Microeconomics Problems

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1. Suppose the market supply and demand curves for wheat are as follows (prices are in dollars, quantities in millions of bushels):

$$\begin{aligned} P^S &= .02Q^S \\ P^D &= 3 - .01Q^D \end{aligned}$$

The government is considering two possible price support policies, (a) or (b).

   (a) The government buys enough wheat so that a market price of \$2.20 is maintained. Wheat bought by the government is stored, destroyed, or given away abroad.

      i. How much wheat does the government buy, how much is domestically consumed, and what is the cost to the government of this policy?

   (b) The government subsidizes wheat by \$x per bushel and buys no wheat itself. Calculate the subsidy needed if farmers are to receive \$2.20 per bushel in the new equilibrium.

      i. Under this policy how much wheat will consumers buy? How much will the government have to pay out?

2. The phone company charges \$20 a month for basic service, which includes 100 free calls per month. After the first 100 calls, each call costs 10 cents. Fred Bloggs makes 200 calls per month in this situation.

   (a) Now a new phone company offers a different payment plan, charging 15 cents for each call, with no monthly service charge, and no free calls.

      i. Will Fred change his phone company? If he decides to change, or if he is forced to change, will he make more phone calls or less?

   (b) The new phone company now raises its price to 17 cents per call. Is it obvious that Fred will choose the old phone company in this situation?

3. A consumer's preference ordering on $\mathbb{R}^2_+$ is represented by the utility function

$$u(x) = \frac{1}{2}x_1^2 + \ln(x_2)$$

   (a) Suppose the prices are $p_1 = 2$, $p_2 = 1$. If income is $I = 4$, what is the optimal consumption plan?

   (b) Suppose that income increases to 5. Now what is the optimal consumption plan?

   (c) Can you find a price vector and an income level such that $x_1(p, I) = 1$?

4. A consumer makes choices in three different (perhaps hypothetical) situations, labeled A, B, C. There are three goods, and the consumer's income in each situation is \$800. The price vectors and the corresponding choices are

$$\begin{aligned} p^A &= (2,1,2), & x^A &= (100,200,200) \\ p^B &= (2,2,1), & x^B &= (200,100,200) \\ p^C &= (1,2,2), & x^C &= (200,200,100) \end{aligned}$$

Would a rational consumer make these choices?

5. **The Likelihood Ratio Test** (Neyman-Pearson Lemma)
A statistician expects to obtain a vector of data generated by one of two probability distributions, p or q. The vector will lie in a finite set

$$X = \left\{x^1, x^2, ..., x^i ..., x^S\right\}$$

called the sample space, and the probabilities (or likelihoods) associated with the points in this set are either $p_1, p_2, ..., p_i, ..., p_S$ or $q_1, q_2, ..., q_i, ..., q_S$, according to whether p or q is the true distribution. The statistician wishes to design a test of the hypothesis H_0 that the true distribution is p. The test involves dividing X into two parts, R and A, rejecting H_0 if the observed data lie in R and accepting H_0 if the data lie in A. Two types of error are possible here: the test might reject H_0 when it is actually true, or it might accept H_0 when it's false. The statistician is primarily concerned about errors of the first type. Provided that these can be held to an acceptable level it is also desirable to avoid errors of the second type. Specifically, the aim is to maximize the probability that H_0 will be rejected when it's false (the statistician calls this the *power* of the test), subject to the constraint that the probability of type I error should not exceed some number α (called the *size* of the test).

   (a) Translate this problem into the language of consumer theory. (What is the utility function? What is the budget constraint?).
   (b) Solve the problem.
   (c) Translate the solution back into language that the statistician can understand. (Note the title of the problem).

6. (Jevons, 1871) Your ship is overdue in port and the beer is running out. The remaining supplies are divided up and you get 22.5 fluid ounces. The ship will not reach port before tomorrow morning, and there is a 60% chance that it will arrive then. You can't take beer with you when you leave the ship, so you could drink it all today, to make sure it isn't wasted. On the other hand, there is a 40% chance that you will still be afloat all day tomorrow, and a 10% chance that you will be afloat the day after that. You could save some beer in case you need it for the second day, or the third. It is certain that you will reach port before the fourth day.

   (a) You are an expected utility maximizer, and your utility function is $u(B) = 6000B - 250B^2$, where B is daily beer consumption. How much beer should you drink today?

7. State whether the following assertions are true, false or ambiguous, and explain why.

   (a) A risk averse person buys full insurance if the price is actuarially fair, and buys less than full insurance if the price is not actuarially fair.
   (b) Constant relative risk aversion implies that the demand for insurance is a decreasing function of wealth.

8. An individual has w eggs and two baskets. One basket just keeps the eggs safe. The eggs in the other basket might increase and multiply, or they might rot. Each egg in this basket yields Z eggs (the increase being $Z-1$), where Z is a random variable that is uniformly distributed on the interval $\left[\frac{2}{3}, \frac{5}{3}\right]$. The individual maximizes expected utility, and the utility function is $u(c) = \log(a+c)$, where c is the number of eggs consumed, and $a \geq 0$.

   Will this person put all of the eggs in one basket?

   (a) **Solution**

$$\max_x E \log(a + xwZ + (1-x)w)$$

expected marginal utility is $E\frac{1}{a+c}$. Given the uniform distribution for Z, the objective is

$$\max_x \int_{z_0}^{z_1} \log(a + w + xw(z-1))\, dz$$

The first-order condition is

$$w\int_{z_0}^{z_1}\frac{z-1}{a+w+xw\,(z-1)}dz=0$$

If all of the eggs are in the safe basket ($x=0$), utility is $\log(a+w)$. This can't be optimal, because the expected value of Z is greater than 1.

$$w\int_{z_0}^{z_1}\frac{z-1}{a+w}dz>0$$

If all of the eggs are in the other basket, expected utility is $\int_{z_0}^{z_1}\log(a+wz)\,dz$. For this to be optimal, the first-order condition requires

$$w\int_{z_0}^{z_1}\frac{z-1}{a+wz}dz>0$$

If $a=0$ then w is irrelevant, because it just multiplies the objective function. So the problem is

$$\max_x\int_{z_0}^{z_1}\log(1+x(z-1))\,dz$$

and the derivative with respect to x is

$$\begin{aligned}\int_{z_0}^{z_1}\frac{z-1}{1+x(z-1)}dz &= \frac{1}{x}\int_{z_0}^{z_0+1}\left(1-\frac{1}{1+x(z-1)}\right)dz\\ &= \frac{1}{x}-\frac{1}{x^2}\left[\log(1+x(z-1))\right]_{z=z_0}^{z=z_1}\\ &= \frac{1}{x}-\frac{1}{x^2}\log\left(\frac{1+xz_0}{1+xz_0-x}\right)\\ &= \frac{1}{x}-\frac{1}{x^2}\log\left(1+\frac{x}{1+xz_0-x}\right)\end{aligned}$$

differentiating this again shows that the slope is increasing as long as z_0 is beyond the positive root of the equation

$$z^2+z=1$$

(the root is at .618)

9. Suppose a union and an employer start to negotiate on January 1, 1994 over wages to be paid for the year 1994. There will be 50 paid weeks in the year (the rest being unpaid vacation time). No work will be done until they reach an agreement.

   The employer's net revenue, after paying all costs other than wages (including a normal return on capital), is \$500 per worker per week. The workers can earn \$240 per week if they leave this employer and go to work elsewhere.

   While negotiations continue workers can collect \$130 per week in unemployment benefits (this is a straight subsidy that does not have to be repaid). The employer has retained an expert negotiator who charges \$200 per week per worker.

   The negotiations proceed as follows. At the beginning of each week the employer's and the union's negotiators meet, and one side proposes a wage for the rest of the year. If this proposal is accepted work begins immediately. If not, the workers collect unemployment benefits for the week, the employer's negotiator is paid for the week, and nothing happens for the rest of the week; next week there is a new meeting and a new proposal.

   At the first meeting a coin is tossed to determine which side makes the proposal for that week. In subsequent meetings they take turns: first one side makes a proposal, then the following week the other side makes a proposal, and so on.

   How long will these negotiations take? What wage agreement will be reached?

10. Consider the following argument:

    If half of the houses in California were destroyed by an earthquake, the price of a house would rise so high that the remaining half would be more valuable in economic terms than was the original total. Thus an economist would advise us not to wait for the earthquake, but to destroy half the houses on purpose, which shows that economists give absurd advice."

    (a) If half of the houses were destroyed would the value of the remainder actually exceed the value of the original total? Explain your answer.
    (b) Would a good economist advise that half of the houses be destroyed on purpose?
    (c) If you personally owned all of the houses, would you destroy half of them?

11. Suppose that the long-run total cost curve of a typical firm in a competitive industry is given by the formula:
$$C = 1200Q - 60Q^2 + Q^3$$
    The industry demand curve is
$$P = 375 - .025Q$$
    (a) Find the long-run equilibrium price. How many firms will there be, and how much will each firm produce?
    (b) The government wishes to raise \$45,000 by taxing this industry. Two different kinds of taxes are under consideration:
        i. A sales tax of \$15 per unit;
        ii. A lump-sum tax of \$450 per firm, regardless of how much the firm produces
    (c) Will either of these taxes generate \$45,000 in revenue for the government? Will the new long-run equilibrium price be the same for both tax schemes? If not, will the price increase be greater under the sales tax or the lump-sum tax? Draw a diagram summarizing your answer.

12. Suppose a consumer has the following utility function
$$U(x_1, x_2) = \alpha x_1 + v(x_2)$$
    where α is a positive parameter, and the function v is concave.

    (a) Is it true that x_1 absorbs all income effects: the income elasticity of demand for each of the other goods is zero? Explain.
    (b) Is it true that the cross price elasticities of demand are zero? Explain.
    (c) Under what conditions would $x_1 = 0$ be optimal?

13. Suppose that the long-run total cost curve of a typical firm in a competitive industry is given by the formula:
$$C = \frac{80}{3} + 4q + \frac{q^2}{5} + \frac{q^3}{300}$$
    The industry demand curve is
$$P = 40 - \frac{Q}{200}$$
    (a) Find the long-run equilibrium price. How many firms will there be, and how much will each firm produce?

14. Three people go to dinner at a restaurant. They have different preferences, over food, f, and other stuff, y, represented by the utility functions
$$\begin{aligned} u^1(f,y) &= fy \\ u^2(f,y) &= \log(f) + \log(y) \\ u^3(f,y) &= 2\log(f) + \log(y) \end{aligned}$$
    They all have the same amount of money to spend, I. The price of food is p (relative to other stuff).

(a) Suppose each person orders independently, and each person pays an equal share of the total cost of the meal. Find a Nash equilibrium of this game (where each person's strategy is the cost of the food that this person orders).

(b) Compare the Nash equilibrium outcome with the outcome when each person pays separately for their own meal.

(c) **Solution**
When each person pays separately

$$
\begin{aligned}
F_1 &= \frac{I}{2} \\
F_2 &= \frac{I}{2} \\
F_3 &= \frac{2I}{3}
\end{aligned}
$$

For 1 and 2, expenditure on food and other stuff is equal; for 3 expenditure on food is twice the expenditure on other stuff.
When each person pays an equal share of the bill, the effective food price from the point of view of each person is one-third of the actual price. Equal expenditure then means the amount of food ordered is three times the amount spent on other stuff; this applies to the first two people, while the third person orders six times the amount spent on other stuff.
Since they all have the same income, and pay the same amount for food, y is the same for all three. Then $F_1 = F_2 = 3y$, and $F_3 = 6y$. Also total spending is

$$F_1 + F_2 + F_3 + 3y = 3I$$

This implies $y = \frac{I}{5}$ so

$$
\begin{aligned}
F_1 &= \frac{3I}{5} \\
F_2 &= \frac{3I}{5} \\
F_3 &= \frac{6I}{5}
\end{aligned}
$$

15. Consider an isolated economy which produces just one crop, corn, using a fixed quantity of land and variable quantities of labor and tractors. Labor is also used to produce the tractors.

(a) Which of the following conditions are needed for Pareto optimality:

i. The marginal rate of technical substitution of tractors for labor in the production of corn is equal to the marginal product of labor in the production of tractors.
ii. The marginal utility of leisure is equal to the marginal product of labor in corn multiplied by the marginal utility of corn, for each individual.
iii. The marginal rate of substitution of leisure for corn is equal for all consumers.

(b) Which of the above conditions would be satisfied in a competitive market economy? How?

(c) **Solution**
(iii) is obviously needed, and it is achieved by having all consumers respond to the same relative price; (i) is also needed, because it involves two different ways to exchange tractors for labor (or leisure); this equality is achieved bcause the corn producers set the $MRTS$ equal to the relative price of tractors and labor, while tractor producers equate the marginal product of labor to the same relative price; (ii) is true because the MRS between corn and leisure has to be equal to the MPL in corn; both are equated to the real wage

16. Thirty-six people live and work on the island of Beesare. There are 17 fishermen, 6 dairy farmers and 13 bakery workers. The bakery is a monopoly which is owned by the fishermen: there are 170 shares of stock outstanding, and each fisherman owns 10 shares. The only economic activities on the island are the production and consumption of fish, bread and milk.

    (a) The Beesare unit of currency is the clam, and 520 clams are in circulation.
    (b) The bakery workers will supply a full day's work for 20 clams. If the wage is less than 20 clams they will not work at all.
    (c) When the bakery employs 13 workers it produces 70 pounds of bread per day, and uses 6 gallons of milk in the production process.
    (d) The bakery is regulated by the government. It is required to produce and sell 70 pounds of bread every day. Otherwise it is free to maximize profit.
    (e) Bakery workers spend 50% of their income on bread, 25% on fish and 25% on milk. Fishermen spend half of their income on bread and the other half on milk. Dairy farmers spend half on bread and half on fish.
    (f) Each fisherman catches and sells 5 pounds of fish per day. Each farmer produces and sells 7 gallons of milk per day.
    (g) Daily transactions occur as follows.
        i. Every morning the milk market opens at 8 a.m. and closes at 9 a.m. Each farmer sells 7 gallons of milk at the market price to the various buyers, and receives clams in exchange.
        ii. The bread market is open from 10 a.m. to 11 a.m. The bakery sells 70 pounds of bread at the market price, and receives clams in exchange.
        iii. The fish market is open from 4 p.m. to 5 p.m. Each fisherman sells 5 pounds of fish at the market price, and receives clams in exchange.
        iv. At 6 p.m. each bakery worker is paid 20 clams, and the bakery's profits for the day are distributed evenly to the shareholders.
        v. Next day, everything is repeated. Each day is an exact copy of the day before.
    (h) Clams are held solely for transactions purposes. There is no saving, and no speculation.

    Questions

    i. Analyze the demand for money. How many clams does each fisherman hold overnight? Give a complete account of money holdings, showing where all 520 clams are held overnight, and how they change hands during the day.
    ii. Find the equilibrium prices of fish, bread and milk.
    iii. **Solution**
    Clams are held by bakery workers (260) and fishers (Y_h), and the bakery holds enough to buy milk in the morning, so

$$260 + Y_h + 6p_m = 520$$

Total milk supply is 42; the bakery buys 6, leaving 36 for bakery workers and fishers so

$$\frac{1}{4}260\frac{1}{p_m} + \frac{1}{2}\frac{Y_h}{p_m} = 36$$

which implies

$$130 + Y_h = 72p_m$$

Then (subtracting to eliminate Y_h)

$$130 - 6p_m = 72p_m - 260$$

which implies $78p_m = 390$ so $p_m = 5$, and then $Y_h = 230$. This determines overnight holdings. Farmers spend half their income on fish, and bakery workers spend $\frac{260}{4} = 65$, so 85 pounds

of fish are sold for $65 + 105 = 170$, so $p_f = 2$.
Each farmer's income is $7p_m = 35$, and there are 6 farmers, so total income of farmers is $Y_f = 210$, and they spend half of this on bread. Total incomes are $Y_w = 260, Y_h = 230, Y_f = 210$. Total bread production is 70 pounds, and total revenue is $130 + 115 + 105 = 350$, so the price of bread is $p_b = 5$, and the bakery profit is $350 - 30 - 260 = 60$.

17. An inventor has discovered a new method of producing a precious stone, using spring water found only in Venice and Tipton. The process is patented and manufacturing plants are set up in both places. The product is sold only in Europe and America. Trade laws are such that the price must be uniform within Europe and America, but the European and American prices may differ. Transport costs are negligible, and there is no second-hand market in the stones because of the risk of forgeries. From the production and marketing data given below, determine the profit-maximizing production and sales plans. In particular, determine the output in Venice and Tipton, sales in America and Europe, quantity shipped from America to Europe or vice versa, and prices in America and Europe.

    (a) Demand:
        i. America: $p = 1500 - \frac{1}{2}Q$
        ii. Europe: $p = 1000 - Q$
    (b) Average Cost:
        i. Tipton: $AC = 150 + \frac{3}{8}Q$
        ii. Venice: $AC = 100 + \frac{1}{2}Q$
    (c) **Solution**

$$\begin{aligned} MR_A &= 1500 - s_A \\ MR_E &= 1000 - 2s_E \end{aligned}$$

$$\begin{aligned} MC_A &= 150 + \frac{3}{4}q_A \\ MC_E &= 100 + q_E \end{aligned}$$

so

$$\begin{aligned} s_A &= 1500 - m \\ s_E &= 500 - \frac{1}{2}m \end{aligned}$$

and

$$\begin{aligned} q_A &= \frac{4}{3}m - 200 \\ q_E &= m - 100 \end{aligned}$$

and

$$\begin{aligned} S &= 2000 - \frac{3}{2}m \\ Q &= \frac{7}{3}m - 300 \end{aligned}$$

but $Q = S$ so

$$2300 = \left(\frac{7}{3} + \frac{3}{2}\right) m$$

Then $m = 600$ and

$$\begin{aligned} s_A &= 900 \\ s_E &= 200 \end{aligned}$$

$$
\begin{aligned}
q_A &= 600 \\
q_E &= 500
\end{aligned}
$$

18. State whether the following assertion is true, false or ambiguous, and explain why.

"There are many firms which produce wooden chairs, and many firms which produce wooden tables. If these firms are all separate, and if they all maximize profits, taking prices as given, then the equilibrium cannot be efficient. This is because when the chair firms increase output they bid up the price of wood, which reduces the profits of the table-producing firms. But the chair firms ignore the effect of their output decisions on the profits of the table firms. An efficient equilibrium would be achieved if all of the firms produced both tables and chairs."

(a) **Solution**
The assertion is false. Efficiency requires that wood is allocated so that it yields equal value at the margin in both uses. In other words consumers' marginal rate of substitution between tables and chairs must be equal to the marginal rate of transformation. But the MRT is the ratio of the marginal products of wood in the production of tables and chairs, and this is equal to the ratio of the prices of tables and chairs if producers are maximizing profits; meanwhile the same price ratio is equal to the consumers' MRS when consumers maximize utility.

19. Suppose the market demand curve for mineral water is

$$P = 30 - \frac{Q}{4}$$

where Q is the annual quantity sold, and P is the price in dollars. There are just two firms which produce mineral water, and the production cost is zero (the water comes from springs).

(a) Find the Cournot equilibrium.

(b) Suppose the Cournot equilibrium has been established, and that the market interest rate is 10% per annum. Now the owner of one of the firms offers to sell out to the other at a cash price of \$5200. Will this offer be accepted? Explain.

(c) **Solution**
The equilibrium price is $P = 10$, and demand at this price is $Q = 80$, so each firm produces $q = 40$, and revenue is 400 (given that the other firm is already supplying 40, the residual demand is $p = 20 - \frac{q}{4}$, and the marginal revenue is $mr = 20 - \frac{q}{2}$, and so marginal revenue is zero when $q = 40$).
A lump sum of \$5200 is equivalent to an annual flow of \$520 if the interest rate is 10%, and since this is higher than the annual profit, an offer to buy the other firm at this price would be accepted. But an offer to sell at this price would only be accepted if the monopoly profit is above \$920. The monopoly price is 15, with $Q = 60$, so a monopoly would makes a gross annual profit of \$900, and a net profit of only \$380 allowing for the flow cost of buying out the other firm, and this is less than the profit in the Cournot equilibrium. So this offer would be rejected.

20. Suppose a profit-maximizing firm produces output according to the following production function

$$Q = 30L - \frac{L^2}{1000}$$

where Q is output and L is labor. The product is sold in a competitive market at a price of \$10 per unit. This firm is the only employer around. Male and female workers are equally productive and their labor supply curves are given by

$$
\begin{aligned}
w_m &= \frac{L_m}{40} + 80 \\
w_f &= \frac{L_f}{10} - 20
\end{aligned}
$$

where the subscripts refer to male and female, and w_m and w_f are the respective wages. If the firm is allowed to pay different wages to men and women, will men be paid more than women, or *vice versa*? If the wages are different, does this mean that the employer is prejudiced against women or against men?

21. A worker is searching for a job that will last two years (no matter how long it takes to find it). Each month, the worker receives exactly one job offer, and the cost of job search for a month is \$800. The best possible job pays \$40,000 a year, the worst pays \$20,000, and any wage between these extremes is equally likely (e.g. there is a 60% chance that any given job pays at least \$28,000).

    (a) What search strategy would maximize the expected wage, net of search costs?

    (b) If an unemployment benefit of \$600 per month is available while the search continues, what happens to the average search duration?

22. A person chooses between leisure and consumption. The utility derived from any combination of leisure and consumption is given by the formula:

$$u = LC - 88C$$

where u is utility, L is the number of leisure hours per week, and C is the number of dollars spent on consumption per week. This person can work as many hours as desired each week, at a wage of \$5 per hour.

    (a) If there is no other source of income, how many hours does this person choose to work?

    (b) Now suppose that overtime is offered at \$7.50 per hour after working 40 hours at \$5 per hour. Will this person accept the overtime, and, if so, for how many hours?

23. A coal-mining company is the only employer in town, and faces this supply curve for labor:

$$w = 48 + \frac{72}{2000}L$$

where w is the daily wage, in dollars, and L is the number of workers employed. The company faces this demand curve for coal:

$$p = 60 - \frac{9}{4000}Q$$

where p is the price of coal, per ton, and Q is the number of tons sold per day. Each miner produces 8 tons of coal per day, regardless of the number hired. The company maximizes profit.

    (a) How many workers will be hired, and how much profit will be made?

    (b) Suppose a union is formed, which sets a wage of \$120 per day. At this wage, according to the supply curve given above, 2000 miners are willing to work, and the company is free to hire as many of these as it wants. How many will be hired, and how much profit will be made?

24. A monopolist faces the demand curve

$$P = 120 - 2Q$$

where Q is the annual quantity sold, and P is measured in dollars. Labor is the only input, and the labor supply curve is perfectly elastic at a wage of \$2 per hour. The production function is

$$Q = \sqrt{2L - 200}$$

where L is total hours worked.

    (a) Find the profit-maximizing price and quantity.

    (b) Suppose a union imposes a wage of \$4 per hour. How many workers will lose their jobs? What will happen to total labor income?

25. A monopolist faces two types of consumers, with demand curves given by

$$\begin{aligned} p_1 &= 3 - q_1 \\ p_2 &= 4 - q_2 \end{aligned}$$

There are 100 consumers of each type. The (constant) marginal cost of production is 1. The monopolist cannot discriminate between the two consumer types, but can charge a two-part tariff: a buyer can buy q units by paying $A + pq$, or buy nothing and pay nothing. Does the profit-maximizing two-part tariff (A, p) involve charging a price p below marginal cost? Explain why or why not.

26. A winemaker has produced 10 bottles of fine wine. There is one potential consumer, who values the wine at \$100 (per bottle), if consumed now. Next year, the wine will be even better, and the consumer would then be willing to pay \$120 for it. The winemaker has no use for the wine, but has other ways to spend money. The consumer discounts future consumption at 20% per year (meaning that \$10 worth of consumption next year is worth \$8 now). The winemaker discounts future consumption at 10% per year (meaning that \$10 worth of consumption next year is worth \$9 now). The consumer has \$2,000 to spend (on wine, or other things). Money can be held until next year, but the interest rate is zero.

   Suppose the winemaker sells all of the wine now for \$95 a bottle. Is this efficient? What is the set of Pareto optimal allocations?

   (a) If the wine is stored for a year, it appreciates by 20%. The consumer is indifferent between consuming it now or later. The winemaker is more patient than the consumer. So efficiency requires that the wine is consumed later. The price can be anywhere between 0 and \$120. Selling now for \$95 is dominated by selling next year for \$114, because the winemaker is better off, and the consumer is indifferent.

27. The Phoenix Moons, a professional football team, has a stadium which seats 30,000 people. All seats are identical. The optimal ticket price is \$5, yet this results in an average attendance of only 20,000.

   (a) Explain how it can be profitable to leave 10,000 seats empty.
   (b) Next week the Moons play the Tucson Turkeys, who have offered to buy an unlimited number of tickets at \$4 each to be resold only in Tucson. How many tickets should be sold to Tucson to maximize profits? (i) 30,000, (ii) more than 10,000, (iii) 10,000, (iv) less than 10,000, (v) none. Explain your answer.
   (c) Given your answer to part (b) above, what price should the Moons charge their own fans, to maximize profit? (i) \$5, (ii) more than \$5, (iii) between \$4 and \$5, (iv) \$4, (v) less than \$4. Explain your answer.

28. Modify the Bertrand duopoly model to allow different marginal costs for the two firms. Can you find an equilibrium in this model? If so, is it unique?
   If the two firms are charging different prices, consumers all buy from the low-price firm. This price could be the monopoly price, and if this is higher than the cost of the other firm, this is an equilibrium. If both marginal costs are below the monopoly price, then both firms must be charging the same price, because otherwise the low-price firm could raise its price slightly, and this would increase profit.
   The price can't be lower than c_2, where $c_1 < c_2$, because otherwise the high-cost firm has negative profit. If the price is higher than c_2, then either firm can increase profit by reducing the price slightly. So if both firms are charging the same price, then $p = c_2$. If firm 1 gets all of the demand at $p = c_2$, this is an equilibrium. But if not, firm 1 can do better by charging $p = c_2 - \varepsilon$. So both the consumers' and the firms' strategies must be specified to get an equilibrium.

29. Design a contract to maximize the expected profits received by a risk-neutral principal who will hire a risk-averse agent. The agent's utility function is

$$u(c, e) = \log(c) - e$$

where e is effort (high or low), and c is consumption, which is equal to the wage payment specified in the contract. The principal can observe gross revenue, but cannot observe the agent's effort. The agent has an outside option that is a sure thing worth $-\frac{1}{2}$ (in units of utility). The low effort level is zero, and the high effort level is $\frac{1}{2}$.

Gross revenue depends on the agent's effort level. If effort is high, revenue R is distributed on the set $\{10, 20, 40\}$ with probabilities $P(10) = .4, P(20) = .2, P(40) = .4$. If effort is low, R is distributed on the same set with probabilities $P(10) = .2, P(20) = .6, P(40) = .2$.

30. Suppose an art dealer wishes to auction a painting so as to maximize expected profit. There are two potential buyers, with independent valuations of the painting which are uniformly distributed between 0 and \$6,000. The dealer has no other use for the painting. The dealer is considering two alternative auction procedures: a first price auction, or a Vickrey (second-price) auction. Suddenly a third buyer appears, and makes a single take-it-or-leave-it offer of 2,200 for the painting. Will the dealer accept this offer? If not, which auction procedure will the dealer use (after the third buyer has left)?

31. Consider an exchange economy with two consumers, California and Wisconsin. California has a stock of 1 gallon of wine, and Wisconsin has 10 gallons of beer. California's utility function is $u(b, w) = 2bw$, where w is wine consumption and b is beer consumption. Wisconsin's utility function is the same. The income of each consumer is the revenue derived from selling wine or beer, so it depends on the prices of wine or beer.

    (a) Draw indifference curves representing utility levels of 5, 10 and 20.

    (b) Let the price of beer be fixed at 50 cents per gallon. Draw budget constraints for California when the price of wine is (i) \$5 per gallon, (ii) \$10, (iii) \$20.

    (c) Find California's optimal consumption plan for each of the three budget constraints.

    (d) Repeat parts (a), (b), (c) for Wisconsin.

    (e) Identify the competitive equilibrium price list (assuming each consumer takes prices as given). Choose units so that the total endowment is one unit of each good. Then since the utility functions are the same, the price has to be 1, meaning that 1 gallon of wine is worth 10 gallons of beer, so if beer is 50 cents, then wine is \$5.

    (f) If the price of beer had been fixed at \$1 instead of 50 cents how would the competitive equilibrium be affected?

32. There is a town with 100 families who do not have cable tv. The cost of supplying cable tv would be \$20 per month for each family connected to the system, plus \$2000 per month in overhead costs which do not depend on the number of families connected.

    50 families live in houses, and the other 50 live in apartments. The demand price of a cable connection varies from one family to the next, in such a way that the demand curve for all those who live in houses is $p = 100 - 2q$, where p is the price charged per month, and q is the number of houses buying connections. The demand curve for those who live in apartments is also $p = 100 - 2q$.

    The town has granted the right to supply cable tv to a profit-maximizing monopoly firm. How will this firm set prices? Will the outcome be efficient? If not, what advice would you give the town as to the possibility of achieving a better outcome?