Very Rough Notes: Games

September 2014 John Kennan

Prisoner's Dilemma. Each prisoner decides whether to confess to a crime, given that communication with the other prisoner is impossible. The payoffs are as follows.

		Two	
		Confess	Don't Confess
One	Confess	(c, c)	(a, d)
	Don't Confess	(d, a)	(b, b)

In the standard case these payoffs are in alphabetical order: $a > b > c > d$. The best outcome for each individual is when that individual confesses, and the other doesn't.

Confess is a dominant strategy for each player, so the only Nash equilibrium is where both Confess. Both would do better if neither confessed, but if this is the anticipated outcome, it pays to defect.

"Battle of the Sexes".

		Man $(\cdot, u^M)$	
		Ballet	Opera
Woman $(u^W, \cdot)$	Ballet	(b, a)	(c, c)
	Opera	(c, c)	(a, b)

If the woman plays B, the man's best response is B; and *vice versa.* In fact the best response to either strategy is to play the same strategy, and this is true for both players.

Strategies. A strategy is a complete specification of the moves the player will make, in every possible contingency. It is like a computer program such that the player can go away on vacation and have the program make all the moves.

A strategy profile is a complete list of strategies, to be played by all of the players.

Best Responses. Given a strategy profile, s_i is a best response if no other feasible strategy gives higher expected utility for player i, taking the other players' strategies as given.

In the Prisoner's Dilemma, if One's strategy is Confess, Two's best response is Confess; if One's strategy is Don't Confess, Two's best response is Confess.

Extensive Form. A game tree lists all of the nodes at which players make decisions. At each node, just one player is on the move. Different moves lead to different nodes at the next stage of the game. A player might not know the current node. For example, in a simultaneous move game, the information set contains several nodes, because each player needs to know which move the other has made in order to determine the node. Friedman has an example of matching pennies, in which there are two rounds, player 2 wins \$5 if the coins match, and the loser of the first round has the option of withdrawing before the second round is played. This is a simultaneous move game. The first node belongs to player 1, and there are two nodes in player 2's information set at the first move.

Normal Form (Strategic Form). A complete list of strategies for each player, and payoff functions for each player showing payoffs for every vector of strategies which might be used by all of the players. In the example of matching pennies, make a list of strategies for each player: I will choose H, then if the other does T I do T, otherwise I do H, etc. Note that many very different-looking games might have the same strategic form.

Example. Penalty Kicks

Sometimes a penalty kick is aimed straight at the goalkeeper, which seems like a stupid thing to do. But often it works, because the goalie dives for the corner and the space where he was standing is left open. Is this because both players are stupid?

The kick can be aimed in 5 possible directions: TopRight, TopLeft, BottomRight, BottomLeft, Center

In practice, the goalie probably can't stop a well-struck ball in the top corner, but there is a good chance that a kick aimed this way will go wide or high.

So assume that if the goalie knows or guesses correctly where the ball will be aimed, then the goalie can stop it. If both players randomize, there is a 20% chance of a save (given what the goalie does, there is a 1 in 5 chance that the kicker will do the same thing).

Chiappori, Levitt, And Groseclose, AER, Sept 02.