Job Search

Suppose the reservation wage is set so that the probability of success is p, and the probability of failure is q, with $p+q=1$. How long will it take, on average, to find an acceptable job?

1 p

2 pq

3 pq^2

4 pq^3

...

So if D is the average duration, then

$$\begin{aligned} D &= p\left(1+2q+3q^2+4q^3+...\right) \\ qD &= p\left(\quad q+2q^2+3q^3+4q^4+...\right) \\ (1-q)\,D &= p\left(1+q+q^2+q^3+...\right) \end{aligned}$$

But

$$p = 1-q$$

$$D = 1+q+q^2+q^3+...$$

and

$$qD = q+q^2+q^3+...$$

$$(1-q)\,D = 1$$

aand

$$D = \frac{1}{p}$$

Example: Roll a pair of dice until a pair of sixes comes up. What is the average number of throws? The answer is 36. It might happen on the first try; it might take 100 tries, but on average it takes 36 tries.

Optimal Search Example

Cost is 100 per search

9000	9500	10000	10500	Jobs
0.3	0.3	0.1	0.1	Frequency
0.8	0.5	0.2	0.1	Cumulative
1.25	2	5	10	Expected Duration
125	200	500	1000	Expected Cost
9500	9800	10250	10500	Expected Gross
9375	9600	9750	9500	Expected Net

Start with a reservation wage of $10,500$. This can't be optimal, because if $10,000$ is offered, it beats the value of continuing. So try $A = 10,000$. The expected value using this policy exceeds 9500; therefore $10,000$ is the optimal choice.

Suppose the wage distribution is uniform on $[L, L+1]$. Say the acceptance wage is $L+a$.

Then the expected cost is $\frac{c}{1-a}$, and the expected wage is $L + \frac{1+a}{2}$.

So the objective is to maximize

$$\max_a \frac{1+a}{2} - \frac{c}{1-a}$$

Setting the derivative to zero gives

$$\frac{1}{2} + \frac{c}{(1-a)^2} = 0$$

so

$$(1-a)^2 = 2c$$

For example, if $c = 8$, then $1 - a = 4$, $a = -3$, meaning that the cost is so high that the optimal policy is to accept anything.

But if $c = 1/18$, then $1 - a = \frac{1}{3}$, so $a = \frac{2}{3}$.

Note that the solution is characterized by equating the acceptance wage $L+a$ with the expected net value of continued search, using this acceptance wage, which is

$$L+a=L+\frac{1+a}{2}-\frac{c}{1-a}$$

This can be solved by picking a trial value.

Suppose the gap between L and H is $100K$, and c is $2K$. In terms of the above notation, the unit is $100K$, and $c=.02$. Then $(1-a)^2=.04$ so $1-a=.2$

Then $a=\frac{4}{5}$ (i.e. $80K$).

Try a = ½ (i.e. 50K). Then the expected net value is $3/4-2/50=71/100$. So it is foolish to accept ½, because continuing the search yields a value of 71.

If a=71, the expected value is 1.71/2 - (1/.29)(1/50) = 1.855 - .06 approximately.

Take the distribution as uniform on [L,H], with G = H-L. Then 2cG = (H-A)^2. For example, c = 2, G = 100 (measured in thousands of dollars) gives H-A = 20.

A decrease in search cost implies an increase in the acceptance wage. For example, if H = 200 then A = 180 when c = 2. But if c = ½, then A = 190.