December 17, 2005, 3-5pm. **Economics 711: Final Exam** John Kennan

Answer **four** questions. Each question is worth 25 points, unless you say otherwise: you can reallocate points subject to the constraint that each question is worth at least 20 (e.g. 40/20/20/20). Explain your answers carefully, using diagrams where appropriate. Write as if you are trying to convince an intelligent person who does not already know the answers. Be as precise as you can, but remember that an imprecise answer is better than nothing, and intuitive reasoning can sometimes be convincing. If the problem is too hard, answer a simplified version of it, and then try to sketch an argument for the more general version.

---

**1.** Consider an economy with two goods, fish and cheese, and two people, One and Two. Each person is endowed with .5 pounds of fish and .5 pounds of cheese. One's utility function is $u(f,c) = f+c$, and Two's utility function is $u(f,c) = f^2+c^2$. There is a publicly available production technology defined by the production set $Y = \{(f,c) : f+c = 0\}$.

**(a)** Can you find a competitive equilibrium for this economy? If so, is it Pareto optimal? If not, explain which assumptions of the First Welfare Theorem are violated.

**(b)** What is the set of Pareto optimal allocations in this economy? Can all these allocations be supported as competitive equilibria? Explain.

**2.** Suppose that a small country produces two consumption goods, with production functions

$$\sqrt{Q_1} = \sqrt{K_1} + \sqrt{L_1}$$
$$\frac{1}{Q_2} = \frac{1}{K_2} + \frac{1}{L_2}$$

where $K_1$ is the amount of capital used in the production of good 1, etc. The economy is competitive, with prices $p_1$ and $p_2$ for the consumption goods, and v and w for capital and labor.

**(a)** Find the relationship between the factor price ratio $\phi = w/v$ and the efficient capital-labor ratio in each industry.

**(b)** Find the relationship between the ratio of the product prices ($p_1/p_2$) and the factor price ratio.

**(c)** Suppose that product prices are determined on the world market, and the price ratio is $p_1/p_2 = 11.25$. The country is endowed with 0.38 units of capital per worker. Can you determine the equilibrium levels of output per worker for each product?

Hint: if you have trouble solving the equations, try the transformation $x = \sqrt{\phi} + 1/\sqrt{\phi}$.

**3.** Consider an economy in which there are two kinds of workers, A and B, and two kinds of jobs, good and bad. Each employer has an unlimited number of vacancies in both kinds of jobs. Some workers are qualified for the good job, and some are not. If a qualified worker is assigned to the good job the employer gains \$71, and if an unqualified worker is assigned to the good job the employer loses \$351. When any worker is assigned to the bad job, the employer breaks even.

Workers who apply for jobs are tested and assigned to the good job if they do well on the test. Test scores range from 0 to 1. The probability that a qualified worker will have a test score less than t is $t^2$. The probability that an unqualified worker will have a test score less than t is $2t-t^2$. These probabilities are the same for A-workers and B-workers.

There is a fixed wage premium of \$27 attached to the good job. Workers can become qualified by paying an investment cost, and this cost is higher for some workers than for others: the distribution of costs is uniform between 0 and \$13, for both A-workers and B-workers. Workers make investment decisions so as to maximize earnings, net of the investment cost.

The total number of workers in the economy is 633,000. The number of A-workers is 608,000.

**(a)** Can you find an equilibrium in which A-workers are over-represented in the good jobs?

**(b)** Now suppose that employers are subject to a rule that requires the proportion of A-workers assigned to the good job to be the same as the proportion of B-workers. Otherwise employers maximize expected profits. What is the effect of this rule?

**4.** Consider an economy with n agents and two goods: a private good, x and a public good, g. Consumer i has an endowment of $\omega_i$ units of the private good, and there is a technology that transforms the private good into the public good.

**(a)** Suppose there are m firms that have access to the public good technology, and each consumer owns equal shares of each firm. How would you define a "Walrasian" (competitive) equilibrium for this economy?

**(b)** Now suppose the public good technology has constant returns to scale, at a rate of two units of x per unit of g.

i. What is the Walrasian equilibrium price ratio?

ii. Are the Walrasian allocations Pareto efficient? Explain.

iii. Relate your answer to the First Welfare Theorem.

**5.** Prove, under suitable assumptions, that any competitive equilibrium of an exchange economy is in the core.

**6.** Consider an economy with two goods and two consumers and one producer. The aggregate endowment vector is (1,2). The first consumer's utility function is $u(x,y) = xy$, where x and y are the consumption levels of the two goods. The second consumer's utility function is $u(x,y) = \sqrt{x} + \sqrt{y}$. The producer's technology is $Y = \{(x,y) : x+y \leq 0\}$

Find the utility possibility frontier.