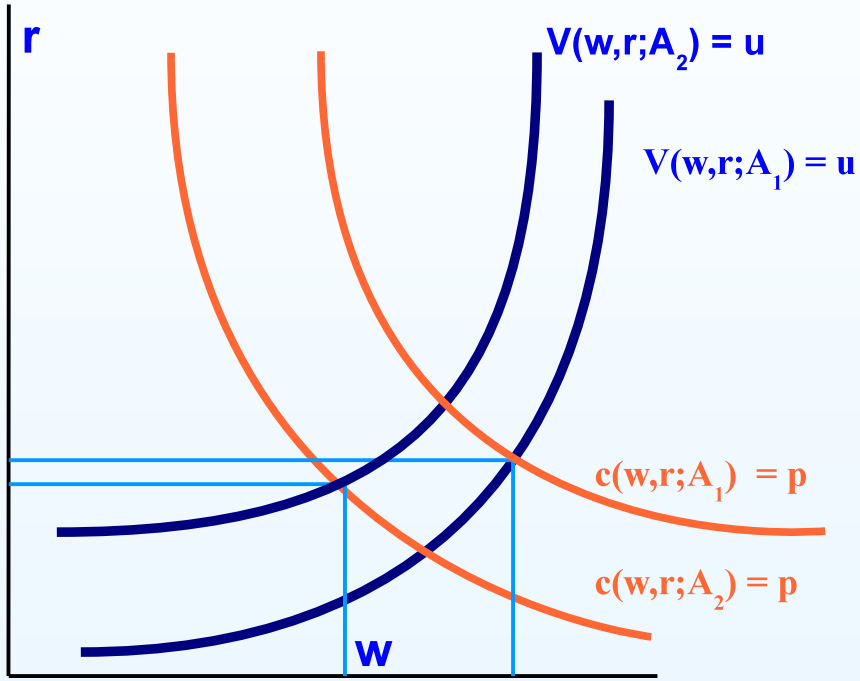
Internal and International Migration: Models and Empirics

John Kennan
University of Wisconsin-Madison and NBER

April 2014

Spatial Equilibrium: Rosen-Roback



Two locations

Amenities attract workers; also affect production costs

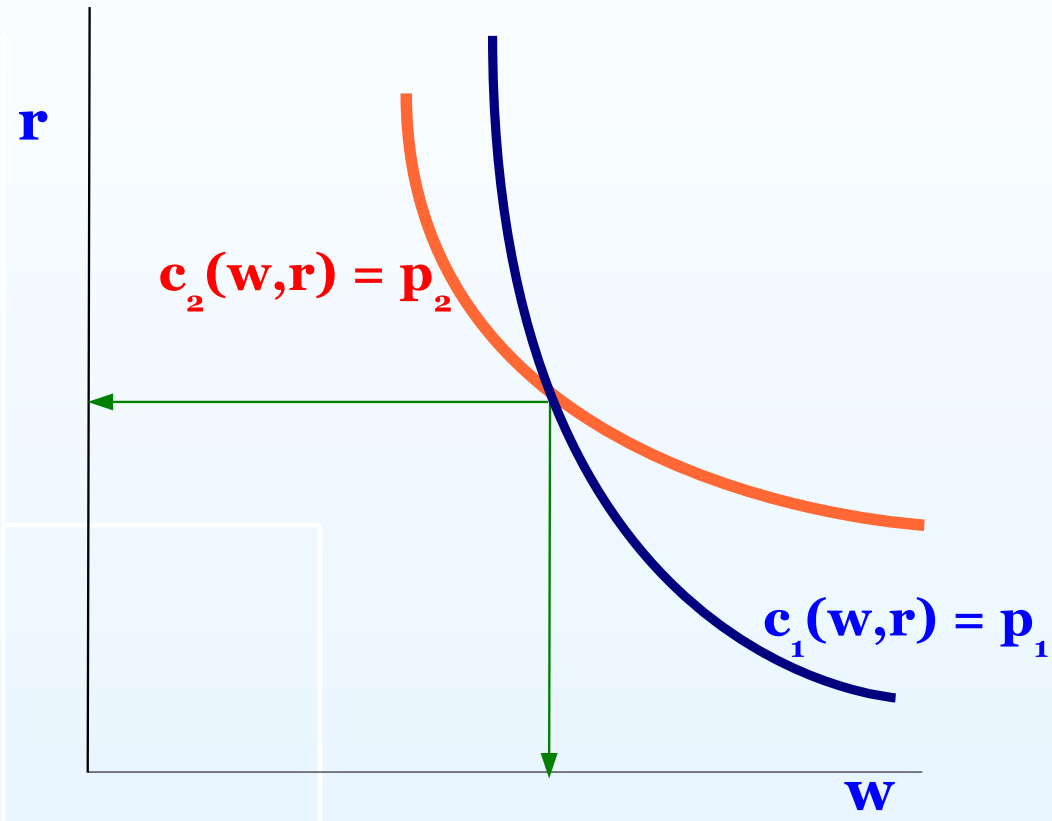
Workers like higher wages (w) and lower land rents (r)

Employers like lower wages and lower land rents

Equilibrium: workers and employers indifferent between (w_1, r_1) and (w_2, r_2)

Two orderings of (w, r): Two prices needed to get indifference

Factor Price Equalization



Two locations, Two products

Producers like lower wages (w) and lower capital prices (r)

Equilibrium: producers of each good indifferent between (w_1, r_1) and (w_2, r_2)

Two orderings of (w, r): Two prices needed to get indifference

Spatial Equilibrium: Empirical Evidence

Blanchard and Katz (1992)

Regional VAR with three variables, all in logs, relative to the national average:

(1) change in employment: Δe

(2) employment/LF: $1 - u$,

(3) LFPR

Annual data, 1978-1990. estimated only at the level of (9) Census regions

The current value of Δe enters the other two equations

idea: a labor demand shock changes employment, this affects participation

Empirical result:

- fall in employment associated with a rise in local unemployment initially,and a decline in participation,
  - employment fall is permanent
  - unemployment and participation return to normal after a while.

Interpretation: most of the adjustment involves net migration.

Migration Decisions

Kennan-Walker (2011) [internal migration in the U.S.]

Rebecca Lessem (2011a,b) [MX-US; Puerto Rico-US]

Maximize PV of lifetime income

w_{ij} individual i's earnings in location j – local price of individual's skill bundle

Wage in current location is known

Wages in other locations can be learned only by moving there

$$w_{ij}(a) = X_i\beta + \mu_j + \upsilon_{ij} + G(X_i, a) + \varepsilon_{ij}(a) + \eta_i$$

$w_{ij}(a)$ Wage of individual i in location j at age a

μ_j Mean wages in location j (known)

υ_{ij} location match effect (permanent)

G age-earnings profile

η_i individual effect, fixed across locations (known to the individual)

ε_i transient effect, iid over time

Migration decisions depend only on μ and υ

Migration

Location choice

$$V(x,\zeta) = \max_j \left(v(x,j) + \zeta_j\right)$$

x: state vector (Includes home location, current and previous location, age)

ζ: payoff shock (preferences or moving costs)

Continuation value

$$v(x,j) = u(x,j) + \beta \sum_{x'} p(x'|x,j)\bar{v}(x')$$

Expected continuation value

$$\bar{v}(x) \quad = \quad E_\zeta V(x,\zeta)$$

Choice Probabilities

$$\rho(x,j) = \exp\left(v(x,j) - \bar{v}(x)\right)$$

State Variables and Flow Payoffs

Flow payoff

$$\tilde{u}_h(x, j) = u_h(x, j) + \zeta_j,$$

$u_h(x, j)$ payoffs associated with observable states

$$u_h(x, j) = \alpha_0 w\left(a, \ell^0, \omega\right) + \sum_{k=1}^{K} \alpha_k Y_k\left(\ell^0\right) + \alpha^H \chi\left(\ell^0 = h\right) - \Delta_\tau(x, j)$$

ζ_j a preference shock or a shock to the cost of moving
ω location match component of wages
α^H attachment to home location

Moving Costs

Cost of moving to location $j \neq \ell^0$ in state x

$$\Delta_\tau(x, j) = \gamma_{0\tau} + \gamma_1 D\left(\ell^0, j\right) - \gamma_2 \chi\left(j \in \mathbb{A}\left(\ell^0\right)\right) - \gamma_3 \chi\left(j = \ell^1\right) + \gamma_4 a - \gamma_5 n_j$$

$\gamma_{0\tau}$ base cost (disutility) of moving, for someone of "type" τ
$D\left(\ell^0, j\right)$ distance from ℓ^0 to j
γ_2 cheaper to move to an adjacent location
$\mathbb{A}\left(\ell^0\right)$ the set of locations adjacent to ℓ^0 (e.g. States that share a border)
γ_3 cheaper to move to a previous location
γ_4 moving cost rises with age
γ_5 cheaper to move to a large location (n_j is the population in location j)

How Big are the Moving Costs?

Most people don’t move (e.g. from Puerto Rico to the U.S.)

The gains from moving are very big

So moving costs must be huge

But ...

How Big are the Moving Costs?

Most people don’t move (e.g. from Puerto Rico to the U.S.)

The gains from moving are very big

So moving costs must be huge

But ...

Some people do move (so the cost can’t be that big)

Many people move in the wrong direction

and many people return to a low-wage location (MX, PR)

after moving to a high-wage location (US)

How Big are the Moving Costs?

Most people don’t move (e.g. from Puerto Rico to the U.S.)

The gains from moving are very big

So moving costs must be huge

But ...

Some people do move (so the cost can’t be that big)

Many people move in the wrong direction

and many people return to a low-wage location (MX, PR)

after moving to a high-wage location (US)

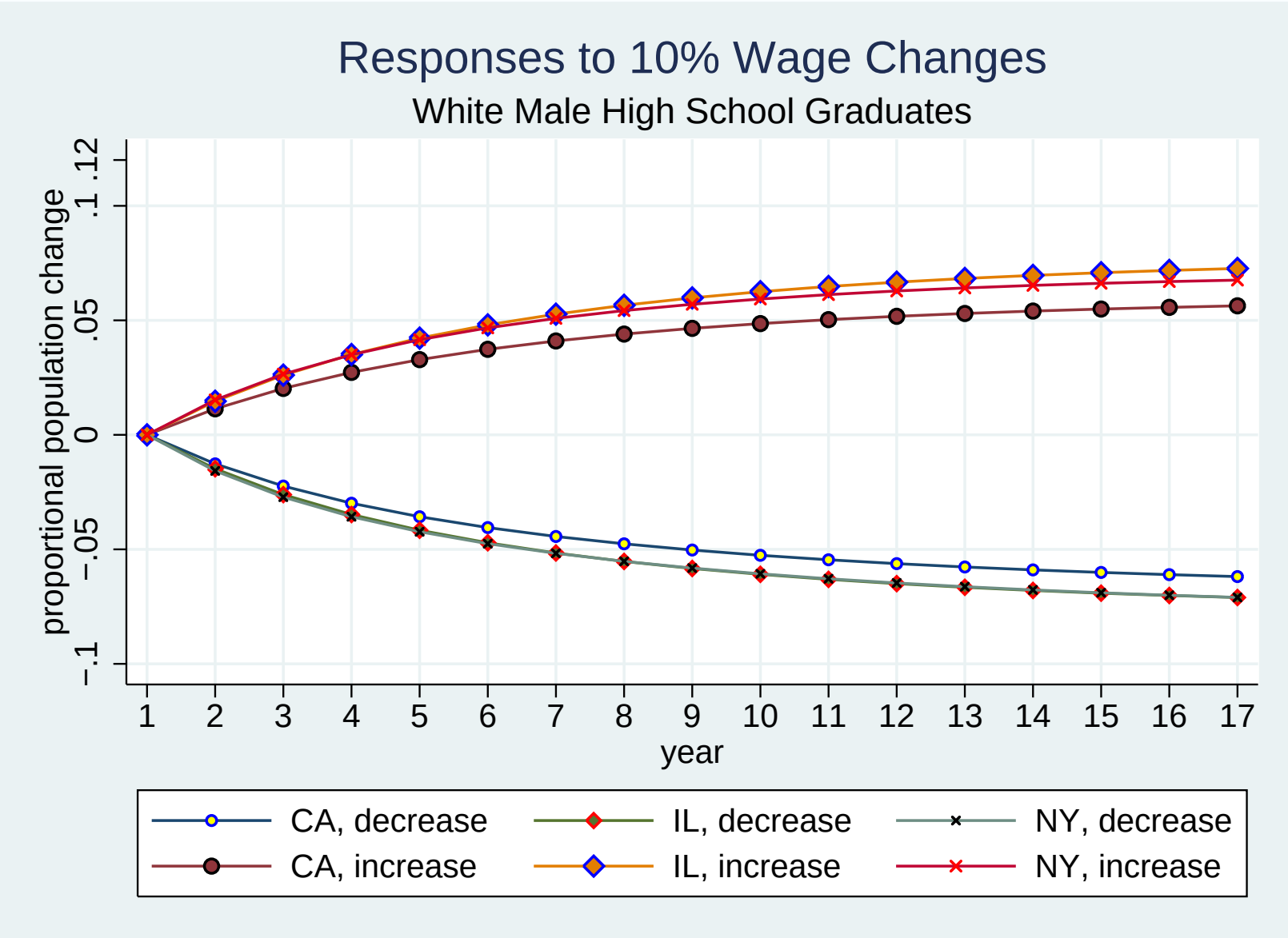
A lot of migration has nothing to do with income (“payoff shocks”)

Moving costs are heterogeneous

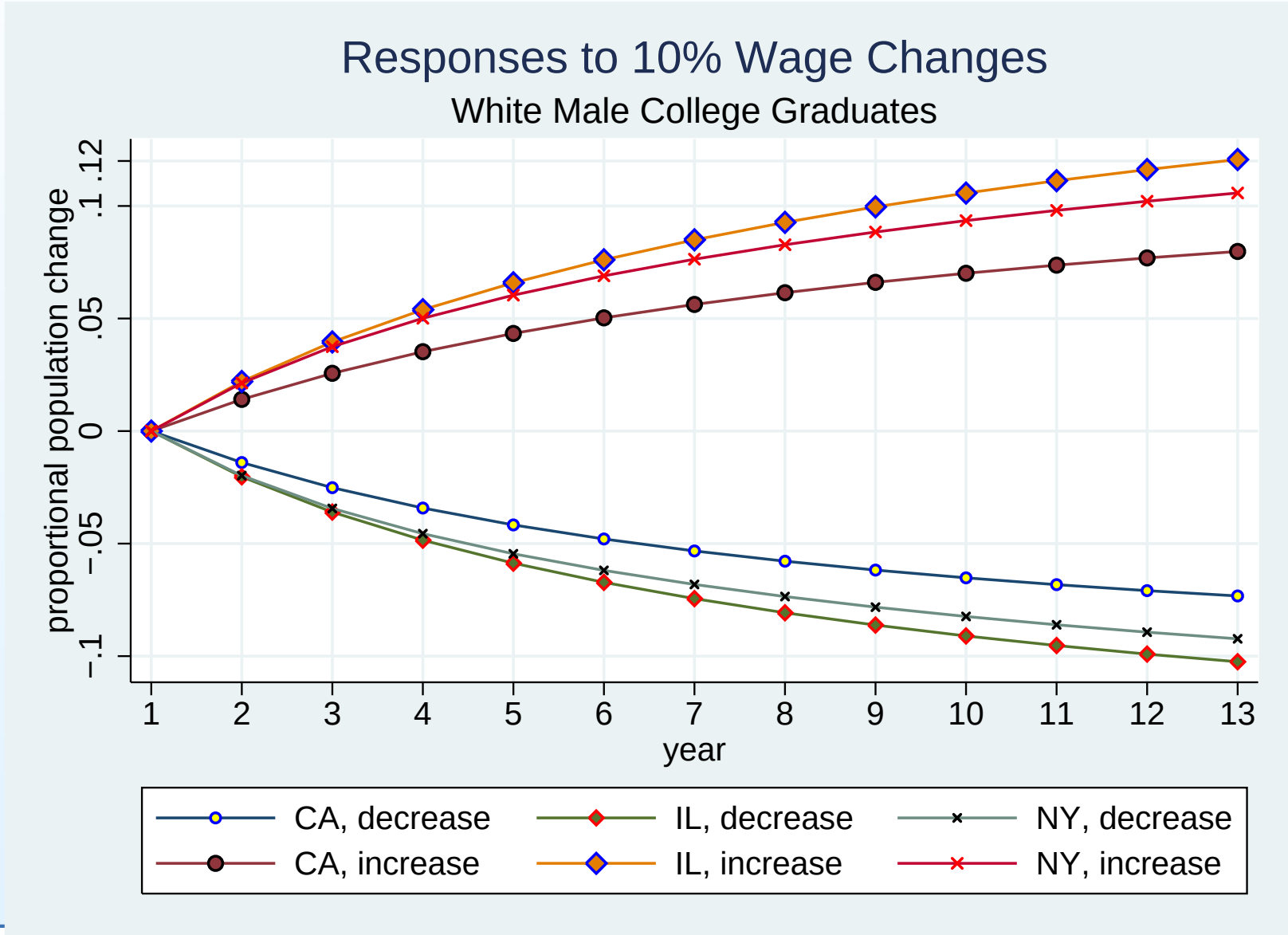
Average cost for those who move is low

Cost of a forced move would be high

Geographical Labor Supply Elasticities



Geographical Labor Supply Elasticities



Spatial Equilibrium: Empirical Models

Rebecca Diamond (JMP 2013)

A static model of location choice (ignoring repeat and return migration).

The difference in wages across cities understates the difference in welfare because high-wage cities have better amenities.

Technology

$$N^\rho = \theta^H H^\rho + \theta^L L^\rho$$

where $\theta^H = \left(\frac{\bar{H}}{\bar{L}}\right)^{\gamma_H}$ and $\theta^L = \left(\frac{\bar{H}}{\bar{L}}\right)^{\gamma_L}$

Firms in cities with a high proportion of skilled workers are more productive

- (even if the firms themselves hire mostly unskilled workers)

Spatial Equilibrium: Empirical Models

Estimation by BLP

Value of each choice for person i

$$v_{ij} = \delta_j + \zeta_{ij}$$

Choice probabilities

$$\rho_j = \frac{\exp(\delta_j)}{\sum_k \exp(\delta_k)}$$

Normalize $\delta_0 = 0$ (because choices only depend on differences). Then

$$\exp(\delta_j) = \frac{\rho_j}{\rho_0}$$

Spatial Equilibrium: Empirical Models

Infer the desirability of each alternative from the proportion of people who choose it

Then analyze how values depend on characteristics (of locations)

$$\delta = \beta^w W - \beta^r R + \beta^{st} x^{st} + \beta^{div} x^{div} + \beta^{col} \frac{H}{L}$$

Wages, rents, home location effects, joy of living with smarter people

Labor demand shocks are measured using the Bartik instruments

Results

Relative productivity changes drew more high-skilled workers to cities (1980-00)

Local amenities in cities increased

Low-skilled workers priced out of cities

True real wage inequality increased more than measured inequality because amenity differences increased

Spatial Equilibrium: Empirical Models

Suphanit Piyapromdee (JMP 2014)

Migration flows generated by spatial wage differentials

moderated by congestion in housing markets, and home-biased locational preferences

different kinds of workers are imperfect substitutes.

Estimated model: wages, employment for different cities and worker types are equilibrium outcomes

mix of IV, GMM and ML

used to recover technology and preference parameters.

Technology is Cobb-Douglas in capital and composite labor

TFP differs across cities, constant returns everywhere

Spatial Equilibrium: Empirical Models

Labor composite is nested CES

Ordering of Labor components:

education (2) then sex (2), then age (2), then birthplace (2)

Perfectly elastic capital supply

Preferences

$$U_i = \zeta_i \log(Q) + (1 - \zeta_i) \log(G) + u_i(N_c)$$

Q is housing, G is consumption (composite good), N is amenities.

Workers of type z choose locations to maximize utility subject to

$$PG + R_c Q = W_c^z$$

Spatial Equilibrium: Empirical Models

Results

even large increases in immigration have small effects on wages

- (constant returns, perfectly elastic capital supply)

interesting adjustments in spatial wage differentials

- immigrants tend to move to cities that already have many immigrants
- native workers tend to stay in the place where they were born.
- substantial negative effects on the wages of unskilled workers in Miami when the increased immigrant flow is restricted to unskilled workers.

A GE model that subsumes Diamond's model (without the amenity externalities), and the Ottaviano-Peri analysis of substitution between immigrants and native workers

Interstate Migration

Kaplan and Schulhofer-Wohl (2013)

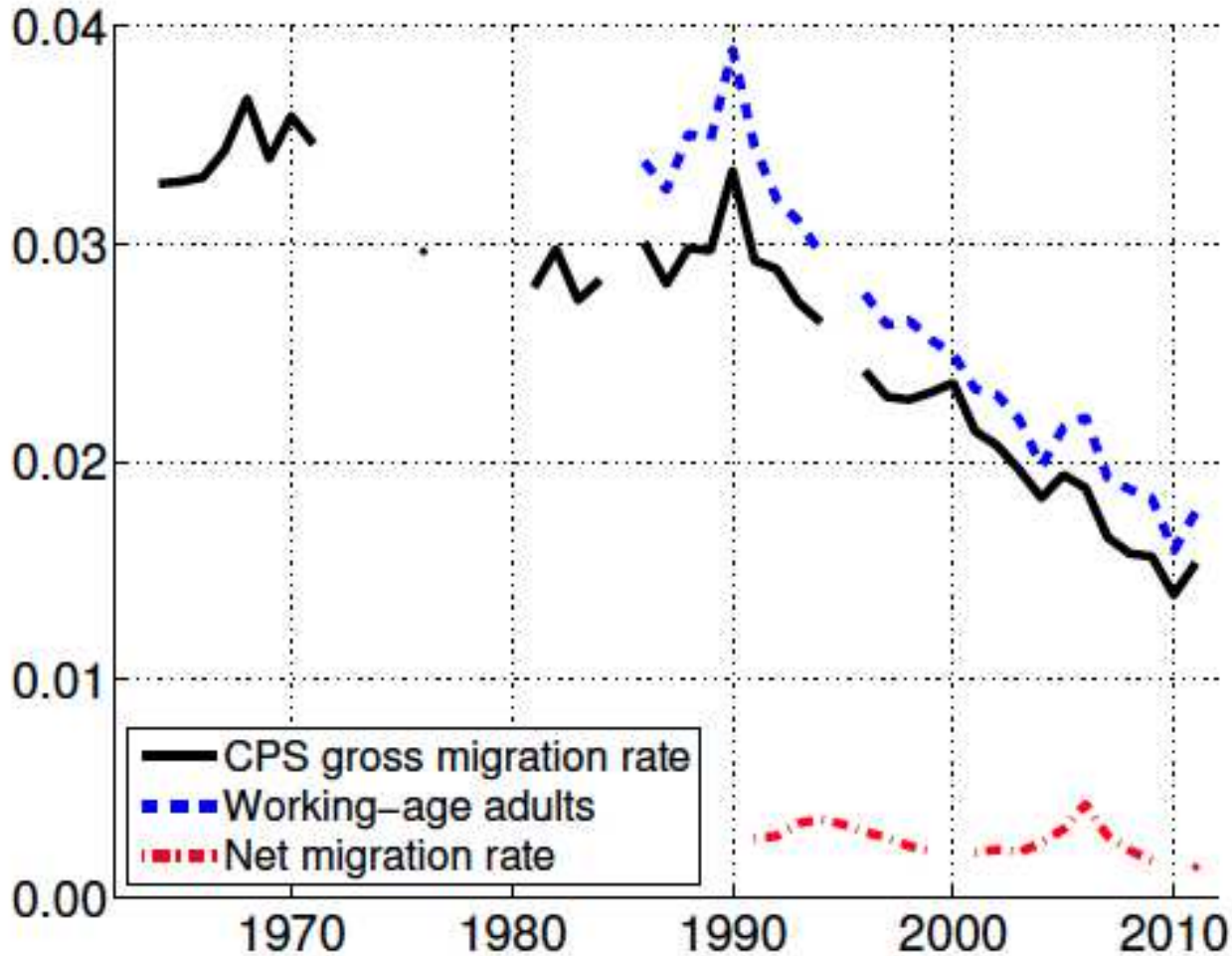
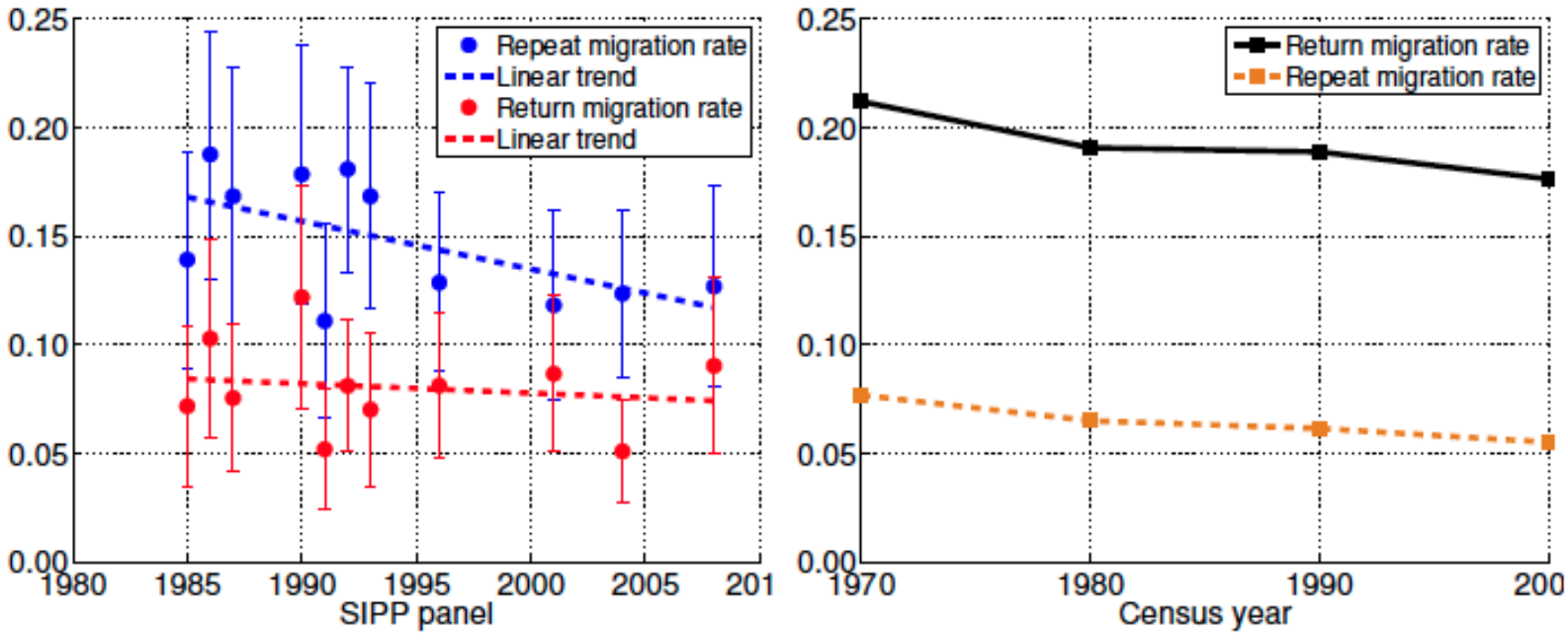


Figure 1: Gross and net interstate migration.

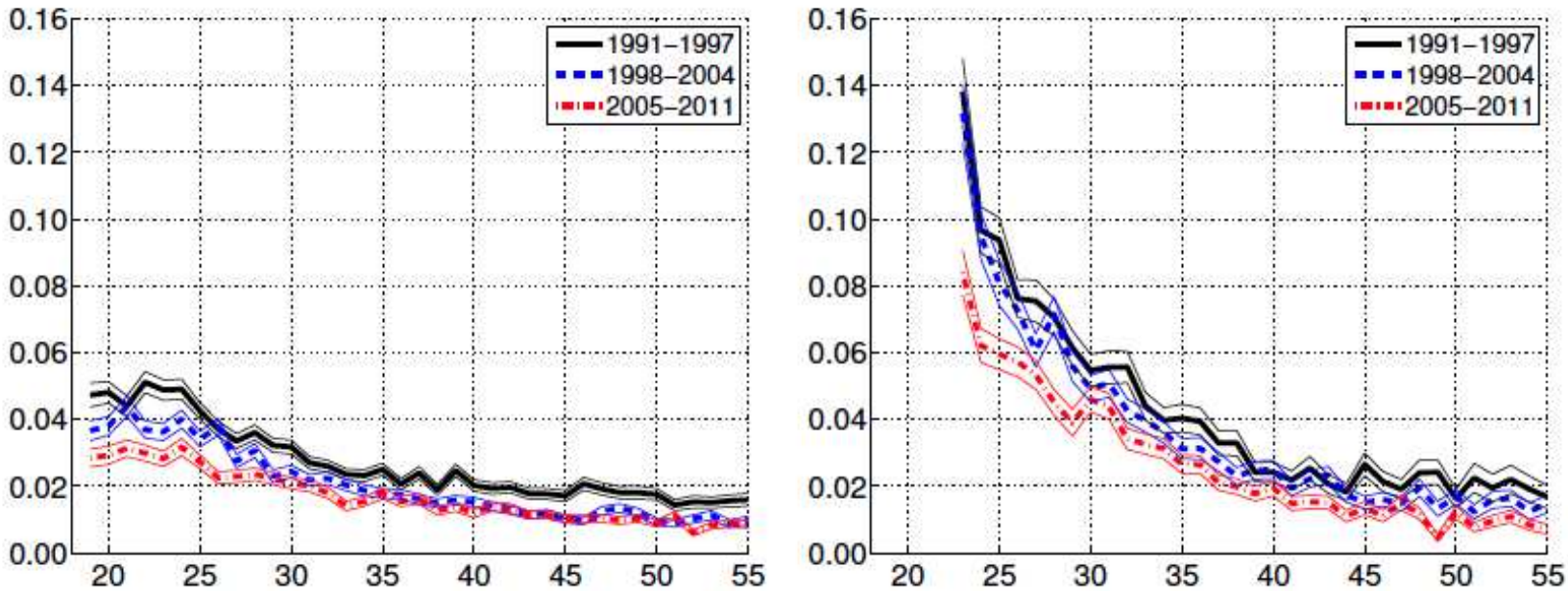
Interstate Migration



(a) Repeat migration in the SIPP (b) Repeat and return migration in the census

Figure 13: Repeat and return interstate migration rates.

Interstate Migration



(a) No bachelor's degree (b) Bachelor's degree

Figure 4: Age profile of interstate migration.

The Economics of Immigration

A huge literature, addressing a limited set of questions

1. Assimilation
2. Selection
3. Effects on Wage Levels and Skill Premia in Host Countries

These questions are interesting

But the most interesting question is largely ignored

The economy of Zimbabwe is a disaster.

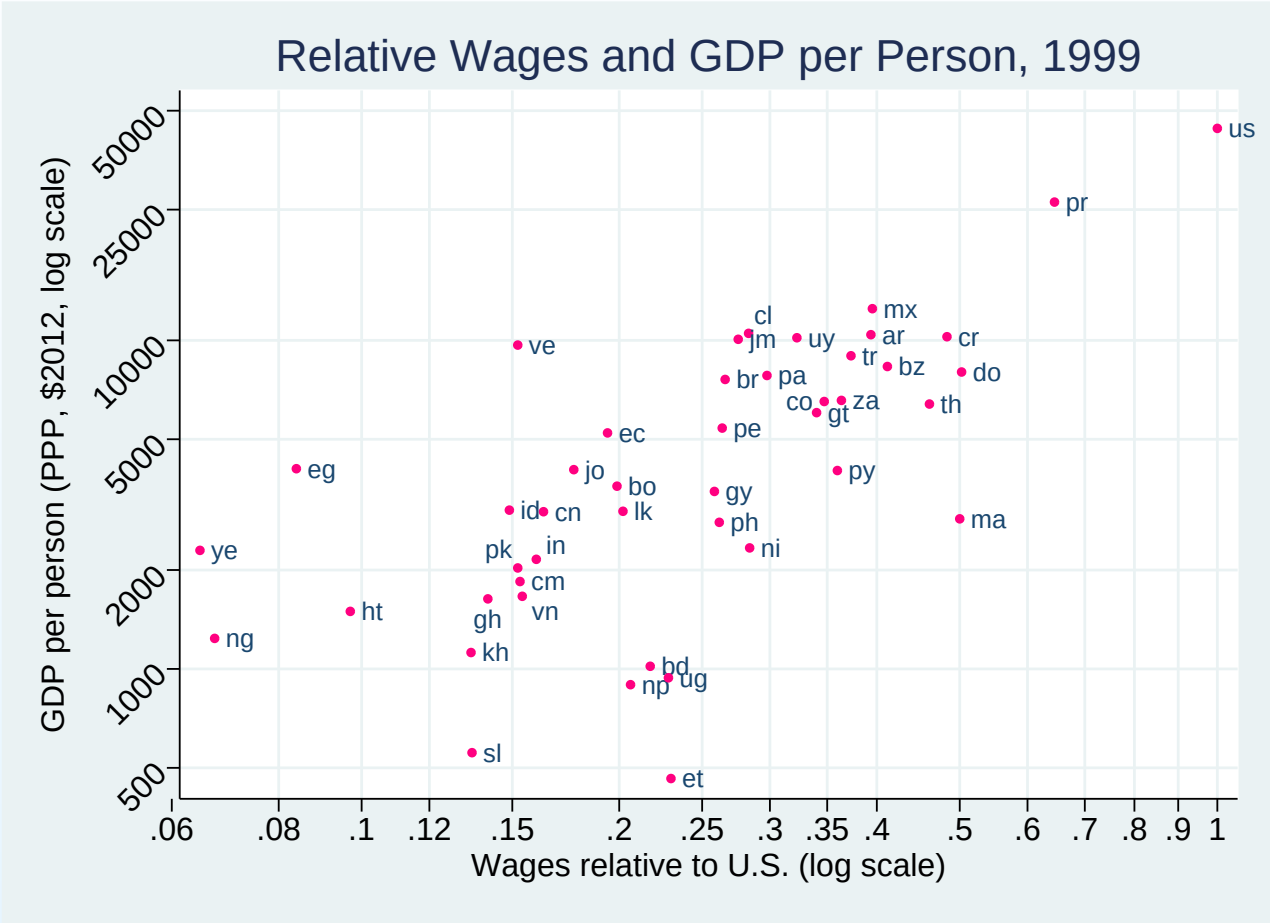
Why worry about how to raise income in Zimbabwe?

Why not just let people move to better places?

What would happen if we just let people choose where they want to live?

- The immigrants who would not otherwise have moved would be better off.
- By how much?
- Who would lose, and how much?

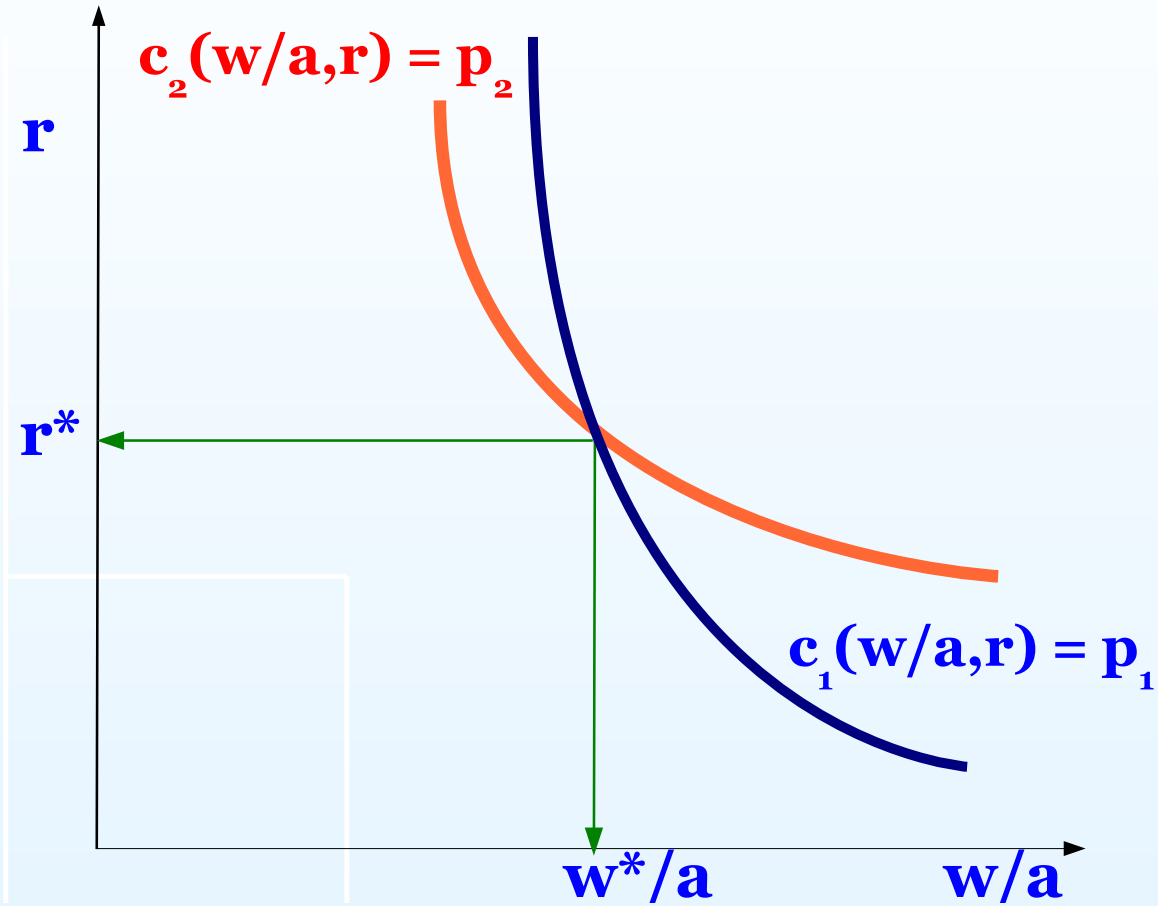
International Wage Differentials



Clemens, M.A. and Montenegro, C.E. and Pritchett, L., “The place premium: wage differences for identical workers across the US border” (2008).

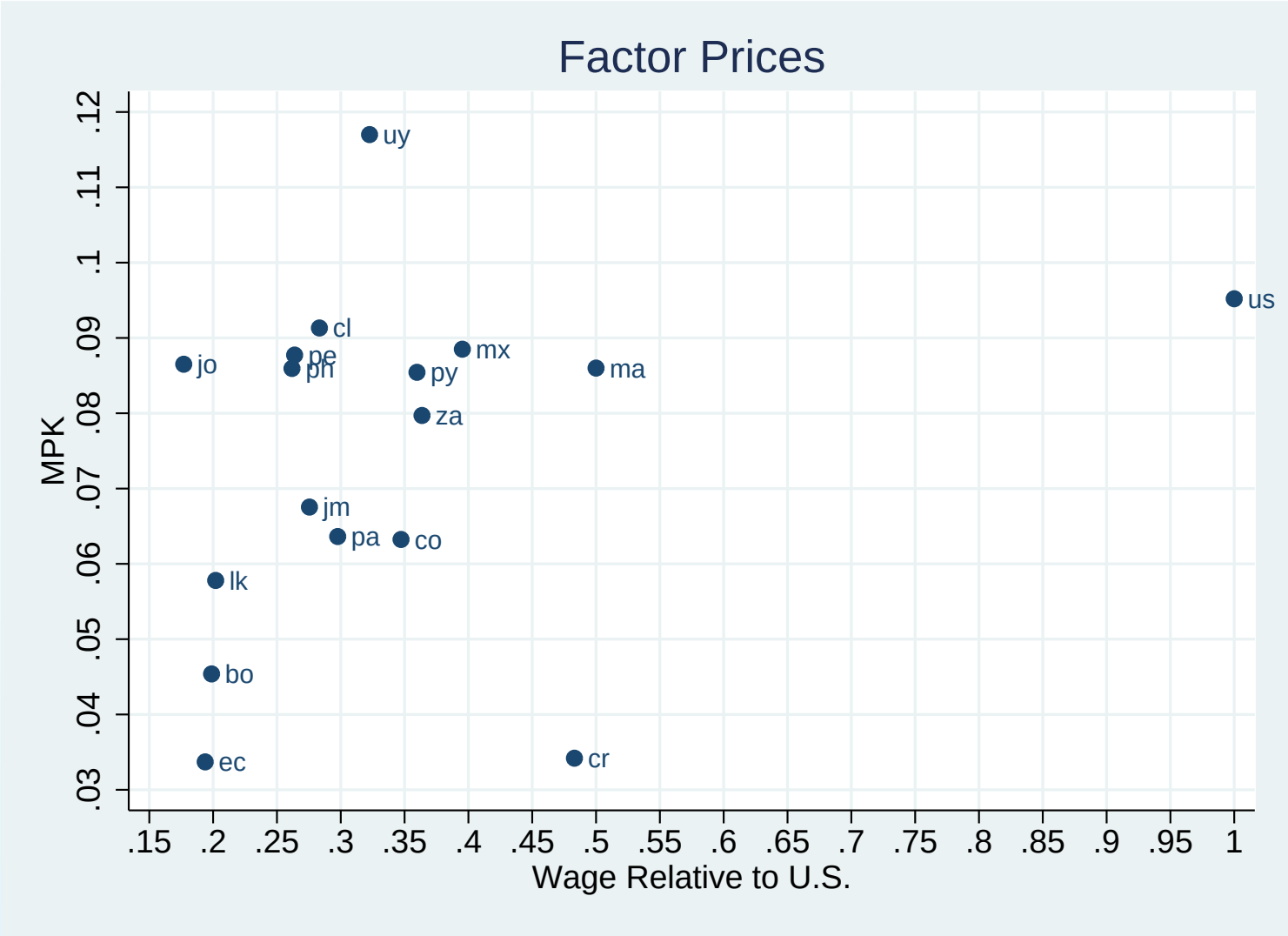
Foreign-born, foreign-educated workers in the U.S. Census compared with similar workers in 42 home countries

Factor Price Equalization with Productivity Differences



$\frac{w}{a}$: wage per efficiency unit of labor

Wages and the Marginal Product of Capital



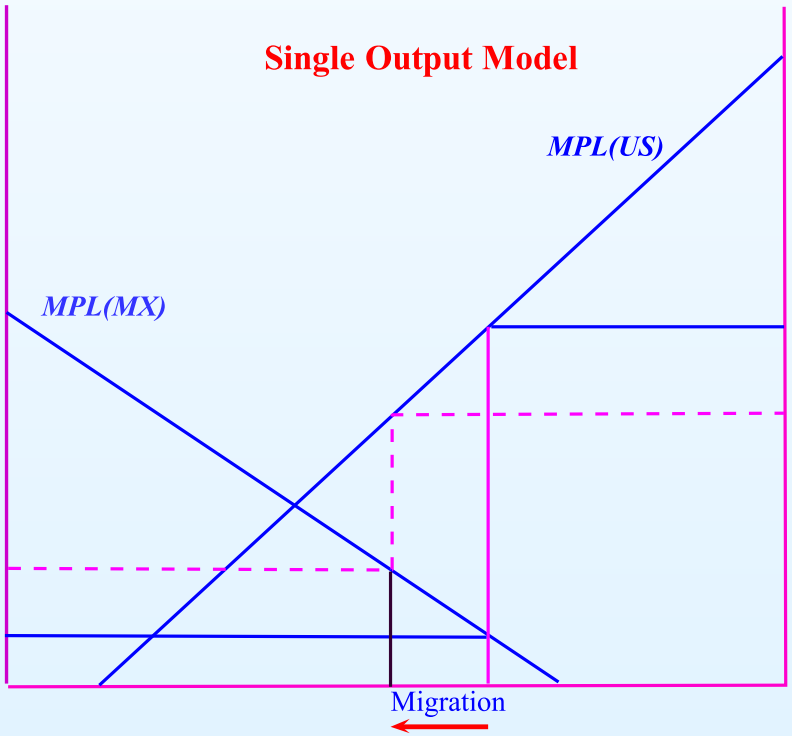
MPK: Caselli and Feyrer, "The Marginal Product of Capital", *QJE* (2007)

Implications

"the very large wage ratios we observe for many countries are sustained by policy barriers to movement" [Clemens et al, (2008)]

"In theory, moving labor from a poor to rich country ... lowers (raises) incomes for laborers in the receiving (sending) country" [Hanson (2010)]

Not in the HO model: removing the barriers has no effect on wage ratios; emigration does not raise wages



Labor Supply and Wages with Open Borders: Magnitudes

Simple Model

Proportion of people who do not move is equal to the relative wage – the ratio of income at home (y_j) to the highest income elsewhere (y_0)

Derivation: Assume log utility. Stay if

$$\log(y_0) - \delta \leq \log(y_j)$$

δ: disutility of moving (attachment to home), randomly distributed over people

Assume the distribution of δ is the unit exponential: $\mathrm{Prob}(\delta \geq x) = e^{-x}$

Then the probability of staying is

$$\mathrm{Prob}\left(\delta \geq \log\left(\frac{y_0}{y_j}\right)\right) = e^{-\log\left(\frac{y_0}{y_j}\right)} = \frac{y_j}{y_0}$$

Immigration and Wages

- A relaxation of immigration restrictions leads to a fall in the real wage
- The wage effect is the same in all (both sending and receiving) countries
- but migration reduces the wage per efficiency unit (and so reduces the wage of all non-migrants)
- Prices of labor-intensive goods fall relative to capital-intensive goods
- but the real wage falls regardless of the composition of consumption
- If $\bar{L}$ doubles the factor price ratio also doubles (Cobb-Douglas)
- So if the capital share for good s is $\alpha_s = \frac{1}{3}$,
- the real wage falls by about 20% when measured in terms of good s.
- Migration increases the wages of (most) migrants

Factor Price Equalization with Productivity Differences

J countries, with different productivity levels.
Productivity differences are labor-augmenting (Harrod-neutral)
(equivalent to TFP differences in the *1-product* Cobb-Douglas case)
Production function for product s in country j

$$Q_s^j = F_s\left(K_s^j, a_j L_s^j\right)$$

a_j: efficiency units of labor per worker in country j (same for all s)
No mobility of capital or labor across countries
Cost function for product s in country j

$$c_s^j\left(v, w\right) = c_s^0\left(v, \frac{w}{a_j}\right)$$

where w is the wage per efficiency unit of labor, and v is the price of capital
c_s^0 is the unit cost function when labor is measured in efficiency units,
$Q_s = F_s\left(K_s, L_s\right)$.

Factor Price Equalization with Productivity Differences

Free trade in product markets, no transport costs

Zero-profit condition implies

$$p_s = c_s^0\left(v_j, \frac{w_j}{a_j}\right)$$

If two products r and s are produced in country j, then

$$\begin{aligned} c_r^0\left(v_j, \frac{w_j}{a_j}\right) &= p_r \\ c_s^0\left(v_j, \frac{w_j}{a_j}\right) &= p_s \end{aligned}$$

These equations determine the factor prices in country j.

If the marginal rates of technical substitution satisfy a single-crossing condition, the factor prices are uniquely determined

Factor Price Equalization with Productivity Differences

If country ℓ also produces these same two products,
the same equations determine factor prices in country ℓ (with a_ℓ in place of a_j)
This implies $v_j = v_\ell$, and

$$\frac{w_j}{a_j} = \frac{w_\ell}{a_\ell}$$

Thus

$$w_j = a_j w_0$$

where w_0 is a reference wage level that can be normalized to 1.
In this model, migration has no effect on relative wages.
If 30 million workers move from Mexico to the U.S.,
it will still be true that the wage in the U.S. is 2.5 times the wage in Mexico.
But migration affects wage levels.

General Equilibrium

Given factor prices, goods prices are determined by the cost functions
Given goods prices, quantities are determined by preferences and total income (where income depends on factor prices)
Given goods quantities, and factor prices, producers choose factor quantities
Given factor demands, factor prices determined by market clearing
If the production function for each good is a CES, the price of good s is given by

$$p_s^{1-\sigma} = \alpha_s \left(\frac{v}{\alpha_s}\right)^{1-\sigma} + \beta_s \left(\frac{w}{\beta_s}\right)^{1-\sigma}$$

w is the wage in efficiency units, σ is the elasticity of substitution, $\alpha_s + \beta_s = 1$
If utility function is loglinear, with inelastic labor supply, quantities are given by

$$p_s Q_s = \theta_s \left(w\bar{L} + v\bar{K}\right)$$

$\bar{K}, \bar{L}$: total amounts of capital and labor (in efficiency units)

General Equilibrium: Factor Demands

Conditional factor demands given by cost function derivatives

$$
\begin{aligned}
K_s &= Q_s c_s^{\sigma} \left(\frac{v}{\alpha_s} \right)^{-\sigma} \\
L_s &= Q_s c_s^{\sigma} \left(\frac{w}{\beta_s} \right)^{-\sigma}
\end{aligned}
$$

Factor market clearing:

$$
\sum_s Q_s c_s^{\sigma} \left(\frac{v}{\alpha_s} \right)^{-\sigma} = \bar{K}
$$

Similar equation for labor (redundant by Walras Law).

Equilibrium Factor Price Ratio

The market-clearing equation for capital reduces to

$$\sum_s \theta_s \xi_s = \frac{v\bar{K}}{v\bar{K} + w\bar{L}}$$

ξ_s: capital share for good s:

$$\xi_s = \frac{vK_s}{vK_s + wL_s}$$

Weighted average of the capital shares matches the capital income share
ξ_s may be an increasing or decreasing function of the $\frac{v}{w}$ (depending on σ)
Cobb-Douglas case:

$$\sum_s \theta_s \alpha_s = \frac{v\bar{K}}{v\bar{K} + w\bar{L}}$$

capital share $\xi_s = \alpha_s$ (a technological parameter)

General Equilibrium: Consumer Prices

The price ratio between any two consumer goods is given by

$$\frac{p_s^{1-\sigma}}{p_t^{1-\sigma}} = \frac{\alpha_s^\sigma \left(\frac{v}{w}\right)^{1-\sigma} + \beta_s^\sigma}{\alpha_t^\sigma \left(\frac{v}{w}\right)^{1-\sigma} + \beta_t^\sigma}$$

In the limit, when σ approaches 1,

$$\log\left(\frac{p_s}{p_t}\right) = (\alpha_s - \alpha_t)\log\left(\frac{v}{w}\right)$$

An increase in the relative price of capital
implies an increase in the relative price of capital-intensive goods.

Immigration and Wages

The effective total supply of labor (aggregated over countries) is

$$\bar{L} = \sum_j a_j L_j$$

When workers move to a country with higher productivity,
the effective supply of labor increases, the capital labor ratio falls
If M_{jk} workers migrate from j to k,

$$\Delta \bar{L} = \sum_j \sum_k (a_k - a_j) M_{jk}$$

Cobb-Douglas: elasticity of $\frac{v}{w}$ with respect to the capital labor ratio is unity
The time it takes to earn one unit of good s is $\frac{p_s}{w}$, determined by

$$\log\left(\frac{p_s}{w}\right) = \alpha_s \log\left(\frac{v}{w}\right) - \alpha_s \log(\alpha_s) - \beta_s \log(\beta_s)$$

Immigration and Wages

A relaxation of immigration restrictions leads to a fall in the real wage

The wage effect is the same in all (both sending and receiving) countries

Factor price equalization holds both before and after the migration of labor,

but migration reduces the wage per efficiency unit

(and so reduces the wage of all non-migrants)

Migration reduces prices of labor-intensive relative to capital-intensive goods

but the real wage falls regardless of the composition of consumption

10% increase in $\bar{L}$ implies a 10% increase in the factor price ratio

So if the capital share for good s is $\alpha_s = \frac{1}{4}$,

the real wage falls by about 2.5% when measured in terms of good s.

When measured in terms of more labor-intensive goods, wage falls less

Migration increases the wages of (most) migrants

Long-Run Wage Effects

Migration increases the return on capital

Steady State

$$f'(k^*) = \rho + \delta$$

f': marginal product of capital

ρ: rate of time preference

δ: depreciation rate of capital

k^* :effective capital-labor ratio

Migration increases effective labor

Capital-labor ratio falls below k^*, MPK rises above $\rho + \delta$

Investment increases, effective capital-labor ratio returns to k^*

Real wage returns to original level

Winners and Losers

Stayers lose in the short run (no change in the long run)

Most migrants gain (all migrants gain in the long run)

World Labor Supply

Each person starts with $x \leq 1$ units of effective labor ($x = 1$ in the U.S.)
The proportion of stayers is x
average supply of effective labor after migration

$$x \times x + (1 - x) = 1 - x + x^2$$

The increase in effective labor per person is $1 - x + x^2 - x = (1 - x)^2$.
For Puerto Rico this gives $\frac{1}{9}$

Labor Supply Estimate

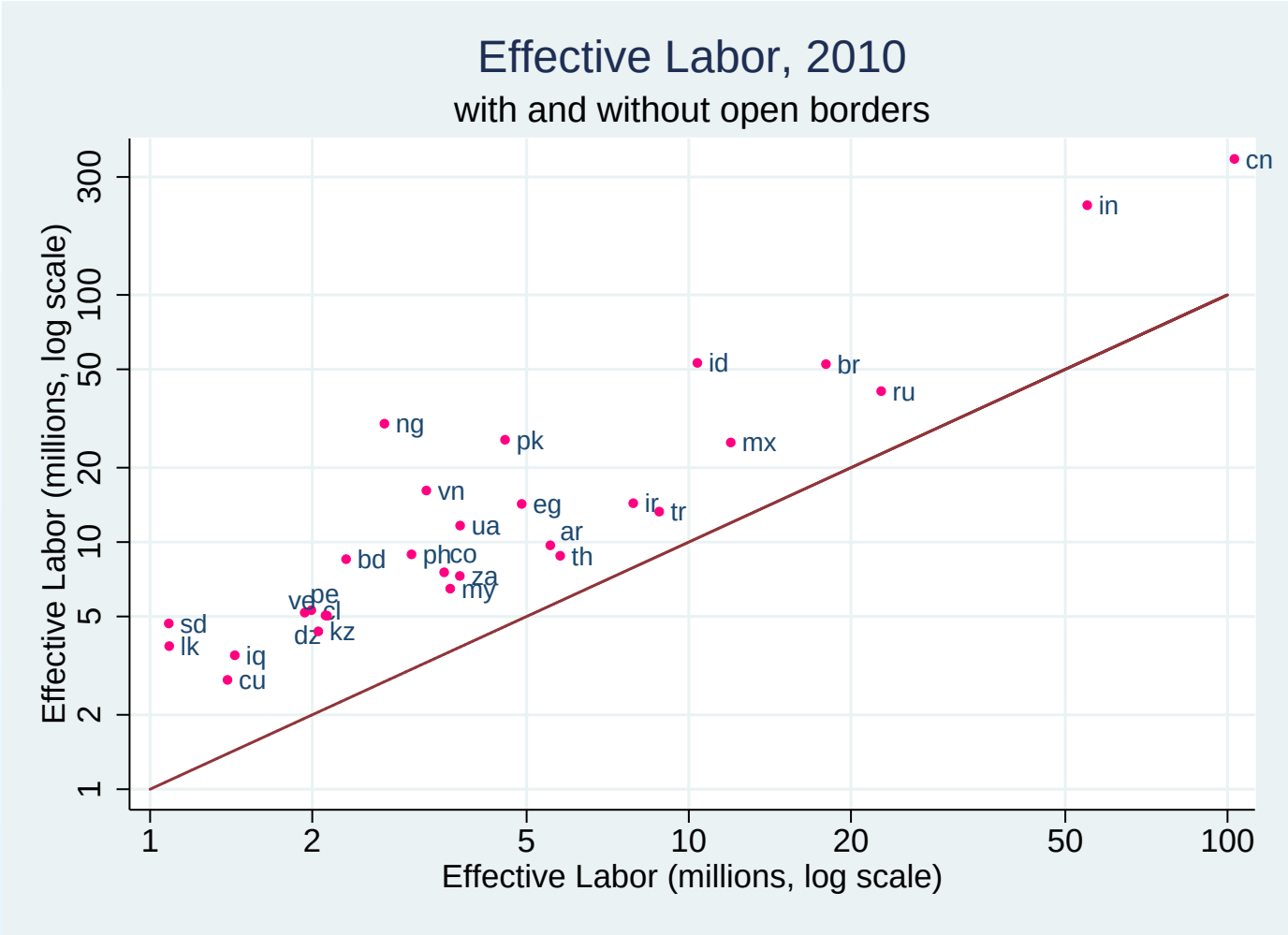
Clemens et al (2008): relative wages in 1999 for 42 developing countries

Effective Labor Supply

$$L(t) = \sum_j n_j(t)\, x_j(t) + \sum_j n_j(t)\,(1 - x_j(t))^2$$

n_j labor force in country j

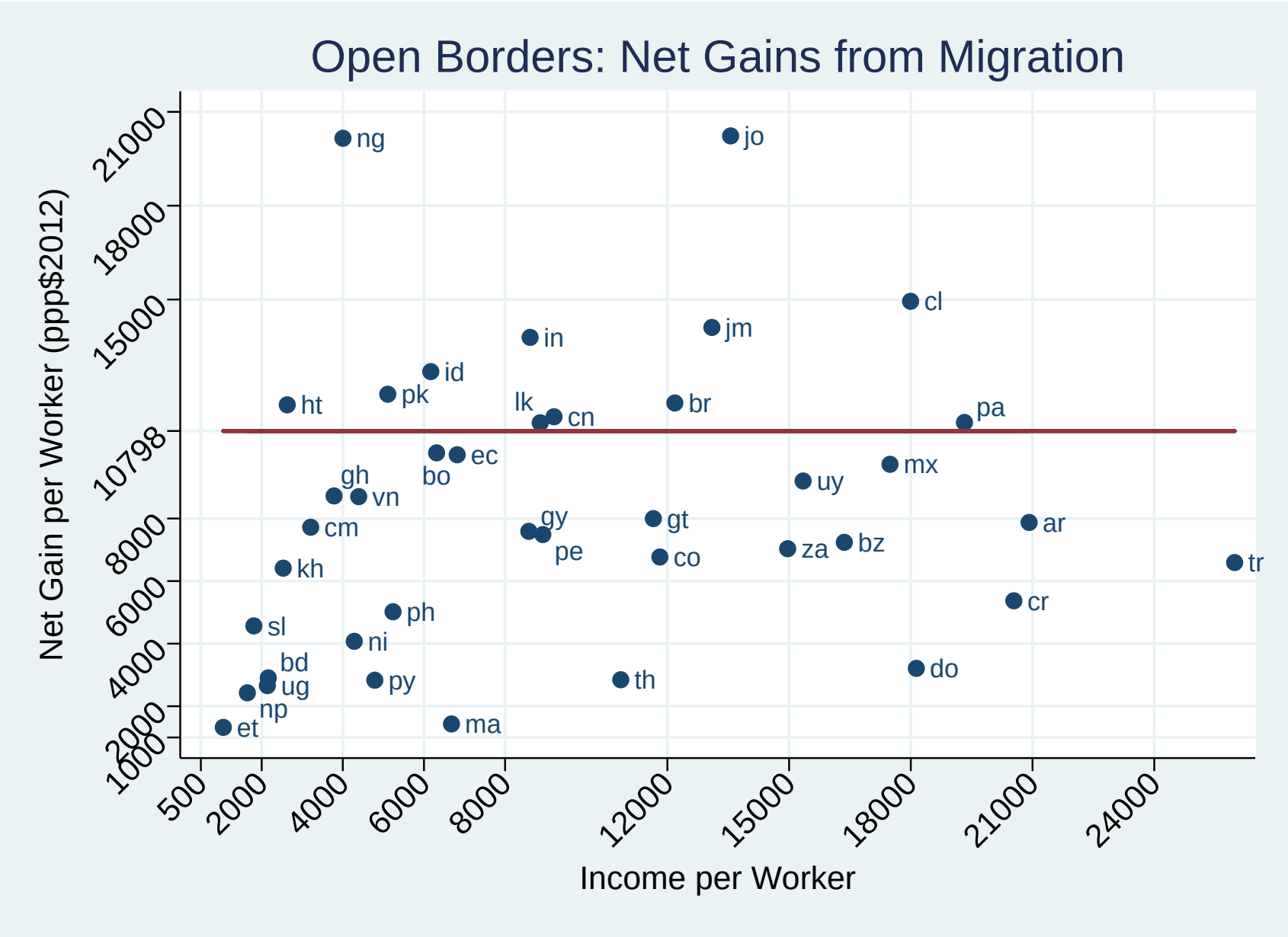
Effective Labor Supply



World effective labor supply increases by **97**%

If capital share is $\frac{1}{3}$, real wage falls by about 20% (short run)

Net Gains from Migration



Net Gains from Migration

Average gain (including stayers): about **$10,798** per worker per year net of moving costs
(for countries with “good” relative wage data)
Average income per person in these countries is $8,633
so the gain is 125% of income.

Average over all countries: $10,135
112%, relative to an average income of $9,079

Heterogeneous Labor: Wage Effects

Two factors A, B enter the production function through the composite X, with

$$Q = F(X, Z)$$

Z is a vector of other factors

Marginal Products

$$\left(\frac{\partial Q}{\partial A}, \frac{\partial Q}{\partial B}\right) = F_X(X, Z)\left(\frac{\partial X}{\partial A}, \frac{\partial X}{\partial B}\right)$$

so the ratio of the marginal products is

$$\frac{\frac{\partial Q}{\partial A}}{\frac{\partial Q}{\partial B}} = \frac{\frac{\partial X}{\partial A}}{\frac{\partial X}{\partial B}}$$

Wage Effects

The composite X is power-linear (CES)

$$X^{\rho} = \gamma A^{\rho} + (1-\gamma) B^{\rho}$$

with $\rho < 1$, where $\sigma = \frac{1}{1-\rho}$ is the elasticity of subsitution
CES Marginal Products

$$X^{\rho-1}\left(\frac{\partial X}{\partial A}, \frac{\partial X}{\partial B}\right) = \left(\gamma A^{\rho-1}, (1-\gamma) B^{\rho-1}\right)$$

$$\frac{\frac{\partial X}{\partial A}}{\frac{\partial X}{\partial B}} = \frac{\gamma}{1-\gamma}\left(\frac{A}{B}\right)^{\rho-1}$$

Competitive factor markets: factors paid their marginal products,
a loglinear relationship between factor price ratios and quantity ratios

$$\frac{w_A}{w_B} = \frac{\gamma}{1-\gamma}\left(\frac{A}{B}\right)^{-\frac{1}{\sigma}}$$

Wage Effects

Card (2009)

> "workers with less than a high school education are perfect substitutes for those with a high school education. ... the impact of low-skilled immigration is diffused across a relatively wide segment of the labor market ... rather than concentrated among the much smaller dropout population ... within broad education classes, immigrant and native workers appear to be imperfect substitutes ... the competitive effects of additional immigrant inflows are concentrated among immigrants themselves, lessening the impacts on natives."

Wage Effects

Are immigrant and native workers perfect substitutes?

Ottaviano and Peri (2012)

Wage and total hours ratios, U.S. Census, 1960-2000, 2006 ACS

Men, less than high school education, 6 years, 8 age groups,

A: immigrants, B natives

$$\begin{aligned} \log\left(\frac{w_A}{w_B}\right) &= \log\left(\frac{\gamma}{1-\gamma}\right) - \frac{1}{\sigma}\log\left(\frac{A}{B}\right) \\ \frac{1}{\sigma} &= .073(.007) \end{aligned}$$

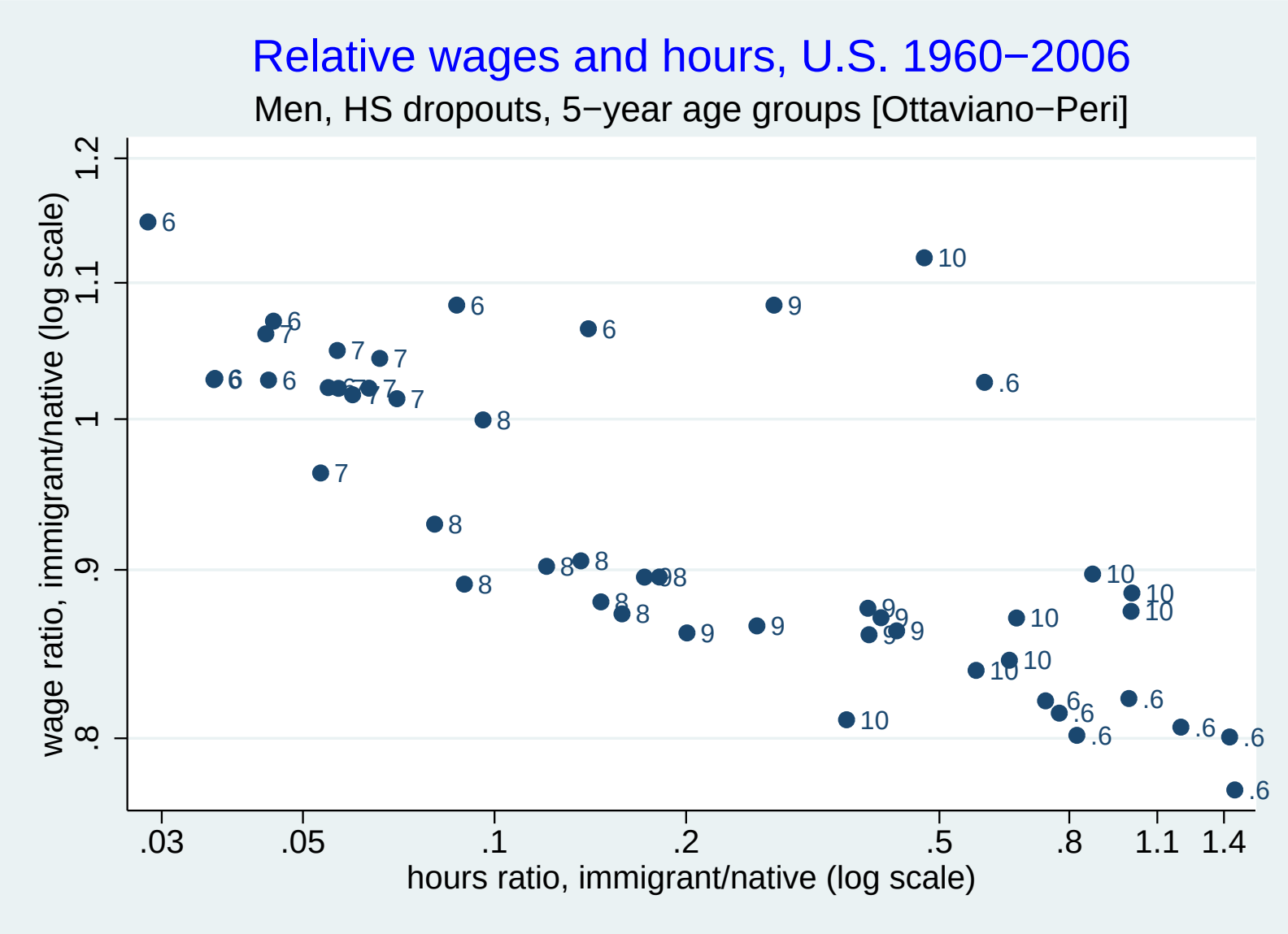
number of immigrant workers changes “exogenously”

if these are not perfect substitutes for natives (within education/age cells),

there will be a change in the relative wages of immigrants and natives.

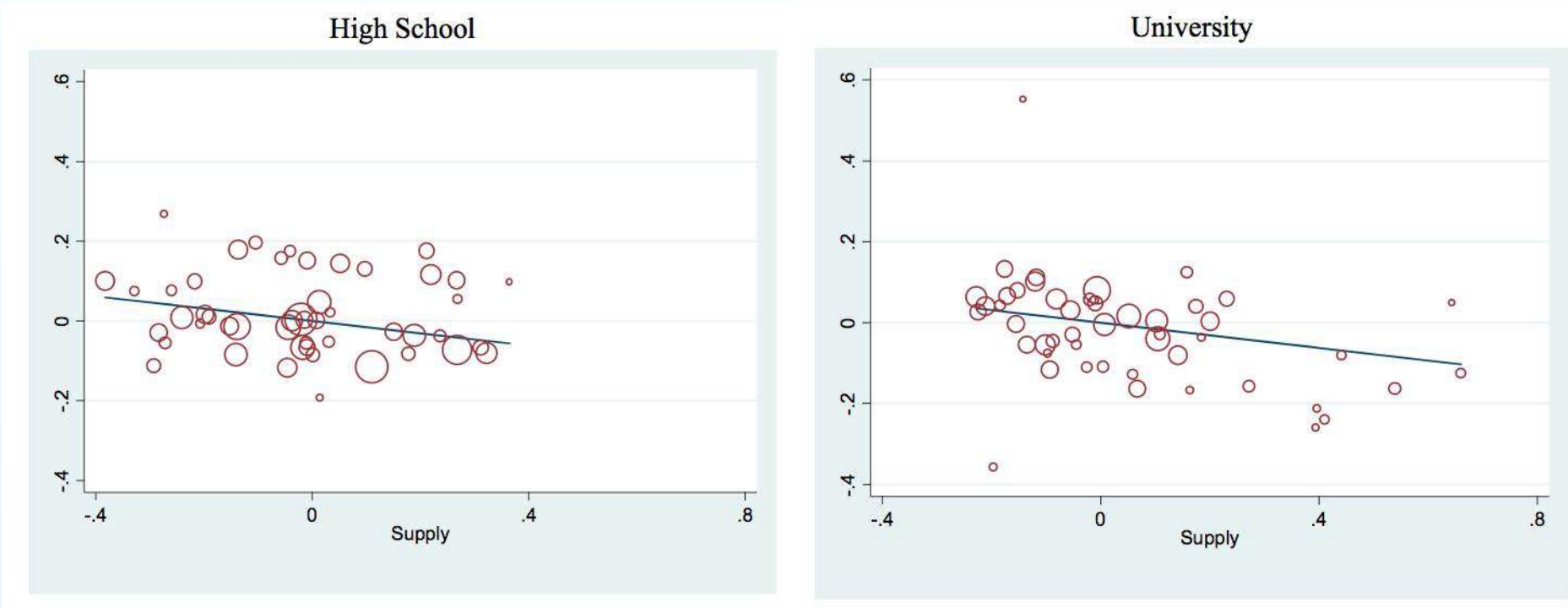
Wage Effects

Are immigrant and native workers perfect substitutes?



Wage Effects

Manacorda, Manning and Wadsworth (2012), UK 1975-2005



(logs, residuals from regressions on time, education and age dummies)

“High School” – left school at age 16-20; “University” – left after age 20

Wage Effects

But what if there is more than one product?
Maybe immigrants and natives are not perfect substitutes in production (because they have different skills).
But the Rybczynski theorem says that an increase in the supply of one factor leads to an increase in the production of goods that use that factor intensively (and a decrease in the production of other products),
with no effect on relative factor prices.
This is in a small open economy that takes product prices as given.
What are the effects of changing the skill mix in a big open economy?

Literature

Daniel Trefler, “International Factor Price Differences: Leontief was Right!”, *JPE* (1993)

Lutz Hendricks, “How Important Is Human Capital for Development? Evidence from Immigrant Earnings”, *AER* (2002)

Donald R. Davis and David E. Weinstein, "The Factor Content of Trade", *Handbook of International Trade* (2004).

Caselli and Feyrer, "The Marginal Product of Capital", *QJE* (2007)

Klein and Ventura, “Productivity differences and the dynamic effects of labor movements”, *JME* (2009)

Gordon H. Hanson, "International Migration and Human Rights", NBER (2010).

Rebecca Lessem, “U.S.-Mexico Immigration: Effects of Wages And Border Enforcement,” jmp (2010)

John Kennan and James R. Walker, “The Effect of Expected Income on Individual Migration Decisions,” *ECMA* (2011)

John Kennan, “Open Borders,” *RED*, April 2013

di Giovanni, Levchenko and Ortega, “A Global View of Cross-Border Migration”, JEEA, forthcoming

Factor Price Equalization: Labor-Augmenting Productivity Differences

J countries, with different productivity levels.
Productivity differences are labor-augmenting (Harrod-neutral)
(equivalent to TFP differences in the *1-product* Cobb-Douglas case)
Production function for product r in country j

$$Q_r^j = F_r\left(K_r^j, a_{j1}S_r^j, a_{j2}U_r^j\right)$$

(a_{js}) efficiency units of labor per worker in country j (same for all products)
No mobility of capital or labor across countries
Cost function for product r in country j

$$c_r^j(v, w) = c_r^0\left(v, \frac{w_j^S}{a_{j1}}, \frac{w_j^U}{a_{j2}}\right)$$

where w is the wage per efficiency unit of labor, and v is the price of capital
c_s^0 is the unit cost function when labor is measured in efficiency units,
$Q_r^j = F_r\left(K_r^j, S_r^j, U_r^j\right)$

Factor Price Equalization with Productivity Differences

Free trade in product markets, no transport costs

Zero-profit condition implies

$$p_r = c_r^0\left(v, \frac{w_j^S}{a_{j1}}, \frac{w_j^U}{a_{j2}}\right)$$

If three products r and s are produced in country j, then

$$\begin{aligned} c_1^0\left(v_j, \frac{w_j^S}{a_{j1}}, \frac{w_j^U}{a_{j2}}\right) &= p_1 \\ c_2^0\left(v_j, \frac{w_j^S}{a_{j1}}, \frac{w_j^U}{a_{j2}}\right) &= p_2 \\ c_3^0\left(v_j, \frac{w_j^S}{a_{j1}}, \frac{w_j^U}{a_{j2}}\right) &= p_3 \end{aligned}$$

These three equations determine the factor prices in country j.

If the marginal rates of technical substitution satisfy a single-crossing condition, the factor prices are uniquely determined

Factor Price Equalization with Productivity Differences

If country ℓ also produces these same three products,
the same equations determine factor prices in country ℓ (with a_ℓ in place of a_j)
This implies $v_j = v_\ell$, and

$$\frac{w_j^S}{a_{j1}} = \frac{w_\ell^S}{a_{\ell 1}}$$

Thus

$$\begin{aligned} w_j^S &= a_{j1} w_0^S \\ w_j^U &= a_{j2} w_0^U \end{aligned}$$

where w_0 is a reference wage level that can be normalized to 1.
In this model, migration has no effect on relative wages.

General Equilibrium

Given factor prices, goods prices are determined by the cost functions

Given goods prices, quantities are determined by preferences and total income (where income depends on factor prices)

Given goods quantities, and factor prices, producers choose factor quantities

Given factor demands, factor prices are determined by market clearing

Technology

Nested CES

Labor is a composite, a power-linear function of skilled and unskilled labor:

$$L^{\kappa} = \gamma \left(g^{S} S\right)^{\kappa} + (1-\gamma) \left(g^{U} U\right)^{\kappa}$$

$\zeta = \frac{1}{1-\kappa} \geq 0$: elasticity of substitution between skilled and unskilled labor

$\gamma \in [0,1]$: skill-intensity (relative importance of skilled and unskilled labor)

Output is a power-linear function of capital and (composite) labor.

$$Y^{\rho} = \alpha \left(g^{K} K\right)^{\rho} + (1-\alpha) \left(g^{L} L\right)^{\rho}$$

$\sigma = \frac{1}{1-\rho} \geq 0$: elasticity of substitution between capital and labor

$\alpha \in [0,1]$: capital-intensity (relative importance of capital and labor)

Technology

Leontief skill mix: $\kappa = -\infty$, $\zeta = 0$

$$L = \min\left(g^S S, g^U U\right)$$

(otherwise $g = 1$, WLOG)

Cobb-Douglas skill mix: $\kappa = 0$, $\zeta = 1$

$$L = AS^{\gamma}U^{1-\gamma}\ , A = \left(g^S\right)^{\gamma}\left(g^U\right)^{1-\gamma}$$

Technology

It is assumed that the elasticities of substitution are the same for all products, but the factor intensities may differ

No loglinear relationship between factor price and (aggregate) quantity ratios.

Prices

The price of good r is given by

$$p_r^{1-\sigma} = \alpha_r \left(\frac{v}{\alpha_r}\right)^{1-\sigma} + (1-\alpha_r)\left(\frac{W_r}{1-\alpha_r}\right)^{1-\sigma}$$

W_r: price of the labor composite in efficiency units determined by the cost function for labor:

$$W_r^{1-\zeta} = \gamma_r \left(\frac{w^S}{\gamma_r}\right)^{1-\zeta} + (1-\gamma_r)\left(\frac{w^U}{1-\gamma_r}\right)^{1-\zeta}$$

Preferences

Utility function is loglinear, with inelastic labor supply
quantities to be produced determined by the expenditure shares θ_r applied to total income

$$p_r Q_r = \theta_r \left(w^S S_0 + w^U U_0 + v K_0 \right)$$

K_0, S_0, U_0: total supplies of capital and labor (efficiency units)

General Equilibrium

Income ratios

$$(x_1, x_2) \quad = \quad \left(\frac{w^S S_0}{v K_0}, \frac{w^U U_0}{v K_0} \right)$$

Labor share for each product

$$\frac{1}{\lambda_r} = 1 + \left(\frac{\alpha_r}{1-\alpha_r} \right)^{\sigma} \left(\gamma_r^{\zeta} \left(\frac{w^S}{v} \right)^{1-\zeta} + (1-\gamma_r)^{\zeta} \left(\frac{w^U}{v} \right)^{1-\zeta} \right)^{\frac{\sigma-1}{1-\zeta}}$$

Share of skilled labor in total labor income (for each product)

$$\frac{1}{\eta_r} = 1 + \left(\frac{1-\gamma_r}{\gamma_r} \right)^{\zeta} \left(\frac{w^S}{w^U} \right)^{\zeta-1}$$

General Equilibrium

Market-clearing equations

$$\begin{aligned}
\sum_r \theta_r \lambda_r \eta_r &= \frac{w^S S_0}{w^S S_0 + w^U U_0 + v K_0} \\
\sum_r \theta_r \lambda_r (1 - \eta_r) &= \frac{w^U U_0}{w^S S_0 + w^U U_0 + v K_0} \\
\sum_r \theta_r (1 - \lambda_r) &= \frac{v K_0}{w^S S_0 + w^U U_0 + v K_0}
\end{aligned}$$

Shares for each product, averaged over products

Compare with values of aggregate factor endowments

General Equilibrium

Two equations

$$\begin{aligned} A_S(x)(1+x_1+x_2) &= x_1 \\ A_U(x)(1+x_1+x_2) &= x_2 \end{aligned}$$

$$x = (x_1, x_2) = \left(\frac{w^S S_0}{vK_0}, \frac{w^U U_0}{vK_0}\right)$$

Aggregate factor shares

$$\begin{aligned} A_S(x) &= \sum_r \theta_r \lambda_r(x) \eta_r(x) \\ A_U(x) &= \sum_r \theta_r \lambda_r(x)(1-\eta_r(x)) \end{aligned}$$

General Equilibrium

Uniqueness

There is a unique equilibrium

Solve two nonlinear equations, two unknowns

This is hard

Proof only for special cases ($\sigma = 1$ or $\zeta = 1$)

But proof using elementary economic arguments is easy [why?]

Uniqueness

1. Any solution of the equations gives a competitive equilibrium.
2. Every competitive equilibrium is Pareto optimal.
3. A Pareto optimum maximizes the utility of an aggregate consumer
   (a) identical homothetic preferences – everyone on the same ray
4. All Pareto optima must have the same total outputs
   (a) strictly convex preferences, convex production set
5. The production function for each good is strictly quasiconcave.
6. All optimal production plans must use the same input vectors.

General Equilibrium: Cobb-Douglas Final Goods

Start with market-clearing equations for the two labor types
Substitute one equation in the other to get a single equation for relative wage

Two cases

1. If labor substitution elasticity is high, equilibrium is at the intersection of an upward-sloping and a downward sloping curve
2. If elasticity is low, equilibrium relative wage is the root of a single-crossing function
    - (slope of this function is negative at any root, so there is only one root)

General Equilibrium: Cobb-Douglas Final Goods

- If skilled and unskilled workers are good substitutes $(\zeta > 1)$,
  - when the (effective) supply of unskilled labor (U_0) increases
    - both wages fall, relative to the price of capital
    - and the skill premium rises
  - an increase in S_0 implies that both relative wages fall, and the skill premium falls.
- If skilled and unskilled workers are not good substitutes $(\zeta < 1)$,
  - an increase in U_0 implies that $\frac{w^S}{v}$ rises, and $\frac{w^U}{v}$ falls, and the skill premium rises
  - an increase in S_0 implies that $\frac{w^U}{v}$ rises, and $\frac{w^S}{v}$ falls, and the skill premium falls

Immigration and Wages

The effective total supply of labor (aggregated over countries) is

$$\begin{aligned} S_0 &= \sum_j a_{j1} S_j \\ U_0 &= \sum_j a_{j2} U_j \end{aligned}$$

When workers move to a country with higher productivity, effective supply of labor increases, capital labor ratio falls
If M_{jk} workers migrate from j to k,

$$\begin{aligned} \Delta S_0 &= \sum_j \sum_k (a_{k1} - a_{j1}) M_{jk}^S \\ \Delta U_0 &= \sum_j \sum_k (a_{k2} - a_{j2}) M_{jk}^U \end{aligned}$$

General Equilibrium: Consumer Prices

The price ratio between any two consumer goods is given by

$$\frac{p_r^{1-\sigma}}{p_t^{1-\sigma}} = \left(\frac{W_r}{W_t}\right)^{1-\sigma} \frac{\alpha_r^\sigma \left(\frac{v}{W_r}\right)^{1-\sigma} + (1-\alpha_r)^\sigma}{\alpha_t^\sigma \left(\frac{v}{W_t}\right)^{1-\sigma} + (1-\alpha_t)^\sigma}$$

where W_r is the price of the labor composite

$$W_r^{1-\zeta} = \gamma_r \left(\frac{w^S}{\gamma_r}\right)^{1-\zeta} + (1-\gamma_r)\left(\frac{w^U}{1-\gamma_r}\right)^{1-\zeta}$$

An increase in the relative price of capital
implies an increase in the relative price of capital-intensive goods.

Immigration and Real Wages

Cobb-Douglas Preferences and Technology

$$\begin{aligned} U(q) &= \sum_r \theta_r \log(q_r) \\ \log(q_r) &= \sum_{i=1}^{n} \alpha_{ir} \log(x_i) \end{aligned}$$

Product Prices (ignoring constants)

$$\log(p_r) = \sum\nolimits_i \alpha_{ir} \log(w_i)$$

Real Wages

$$\begin{aligned} \log(y^*) &= \log y - \sum\nolimits_i \alpha_{i0} \log(w_i) \\ \log(y_k^*) &= \sum\nolimits_i \alpha_{i0} \log(X_i) - \log(X_k) \end{aligned}$$

$\alpha_{i0} = \sum_r \theta_r \alpha_{ir}$

Immigration and Real Wages

If the unskilled labor endowment doubles, the ratio $\frac{w^U}{v}$ is cut in half,

no change in $\frac{w^S}{v}$

If $\sum_r \theta_r (1-\alpha_r)(1-\gamma_r) = \frac{1}{3}$

e.g labor share is $\frac{2}{3}$ $\left(\alpha_r = \frac{1}{3}\right)$,

and the share of skilled labor in the labor composite is $\gamma_r = \frac{1}{2}$,

then the real wage of skilled workers rises by about 25%

and the real wage of unskilled workers falls by about 40%

Simple Migration Model

Proportion of people who move determined by the relative wage
– the ratio of income at home (y_{js}) to the highest income elsewhere (y_{0s})
Utility is loglinear, so indirect utility is $\log(y)$. Stay if

$$\log(y_{0s}) - \delta_s \leq \log(y_{js})$$

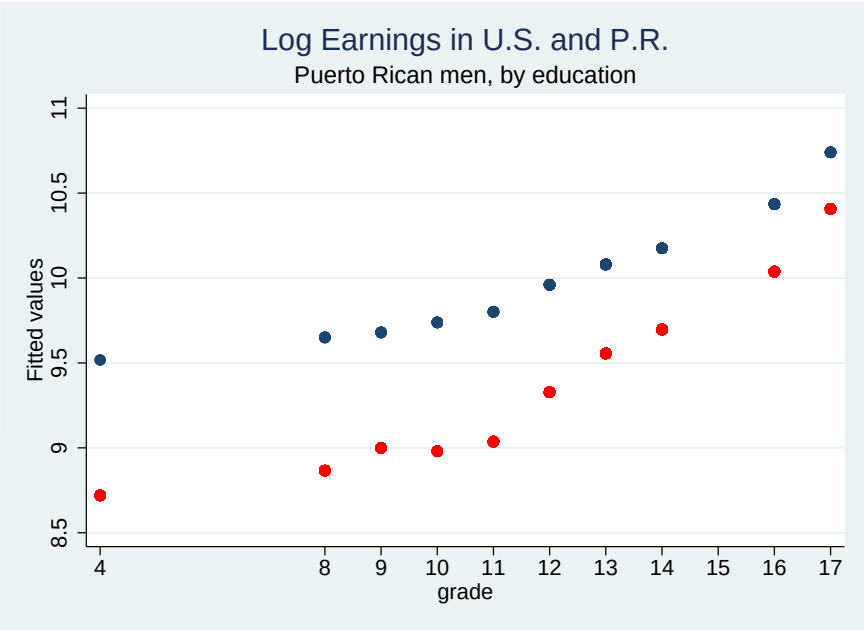
δ_s: disutility of moving (attachment to home), randomly distributed over people
Assume the distribution of δ is exponential: $F_s(t) = 1 - e^{-\varsigma_s t}$
Then the probability of staying is

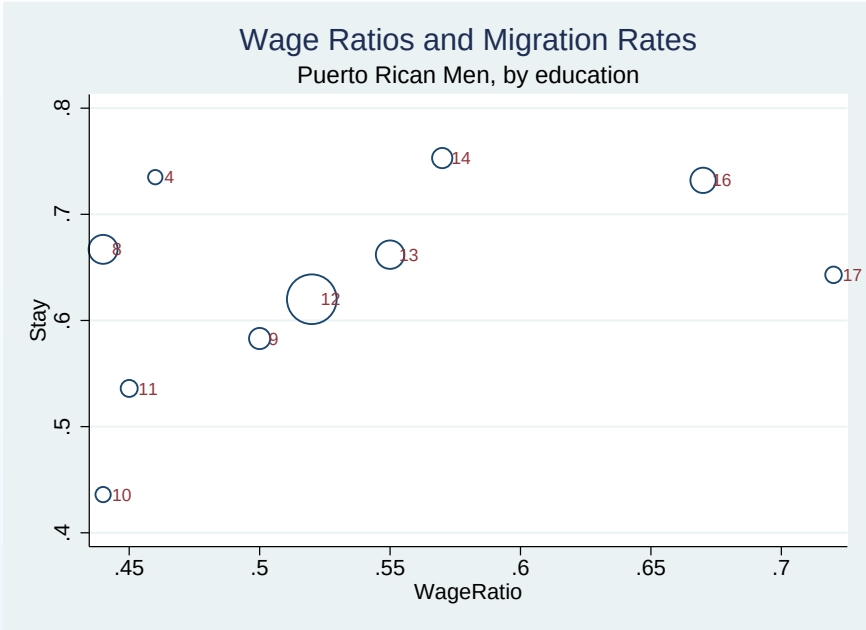
$$\text{Prob}\left(\delta \geq \log\left(\frac{y_{0s}}{y_{js}}\right)\right) = e^{-\varsigma_s \log\left(\frac{y_{0s}}{y_{js}}\right)} = (a_j)^{\varsigma_s}$$

So if the proportion who stay is $\mathcal{S}_{js}$ then

$$\log(\mathcal{S}_{js}) = \varsigma_s \log(a_{js})$$

Skills and Migration Rates: Puerto Rico





Skills and Migration Rates: Puerto Rico

Schooling	0-9	9-11	12	13-15	16	17
Wage Ratio	0.46	0.49	0.53	0.60	0.67	0.72
Stay	0.68	0.53	0.62	0.69	0.73	0.64
ζ	0.49	0.88	0.75	0.72	0.78	1.34
N	218,715	203,138	515,421	254,483	134,023	56,929

Wage (efficiency) ratios vary a lot across education levels

World Labor Supply

Effective labor after migration

$$\left(a_j^{\zeta} \times a_j + \left(1 - a_j^{\zeta}\right)\right) \times y_{0s}$$

Increase in effective labor per person

$$\left(1 - a_j^{\zeta}\right)(1 - a_j)\frac{y_{js}}{a_j}$$

Aggregate increase in effective labor due to migration is

$$\Delta L_0 = \sum_{j=1}^{J}\left(1 - a_j^{\zeta}\right)(1 - a_j)\frac{y_{js}}{a_j}N_{js}$$

N_{js} is the supply of labor at skill level s in country j.

Effective Labor Supply

Data

Barro and Lee (2010): schooling levels (age 20-64; 146 countries)

Clemens, Montenegro and Pritchett (2008): relative wages at three schooling levels (42 countries)

Penn World Table (7.1): real GDP per worker (189 countries)

Bernanke and Gurkaynak (2002) and Gollin (2002) labor shares (63 countries)

Effective Labor Supply

Results

Increase in World Labor Supply			
Schooling Years	**0-8**	**9-12**	**13-16**
Percentage Increase in Effective Labor	149%	101%	42%
Migration from Non-Frontier Countries (millions)	689	870	203
Population in Frontier Countries	113	373	257
Population in Non-Frontier Countries	1,305	1,311	333

- a big increase in labor supply
- a big decrease in the ratio of skilled to unskilled workers
- huge population movements
- but movement is slow
  - when Poland joined the EU, annual migration peaked at 47,000 in 2006
    - about 38,000,000 stayed in Poland

Immigration and Real Wage Changes

Marginal Products, Cobb-Douglas production functions

$$MPL_r = (1-\alpha_r)\frac{Q_r}{L_r}$$

Aggregation with Cobb-Douglas preferences, n factors

$$\log Q = \sum_{i=1}^{n} \alpha_{i0} \log(X_i)$$

X_i: endowment of factor i

$$\alpha_{i0} = \sum_{r=1}^{J} \theta_r \alpha_{ir}$$

Real Wage Changes

$$\frac{w'}{w} = \frac{APL'}{APL}$$

Immigration and Real Wage Changes

Skill Shares

	lo	med	hi
Schooling Years	**0-8**	**9-12**	**13-16**
Effective Labor Supplies	4104	12401	12376
Wages (U.S. Census)	11311	18983	35761
Shares	6.4%	32.5%	61.1%
α_{i0} (capital share $\frac{1}{3}$)	4.3%	21.7%	40.7%

Results

Real Wage Changes			
Schooling Years	**0-8**	**9-12**	**13-16**
Percentage Increase in Effective Labor	149%	101%	42%
Real Wage Change	-44.0%	-30.5%	-1.8%
Population in Frontier Countries	113	373	257
Population in Non-Frontier Countries	1,305	1,311	333

Long-Run Wage Effects

Migration increases the return on capital

Steady State

$$f'(k^*) = \rho + \delta$$

f': marginal product of capital

ρ: rate of time preference

δ: depreciation rate of capital

k^* :effective capital-labor ratio

Migration increases effective labor

Capital-labor ratio falls below k^*, MPK rises above $\rho + \delta$

Investment increases, effective capital-labor ratio returns to k^*

Real wage returns to original level

Questions

General Equilibrium calculations with skill differences are complicated

- effects of differential migration rates depend on elasticities

Cobb-Douglas Benchmark

- big negative real wage effects at lower skill levels

Big incentives to invest in capital

- Effects on skill premia?
- Big incentives to invest in human capital

More General Questions

- What happens with more general substitution elasticities?
- Allow for alternative CES nesting structures
- Aggregation with CES preferences

Opposition to Immigration

Wages in developed countries might fall a lot

- this is not consistent with either theory or data

Immigrants might impose a large financial burden on social welfare systems

- fairly simple tax and timing adjustments could take care of this

Immigrants would dilute cultural identities in receiving countries

- (what would economists know about this?)

References

Card, D. (2009, September). Immigration and inequality. *American Economic Review 99*(2), 1–21.

Ottaviano, G. I. P. and G. Peri (2012). Rethinking the effect of immigration on wages. *Journal of the European Economic Association 10*(1), 152–197.