1. Suppose that crime is a profitable activity for some people, but not for others. A criminal who is caught pays a penalty J, and a criminal who is not caught receives H. The probability of being caught is σ. The alternative to crime is a legal activity that pays x, where x is randomly distributed over the population, with distribution function F. The payoffs J, H and x are measured in utilities. People decide whether to be criminals, after seeing the realization of x, according to whether the expected utility from crime exceeds the utility from legal activities.

The population is made up of two types of people, A and B, and the proportion of A-types is λ. The distribution of the returns to legal activities may be different for the two types, (with distribution functions F_A and F_B) but the payoffs J and H are the same.

Criminals are caught when the police decide to search them. But the police have fixed resources, such that the proportion of people who are searched is s. The police may decide to search A and B types with different probabilities, σ_A and σ_B, subject to the constraint that $\lambda\sigma_A + (1-\lambda)\sigma_B = s$.

a. If the objective of the police is to maximize the number of criminals who are caught, taking as given the number of people who have decided to be criminals, how should σ_A and σ_B be chosen?

b. Suppose the police are required to search A and B types with equal probability, and this is known before people decide whether to be criminals. Would this increase the crime rate?

c. If the objective of a planner is to minimize crime, and if the planner sets policy before people decide whether to be criminals, how should σ_A and σ_B be chosen?

d. If it is observed that in practice the police are more likely to search B types than A types, is it reasonable to infer that the police are prejudiced against B types?

e. Illustrate your answers using specific distribution functions.

σ is the probability of being caught, so the expected payoff from crime is $y = (1-\sigma)H - \sigma J$. The proportion of criminals is the fraction of people with x less than this expected payoff. So from the point of view of the police, the probability of a successful search is F(y). If the police are searching both types, then it must be that $F_A(y_A) = F_B(y_B)$. If only the B type is searched, then $\sigma_B = s$, and $F_A(H) \le F_B\big((1-s)H - sJ\big)$. This would arise if the A types have much better outside opportunities – for many of them crime doesn't pay even if they know they won't be caught, while the alternative opportunities of the B types are so bad that crime pays even if the probability of being caught is relatively high.

In an interior equilibrium, the B types are searched much more often, but they are no more likely than the A types to be actually committing crimes (because they know they are more likely to be searched).

The crime rate is $\lambda F_A(y_A) + (1-\lambda)F_B(y_B) = F(y)$.

To minimize the crime rate, σ_A should be varied so that at the margin, an increase in σ_A and the resulting decreasing in σ_B have exactly offsetting effects on crime.

The marginal utility per dollar condition implies $f_A(y_A) = f_B(y_B)$.

So the policeman equates the distribution functions, but the planner equates the density functions.

The police take crime choices as given, and try to catch the maximal number of people who have committed crimes.

The planner tries to minimize the number of crimes committed. This entails making a commitment to search some people who would not commit crimes if they knew they were likely to be searched, but would commit crimes if a search was unlikely. In order to deter such people, it is necessary to actually search them, ex post, even though that is not the optimal thing to do after the fact (they did not commit the crime, because they expected to be searched, so now it is a waste of resources to actually search them).

If $f_A(x) = 1$, and $f_B(x) = 2x$, with support [0,1], then an interior solution requires $y_B = ½$. In general $y = K(\gamma-\sigma)$, where $K = H+J$, and $\gamma = H/K$. So $\sigma = \gamma - y/K$, and in this case
$\sigma_B = \gamma - 1/(2K)$, and $\sigma_A = (s-(1-\lambda)\sigma_B)/\lambda$, provided that these numbers are between 0 and 1.

It might be better to use specific distribution functions in the question itself.

The question about prejudice is not worth much.

The first part is a question about equilibrium: the police take the crime rate as given, and the criminals make optimal choices. It's not just a question about the best response function for the police. But the question was not well written.

“Racial Profiling, Fairness, and Effectiveness of Policing,” *American Economic Review* 92(5), December 2002, pp. 1472-97.

In this environment, any interior equilibrium has the property that the probability of being searched is higher for the group with inferior alternative opportunities, yet the probability that a search catches a criminal is the same for both groups. Thus it may seem that one group is being searched more often for no good reason. The paper emphasizes that there is a good reason, in the sense that the differential search rates arise in equilibrium when the police are not prejudiced. If fairness is defined as equal treatment, then there is generally a conflict between fairness and effective policing. But it may well be that imposing a fairness constraint actually reduces the crime rate. This is a second-best result: since the police are not acting so as to minimize the crime rate, there is no presumption that a fairness requirement would make things worse.