Very Rough Notes: Imperfect Competition

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Monopoly

Marginal Revenue.

$$\begin{aligned}\frac{dR}{\mathrm{d}Q} &= \frac{d\left(PQ\right)}{dQ} \\ &= P + Q\frac{dP}{\mathrm{d}Q} \\ &= P\left(1 - \frac{1}{\varepsilon}\right)\end{aligned}$$

Here ε represents the magnitude of the demand elasticity:

$$\frac{d\log\left(Q\right)}{d\log\left(P\right)} = \frac{P}{Q}\frac{dQ}{dP} = -\varepsilon$$

The monopoly price always exceeds marginal cost:

$$MC = P\left(1 - \frac{1}{\varepsilon}\right)$$

so

$$\frac{P}{\varepsilon} = P - MC$$

and

$$\frac{1}{\varepsilon} = \frac{P - MC}{P}$$

This is the Lerner Index: the markup as a fraction of the price. If the elasticity of demand is large then the markup is small.

The demand elasticity must be greater than unity (in magnitude); otherwise the monopolist could make infinite profit.

Monopoly output is not efficient, because the demand price of the product exceeds the mc of production. Price discrimination may solve this, to some extent. Regulation may take the form of a price ceiling set at AC, to control profits, or a price ceiling at $p = MC$, to get efficiency. The ideal case sets $p = mc$ where the mc curve cuts the demand curve. Then if $p > AC$ at this point, all is well, and a lump-sum tax can be used to remove excessive profits. But if AC is declining (as in a natural monopoly) marginal cost pricing means losses.

A natural monopoly is a situation where there isn't room for enough firms operating at the efficient scale. The simplest example is a constant marginal cost with a positive fixed cost.

Example: The salaries paid to baseball, football and basketball players have risen drastically in recent years. Are these cost increases passed on to the fans in the form of higher ticket prices?

How should tickets be priced, in order to maximize profit?

How does this calculation change when salaries rise?

Many Markets. Given the total volume of sales, however that is determined, the product should be distributed over markets so as to maximize revenue. This requires equal marginal revenues in each market (not equal prices). Price will be higher in markets with inelastic demand.

The Social Costs of Monopoly. Under monopoly the amount of output produced is too low, and the price that individuals would be willing to pay for an additional unit of output exceeds the marginal cost of production. Thus some potential gains from trade are not being realized.

The Regulation of Monopoly. Price regulation
The Sherman, Clayton, and Robinson-Patman Acts
Department of Justice and FTC Guidelines

Price Regulation with Rising AC. When the price is not too far below the monopoly level, output rises.
If the regulator sets exactly the right price, the outcome is fully efficient.
But this price is hard to find, and it's a moving target.

Price Regulation with Falling AC. If price is regulated at MC the firm takes a loss, and will leave the industry.
Regulator must find a way to spread the fixed cost.
A crude solution is $p = AC$, but this is not fully efficient.

Examples. Set tuition to marginal cost (a low number). Otherwise there is considerable waste: if someone can't afford to pay average cost it is inefficient to force that person to leave.
HBO should be free at the margin. So should computer software. Books should cost only a few dollars. Standby fares on airlines should be very low. International phone calls should be almost free (unless the satellite link is full).

Price Discrimination.

First Degree. Fleece them one at a time
Every customer is treated as a special person ("a special price, just for you")

Second Degree. Nonlinear Pricing, Quantity Discounts
Two-Part Tariff used to extract consumer surplus without reducing sales
Examples: Disneyland, Telephone service, Cover charges

Example [Tirole p146]. Two types of consumers, with demand curves given by

$$\begin{aligned} p_1 &= \theta_1(1 - q_1) \\ p_2 &= \theta_2(1 - q_2) \end{aligned}$$

The marginal cost of production is a constant, c, which is less than both θ_1 and θ_2.
If the monopolist sets a single price p, then demand is the horizontal sum of the two demand curves:

$$\begin{aligned} q_1 &= 1 - \frac{p}{\theta_1} \\ q_2 &= 1 - \frac{p}{\theta_2} \end{aligned}$$

so

$$Q = 2 - Ap$$

where $A = \frac{1}{\theta_1} + \frac{1}{\theta_2}$. Tirole's notation is $\frac{1}{\theta} = \frac{\lambda}{\theta_1} + \frac{1-\lambda}{\theta_2}$ where λ is the proportion of type-1 consumers, and θ is the harmonic mean.

$$R = \frac{Q(2-Q)}{A}$$

and

$$MR = \frac{2-2Q}{A}$$

So

$$Ac = 2 - 2Q^*$$

and the optimal price is determined from the demand curve

$$1 - \frac{Ac}{2} = 2 - Ap^*$$

so

$$p^* = \frac{1}{A} + \frac{c}{2}$$

Then profit is given by

$$\begin{aligned} \pi^* &= (p^* - c)\, Q^* \\ &= \left(\frac{1}{A} - \frac{c}{2}\right)\left(1 - \frac{Ac}{2}\right) \end{aligned}$$

so

$$\pi^* = A\left(\frac{1}{A} - \frac{c}{2}\right)^2$$

Now consider a two-part tariff

$$T(q) = \alpha + xq$$

The monopolist can choose α and x. Ideally, the monoplist could charge different prices for the two types, but it is not possible to tell which is which.

The tariff suggested by Oi sets price equal to marginal cost ($x = c$), and sets α so that the type with the lower demand is indifferent between buying or not. That is, the entry cost extracts all of the consumer surplus from the first type (with $\theta_1 < \theta_2$). The area under the triangle for the first type

The optimal two-part tariff sets price above marginal cost (but below the monopoly price)

$$x^* = \frac{c}{2 - \frac{\theta}{\theta_1}}$$

Third Degree. Completely separate markets, consumers cannot arbitrage.

Equate Marginal Revenues across Markets

Charge high prices where elasticity of demand is low.

$$MR_1 = P_1\left(1 - \frac{1}{\varepsilon_1}\right) = P_2\left(1 - \frac{1}{\varepsilon_2}\right)$$

Oligopoly

Cournot. Suppose there are n identical firms, with constant marginal costs, and the market demand curve is linear. Measure price relative to marginal cost, and choose units so that the maximal price is 1, and the maximal quantity is also 1.

A Nash equilibrium means quantities $\{q_i\}$ that are best responses to each other.

Let Q_{-i} be the total quantity supplied by other firms. Then the relationship between price and quantity for firm i is

$$P = (1 - Q_{-i}) - q_i$$

and marginal revenue for this firm is

$$MR_i = (1 - Q_{-i}) - 2q_i$$

Thus i's best response is

$$q_i = \frac{1 - Q_{-i}}{2}$$

Subtracting $\frac{q_i}{2}$ from each side gives

$$\frac{q_i}{2} = \frac{1 - Q}{2} = \frac{P}{2}$$

So $q_i = P$ for all i, and $Q = nP$, and the demand curve then gives $P = 1 - nP$, so

$$P = \frac{1}{n+1}$$

with

$$q_i = \frac{1}{n+1}$$

for all i. This gives the monopoly solution when $n = 1$, and the competitive solution in the limit as the number of firms increases.

Bertrand. There are two firms, with the same cost function, which is linear: $MC = AC = c$. Consumers view the two products as identical, and they will buy whichever is cheaper.

Consider a Nash equilibrium in which the two firms are charging different prices. Then if one firm is making money, the other is not, and if this firm just matches the other firm's price, profit will be positive.

Assume that the maximal demand price is above cost. There can't be a Nash equilibrium in which either firm is charging below cost, because that firm could do better by selling nothing. There can't be a Nash equilibrium in which both firms are charging a price strictly above cost, because then the firm with the higher price could do better by charging a price slightly lower than the other firm's price (and if the prices are initially equal, either firm could gain by doing this). This implies that at least one firm is setting $p = c$. If the other firm is setting a higher price, this is not an equilibrium because the low-price firm could raise its price. Thus the only Nash equilibrium is

$$p_1 = p_2 = c$$

Hotelling. The example in the text works out the Nash equilibrium in prices with the locations a and b given. Whatever this equilibrium might look like, there will necessarily be a consumer located between a and b who is indifferent between the two sellers. If b moves a little closer to this person while charging the same price, there will be an increase in profit. So there is no Nash equilibrium with the sellers in different places.

Stackelberg. The leader (e.g. firm 1) chooses a profit-maximizing point on the follower's reaction curve. If the demand function is $P = 1 - Q$, the second firm's output is

$$q_2 = \frac{1 - q_1}{2}$$

so

$$\begin{aligned} Q &= q_1 + \frac{1 - q_1}{2} \\ &= \frac{1 + q_1}{2} \end{aligned}$$

and

$$P = \frac{1 - q_1}{2}$$

and

$$MR = \frac{1}{2} - q_1$$

So the leader's output is $q_1 = \frac{1}{2}$. This is also the monopoly output. But the price is not the monopoly price: $P = \frac{1}{4}$. The leader's profit is $\pi_1 = \frac{1}{8}$ (higher than the Cournot profit); the follower's profit is $\pi_2 = \frac{1}{16}$ (much lower than the Cournot profit).