A monopoly airline sells tickets to business travelers (B) and to leisure travellers (L). The proportion of B types is λ. There are two periods. At the beginning of period one, the traveler privately learns his type, which determines the probability distribution F_B or F_L that will determine his valuation for the ticket (where for example $F_B(v)$ is the probability that the B type will draw a valuation of v or less).

The seller and the traveller contract at the end of period one. At the beginning of period two, the traveller privately learns his actual valuation for the ticket, and then decides whether to travel. Each ticket costs the seller c. The seller and the traveller are risk-neutral, and there is no discounting. The reservation utility of each type of traveller is normalized to zero.

A partially refundable ticket contract consists of a pair (a, r), where a is an advance payment at the end of period one and r is a refund that can be claimed at the end of period two if the ticket is not used. The traveler's payoff under this contract is $v - a$ if the ticket is used, and $r - a$ if it is not. The seller offers two contracts (a_1, r_1) and (a_2, r_2), and the four parameters describing these two contracts are chosen so as to maximize expected profit. Since the seller does not know the traveler's type, each traveler can choose either contract.

Suppose $\lambda = \frac{2}{3}$, $c = 50$, F_B is a uniform distribution on the set $[0, 50] \cup [100, 150]$, and F_L is a uniform distribution on the set $[50, 100]$.

1. (a) A simple strategy for the seller is to just charge a single ticket price p, that is fully refundable. This can be implemented by setting $a_1 = a_2 = r_1 = r_2 = p$. What is the optimal ticket price in this case, and how much profit does the seller make?

   (b) Can you find two contracts that yield more expected profit than the optimal simple strategy?

2. (a) What are the expected profit maximizing choices of (a_1, r_1) and (a_2, r_2)?

   (b) If the seller chooses (a_1, r_1) and (a_2, r_2) so as to maximize expected profit, is the outcome efficient?

1(a) The problem is just a monopolist facing a distribution of valuations which is uniform on $[0, 150]$, with a marginal cost of 50. This is a linear demand curve, so the optimal price is the midpoint between the marginal cost and the intercept of the demand curve, meaning that the price is 100. Then one-third of the customers buy (half of the B types, none of the L types). Profit is 50 per ticket, so total profit is $\frac{50}{3}$ per customer.

1(b) A natural thing to do is to ignore the L types (since they are a minority, and they do not have the highest valuations) and use an efficient two-part tariff for the B types. This means setting $r_1 = 50$, and setting $a_1 = \frac{175}{2}$, and setting $a_2 = a_1$and $r_2 = r_1$. Then half of the L types travel and the other half get a refund, costing 50 in either case, so profit is $\frac{2}{3}\left(\frac{175}{2} - 50\right) = 25$. Alternatively, set $a_1 = a_2 = 75$, with $r_1 = r_2 = 50$. Then both types participate, and profit is again 25 per customer.

2(a) The ideal contract for the seller would have the buyers maximize the trade surplus and hand it over. The surplus is maximal when all of the leisure types buy, and the business types buy if and only if the realized valuation is greater than the cost. There is no reason to offer refunds to the leisure types, since it is already known that their valuations will be above the cost. The expected surplus for the leisure types is the average value of v, which is 75, so set $a_2 = 75$ and $r_2 = 0$.

To induce the business types to make the surplus-maximizing decision set $r_1 = c$. Then the average surplus for those who travel is 125 (the midpoint between 100 and 150), while those who do not travel get their money back, and since the chances are 50-50, the business types are willing to pay $\frac{1}{2}125 + \frac{1}{2}50$ in advance, so set $a_1 = \frac{175}{2}$.

If the business types choose the wrong contract they will lose 50 or gain 50 with equal probability (the average value in the lower half of the distribution is 25, and they have paid 75, while the average value in the upper half is 125). So there is no incentive for the business types to choose the wrong contract.

If the leisure types choose the wrong contract their average gross payoff is 75, but they have paid $\frac{175}{2}$, so they strictly prefer their own contract.

The seller's profit is $\frac{175}{2} - 50$ on the business types (with probability $\lambda = \frac{2}{3}$), and 25 on the leisure types, so total profit is $\frac{2}{3}\frac{75}{2} + \frac{1}{3}25 = \frac{100}{3}$.

This pair of contracts is optimal since the surplus is maximal and the seller gets all of it.

2(b) The optimal contract is efficient, by construction.

Reference @article{CourtyLi00, title={{Sequential screening}}, author={Courty, P. and Li, H.}, journal={Review of Economic Studies}, volume={67}, number={4}, pages={697–717}, year={2000}}