected profit from a-workers is:
\[
\text{Profit}(t_a) = 1000N \left( \frac{2}{3} \right) (1-t_a) - 1000N \left( \frac{1}{3} \right) (1-t_a) \left( 2-t_a \right)
\]
Maximizing this expression with respect to \( t_a \), we obtain \( t_a = 0 \), that is, all a-workers are assigned to the good job.

And the expected profit from b-workers is:
\[
\text{Profit}(t_b) = 1000N \left( \frac{1}{3} \right) (1-t_b) - 1000N \left( \frac{2}{3} \right) (1-t_b) \left( 2-t_b \right)
\]
Maximizing this expression with respect to \( t_b \), we obtain \( t_b = 3/4 \).

2. (OPTIONAL)
In this case we need to ensure that the proportion of each type of workers assigned to the good job is the same, that is:
\[
(2/3)(1-t_a) - (1/3)(1-t_b) \left( 2-t_b \right) = (1/3)(1-t_a) + (2/3)(1-t_b) \left( 2-t_b \right)
\]
Let us call this constraint (1). Then, what the employer wants to do is to maximize
\[
\text{Profit}(t_a, t_b) = 1000N \left( \frac{2}{3} \right) (1-t_a) - 1000N \left( \frac{1}{3} \right) (1-t_a) \left( 2-t_a \right) + 1000N \left( \frac{1}{3} \right) (1-t_b) - 1000N \left( \frac{2}{3} \right) (1-t_b) \left( 2-t_b \right)
\]
subject to (1). To solve this a computer (or a lot of patience) is needed. The process would be: select a \( t_a \), and find from (1) what \( t_b \) would satisfy the requirement of equal proportions. Then calculate profits. Keep on doing this until you get to the maximum (which, by the way, was something like \( t_a = 0.53 \), \( t_b = 0.45 \)).

C. This is a broad question, and thus there were many different possible answers. Here are some important points.

First note that by competitive industry we referred to competitive in the labor market. Whether the industry is competitive on the product side or not will definitely be important, but notice that a monopoly may face a competitive labor market, and a monopsonist may face a perfectly competitive product market (think, for example, of a mine in a mining town, selling its output in the international market.) By the way, a monopsonist does not have a labor demand curve, since it never faces (in the absence of a union, at least) a given wage to which it reacts, but rather selects the wage-employment combination, subject to the constraint imposed by the labor supply curve.
There are two different aspects to consider. One is whether a union would have a greater bargaining power against a monopoly or against a group of competitive firms. We have not really discussed this in class, but many of you dealt with this issue. Two things to note: in the competitive case, the union (if it organizes the whole industry) can threaten to withdraw labor from any one firm credibly, as that does not imply giving up all labor income (the union would only give up wage income for some of its members, and this could be alleviated, say, by setting up a strike fund,) while against a monopsony withdrawal of labor has more extreme consequences for workers. On the other hand, the same factors that allowed for the existence of a monopsony, may make it easier for the union to ensure that no workers cross the picket line (for example, geographical concentration)

The second aspect is one to which we can give a more definite answer: given a certain union bargaining power, will the effect of the union on wages and employment be different if we start from a competitive firm as opposed to a monopsony?

To answer this question we have to discuss how bargaining will be conducted. For example, if the union is able to set the wage, and employers can then decide the amount of labor to hire, we will expect the union to increase wages, and as a result to employment will decrease. However, in the monopsony case, by increasing wages the union may actually be able to increase employment (same reasoning as with the case of a minimum wage.)

However, if bargaining is efficient, it is harder to tell a priori what will happen to wages and employment. In principle, any change that does not involve a simultaneous decrease of wages and employment (in which case, workers would dissolve the union,) is possible. What change actually takes place will depend on the shape of the union indifference curves, the firm’s isoprofit curves, and the relative bargaining powers.

D. This problem bears some resemblance to the pizza problem on your midterm, though this one is a bit easier since the distinction between hours and workers is not an issue here.

1. The union moves first by setting a wage, being as greedy as possible but recognizing that the employer will then take this wage as given and maximize profits accordingly. So if the union is too greedy, the employer will only demand a very small amount of labor at that wage. This can be thought of as a sequential two-period game in which the union moves first, followed by the employer. As with the backward induction argument we saw in part A, we can start our analysis in the last period by asking what the employer would do. The employer maximizes profits, \( \pi = PB - wL \), taking the wage, \( w \), as given. In this problem, the production function is \( B = L \), which gives production of sixpacks per hour. Substituting this, along with the product demand curve, into the profit equation gives \( \pi = (1200 - 0.16L)L - wL, \) or \( \pi = (1200 - w)L - 0.16L^2 \). The employer chooses \( L \), the number of workers, to maximize profits, subject to the constraint \( L \leq 2000 \). This gives the labor demand curve:
Then we consider the first period, in which the union selects \( w \) so as to maximize the total income of its 2000 members. That is, the union picks \( w \) to maximize \( wL + 320(2000-L) \), treating the employer's labor demand curve as a constraint. Substitute the labor demand curve for \( L \) in the union's maximand. Then take the derivative of the union's maximand with respect to \( w \), set it equal to 0, and solve for \( w \) to get \( w = 760 \) cents. [Technical note: Since \( L = 1375 \) at this wage, the constraint \( L < 2000 \) does not bind or is irrelevant.]

2. As we saw in class, contracts such as the one above are inefficient in that they do not lie on the contract curve. In contrast, contracts in which the wage and employment level are chosen simultaneously are Pareto efficient. Efficient contracts are difficult to enforce, however, because they are unstable. The contract curve lies to the right of the labor demand curve (that is, there are more workers hired at any given wage than the employer would like). So the employer will always be looking longingly at his labor demand curve, and hoping to hop off of the contract curve and onto his labor demand curve whenever the union is not looking. To solve this problem, the union will choose both \( w \) and \( L \) to maximize the total income of its members, being as greedy as possible but recognizing that the employer will reject any contract that will make it earn negative profits. That is, the union maximizes \( wL + 320(2000 - L) \) subject to the constraints \((1200 - 0.16L)L - wL = 0 \) and \( L = 2000 \). The answer is obvious if you consider a graph. Draw the employer's zero-profit isoprofit curve in a graph with \( w \) on the vertical axis and \( L \) on the horizontal axis. This is just the equation \( w = (1200 - 0.16L)L \), the first constraint in the union's maximization problem, and it looks like a hill with a peak at \( L = 3750 \). Now draw a vertical line at \( L = 2000 \), representing the second constraint in the union's problem. The union wants to get on the highest indifference curve it can (best \( w \) and \( L \) combination) that lies on or to the left of the line \( L = 2000 \) and on or below the zero-isoprofit hill. This must occur where the two constraints intersect, at \( w = 880 \) cents and \( L = 2000 \).

Note that the above argument amounts to maximizing total revenues given the constraint that \( L \) cannot be larger than 2000, and setting \( w \) equal to total revenues divided by the number of workers.