104. The prices of consumer goods in Los Angeles and Miami are different: some things are cheaper in Miami, while others are cheaper in Los Angeles. Suppose the price of every consumer good in Houston is exactly halfway between the Miami price of that good and the Los Angeles price. A market research firm surveys 1000 consumers, who have different preferences over consumption bundles, and different incomes. The consumers are asked to rank these three cities in terms of the consumption bundles that they could afford in each place. The result of the survey is that 450 consumers rank Los Angeles first, 350 rank Houston first and 200 rank Miami first. Is this consistent with the Weak Axiom of Revealed Preference?

105. Consider the utility function

$$u = 2x_1^2 + 4x_2^{1/2}$$

(a) Find the demand functions for goods 1 and 2 as they depend on prices and wealth.

(b) Find the compensated demand function h(·).

(c) Find the expenditure function, and verify that $h(p, u) = \nabla_p e(p, u)$.

(d) Find the indirect utility function, and verify Roy's identity.

106. Suppose that a preference ordering on $\mathbb{R}_+^L$ can be represented by the utility function

$$u(x) = \theta_L x_L x_1 + \sum_{i=1}^{L-1} \theta_i x_i x_{i+1}$$

where $\theta \in \mathbb{R}_+^L$

(a) Is this preference ordering homothetic?

(b) Is this preference ordering separable? For example, if x_L is fixed at some level a, the utility function defines a preference ordering on $\mathbb{R}_+^{L-1}$, and if x_L is fixed at b, the utility function defines another preference ordering on $\mathbb{R}_+^{L-1}$. Are these two orderings actually the same?

(c) Find the Walrasian demand function.

107. Suppose that a preference ordering on $\mathbb{R}_+^2$ can be represented by the utility function

$$u(x) = \frac{1}{2}x_1^2 + \ln(x_2)$$

(a) Suppose the prices are $p_1 = 2$, $p_2 = 1$. If wealth is $w = 4$, what is the optimal consumption plan?

(b) Suppose that wealth increases to 5. Now what is the optimal consumption plan?

(c) Can you find a price vector and a wealth level such that $x_1(p, w) = 1$?

108. Suppose that a consumer's preference ordering on $\mathbb{R}^L_+$ can be represented by the utility function

$$u(x) = \sum_{\ell=1}^{L} \frac{\alpha_\ell (x_\ell - \delta_\ell)^{\rho_\ell} - 1}{\rho_\ell}$$

with $\sum_{\ell=1}^{L} \alpha_\ell = 1$, and $\alpha_\ell \geq 0$ and $\rho_\ell < 1$, and $\sum_{\ell=1}^{L} p_\ell \delta_\ell < w$, where $p \cdot x \leq w$ is the budget constraint.

(a) Find the Frisch demand function.

(b) Show how the Walrasian demand function is obtained from the Frisch demand function.

(c) Suppose that $L = 2, \delta_1 = 1, \delta_2 = 2, \alpha_1 = \frac{1}{2}, \alpha_2 = \frac{1}{2}, \rho_1 = \frac{1}{2}, \rho_2 = \frac{3}{4}, p_1 = 1, p_2 = 1, w = 23$. What is the optimal consumption plan?

(d) Suppose the parameters $\rho_\ell, \alpha_\ell, \delta_\ell$ are given. Is it possible that this consumer chooses the same consumption plan from two different budget sets?

(e) Now suppose the assumption $\rho_\ell < 1$ is replaced by $\rho_\ell \leq 1$. How does this small modification affect the Frisch and Walrasian demand functions?

109. A monopoly airline sells tickets to business travelers (B) and to leisure travellers (L). The proportion of B types is λ. There are two periods. At the beginning of period one, the traveler privately learns his type, which determines the probability distribution F_B or F_L that will determine his valuation for the ticket (where for example $F_B(v)$ is the probability that the B type will draw a valuation of v or less).

The seller and the traveller contract at the end of period one. At the beginning of period two, the traveller privately learns his actual valuation for the ticket, and then decides whether to travel. Each ticket costs the seller c. The seller and the traveller are risk-neutral, and there is no discounting. The reservation utility of each type of traveller is normalized to zero.

A partially refundable ticket contract consists of a pair (a, r), where a is an advance payment at the end of period one and r is a refund that can be claimed at the end of period two if the ticket is not used. The traveler's payoff under this contract is $v - a$ if the ticket is used, and $r - a$ if

it is not. The seller offers two contracts (a_1, r_1) and (a_2, r_2), and the four parameters describing these two contracts are chosen so as to maximize expected profit. Since the seller does not know the traveler's type, each traveler can choose either contract.

Suppose $\lambda = \frac{2}{3}$, $c = 50$, F_B is a uniform distribution on the set $[0, 50] \cup [100, 150]$, and F_L is a uniform distribution on the set $[50, 100]$.

(a) Strategies

i. A simple strategy for the seller is to just charge a single ticket price p, that is fully refundable. This can be implemented by setting $a_1 = a_2 = r_1 = r_2 = p$. What is the optimal ticket price in this case, and how much profit does the seller make?

ii. Can you find two contracts that yield more expected profit than the optimal simple strategy?

(b) Profit Maximization

i. What are the expected profit maximizing choices of (a_1, r_1) and (a_2, r_2)?

ii. If the seller chooses (a_1, r_1) and (a_2, r_2) so as to maximize expected profit, is the outcome efficient?

110. A winemaker has produced 10 bottles of fine wine. There is one potential consumer, who values the wine at \$100 (per bottle), if consumed now. Next year, the wine will be even better, and the consumer would then be willing to pay \$120 for it. The winemaker has no use for the wine, but has other ways to spend money. The consumer discounts future consumption at 20% per year (meaning that \$10 worth of consumption next year is worth \$8 now). The winemaker discounts future consumption at 10% per year (meaning that \$10 worth of consumption next year is worth \$9 now). The consumer has \$2,000 to spend (on wine, or other things). Money can be held until next year, but the interest rate is zero.

Suppose the winemaker sells all of the wine now for \$95 a bottle. Is this efficient? What is the set of Pareto optimal allocations?

111. An expected utility maximizer with constant relative risk aversion and wealth w buys α units of insurance at price q against a loss D that occurs with probability π, where $q > \pi$. Find α.

112. State whether the following assertions are true, false or ambiguous, and explain why.

(a) A rise in the wage rate implies a rise in the firm's marginal cost curve.

(b) If a production technology has production isoquants defined by $y^2 = K^2 + L^2$, the cost function has L-shaped contours in the factor prices

(i.e. the combinations of v and w which keep $c(v, w, y)$ constant, for fixed y, lie on an L-shaped isocost curve).

(c) The utility function $u(x_1, x_2) = x_1^2 x_2^2$ is not quasi-concave, so the expenditure function derived from this utility function is not concave in prices.

(d) Constant relative risk aversion implies that the demand for insurance is a decreasing function of wealth.

113. [Prelim, January 2011] Suppose that there are 50 million people in the labor force in Mexico and 150 million in the U.S. All workers prefer to work in their own country, but the extent of this preference varies across people. Assume that Mexican and U.S. workers are perfect substitutes (that is, they are equally productive when working in the same country), and assume that the same product is produced in both countries, and that the product price is 1.
The technology in each country is described by a Cobb-Douglas production function with constant returns:

$$Q_i = A_i K_i^{\alpha} L_i^{1-\alpha}$$

for $i \in \{1, 2\}$, with $\alpha \in (0, 1)$.
The total supply of capital in the two countries is a fixed amount K^0, and capital can be moved from one country to the other at no cost. The owners of the capital act so as to maximize income – the capital is rented to the highest bidder.
All markets are competitive, except that there may be restrictions on migration of labor from one country to the other.

(a) Suppose that immigration is not allowed, and it is observed that the equilibrium wage is \$30 per hour in the U.S., and \$10 per hour in Mexico. Does this imply that U.S. firms are more productive (i.e. that $A_1 > A_2$)? If $\alpha = \frac{1}{3}$, and $A_2 = 1$, do you have enough information to determine A_1?

(b) Now suppose that workers can freely migrate from one country to the other. The home country preference depends on the relative wage, $\omega = \frac{w_1}{w_2}$, and preferences are uniformly distributed. If $\omega = 5$, all Mexican workers would prefer to work in the U.S.; and if $\omega = 2$, 40% of Mexican workers would prefer to work in the U.S., and so on.

   i. How many people will migrate, in the new equilibrium?
   ii. What happens to wages in each country? Explain why.
   iii. Does the relative wage rise or fall?
   iv. What happens to output in each country? What happens to total output (the sum of the outputs in the two countries)?
   v. Discuss the welfare implications of your results.

114. [Prelim, June 2011] Consider a pure exchange economy with three consumers (labeled 1, 2 and 3) and three goods (labeled x, y and z). Agent i's consumption vector is (x_i, y_i, z_i). Each agent is endowed with only one type of good: $\omega_1 = (0,0,3), \omega_2 = (1,0,0)$ and $\omega_3 = (0,2,0)$, where ω_i is i's endowment. The consumers' preferences can be represented by utility functions, as follows:

$$\begin{aligned} U_1 &= \log(y_1) + \log(z_1) \\ U_2 &= \sqrt{y_2 z_2} \\ U_3 &= x_3 y_3 z_3 \end{aligned}$$

(a) Find a competitive equilibrium. Is it unique?

(b) Find the set of Pareto optimal allocations.

115. Suppose J identical consumers have preferences over a basic good, x_0, and a composite y of different varieties of another good. Each consumer starts with μ units of the basic good and has preferences represented by the utility function

$$U(x) = x_0 + y$$

where

$$y = \sum_{i=1}^{n} x_i^{\rho}$$

with $0 < \rho < 1$. The price of good i is p_i, and consumers take these prices as given.
The market supply of the basic good is perfectly elastic, at $p_0 = 1$.
Each variety x_i is produced by one firm, which requires one unit of capital regardless of how much output is produced, and also uses c units of the basic good for every unit of output. The market supply curve of capital is

$$p_K = aK$$

where p_K is the price of capital (per unit of the basic good) and K is the quantity supplied.
Each firm sets a price, and sells whatever consumers want to buy at that price.
The number of varieties, n, is determined by free entry: anyone can introduce a new variety, and there will then be a new firm producing that variety. There is an unlimited supply of potential entrants, and they act so as to maximize profits.

(a) In equilibrium, how many varieties will be produced?

   i. Illustrate your answer using the following parameter values: $J = 100, \rho = \frac{1}{2}, \mu = 1000, c = \frac{1}{10}, a = \frac{1}{8}$.

(b) Is the equilibrium efficient?