Thursday December 12, 2002, 1-3pm. **Economics 711: Final Exam** John Kennan

Answer **four** questions. Each question is worth 25 points, unless you say otherwise: you can reallocate points subject to the constraint that each question is worth at least 20 (e.g. 40/20/20/20). Explain your answers carefully, using diagrams where appropriate. Write as if you are trying to convince an intelligent person who does not already know the answers. Be as precise as you can, but remember that an imprecise answer is better than nothing, and intuitive reasoning can sometimes be convincing. If the problem is too hard, answer a simplified version of it, and then try to sketch an argument for the more general version.

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**1.** **a.** Give an example of a preference ordering that generates a demand function with the following property: the demand for good i increases when the price of good j increases, for any pair of goods with $i \neq j$.

**b.** Consider an exchange economy in which all consumers have preferences satisfying the above property. Show that there can be only one competitive equilibrium in such an economy.

**2.** Suppose there are two consumption goods with production functions $\sqrt{Q_1} = \sqrt{K_1} + \sqrt{L_1}$, $\frac{1}{Q_2} = \frac{1}{K_2} + \frac{1}{L_2}$, where $K_1$ is the amount of capital used in the production of good 1, etc. There are two small countries, and all markets within each country are competitive, and there is free trade in good 1, but neither good 2 nor the factors of production can move between countries. All consumers regard goods 1 and 2 as perfect substitutes, in the sense that 1 unit of good 2 is always worth 2 units of good 1.

Suppose there is an equilibrium in which both goods are produced in both countries. In such an equilibrium, can you determine whether factor prices are equal in the two countries? If so, explain why; if not, explain why not.

**3.** Consider an exchange economy with two goods, fish and cheese, and two people, One and Two. Each person is endowed with .5 pounds of fish and .5 pounds of cheese. One's utility function is $u(f,c) = fc$, and Two's utility function is $u(f,c) = f^2+c^2$.

**(a)** Can you find a competitive equilibrium for this economy? If so, is it Pareto optimal? If not, explain which assumptions of the First Welfare Theorem are violated.

**(b)** What is the set of Pareto optimal allocations in this economy? Can all these allocations be supported as competitive equilibria? Explain.

**4.** An economy contains many identical consumers, with utility functions $u(x) = \sum_{i=0}^{N} \sqrt{x_i}$. Each consumer is endowed with some quantity of good 0, and the other goods are produced using identical technologies which require 0.01 units of $x_0$ to get started, and 2 units of $x_0$ for each unit of $x_i$ produced. Each good $i > 0$ is produced by a single firm that maximizes profits. The number of possible goods, N, is big relative to the number of consumers. There is free entry in the production of all goods.

**(a)** How many goods will be produced in equilibrium?

**(b)** If there is technical progress such that the marginal cost of $x_i$ falls from 2 to 1, what happens to the equilibrium number of firms? Give an economic interpretation of your result.

**5.** Consider an economy in which there are equal numbers of two kinds of workers, A and B, and two kinds of jobs, good and bad. Each employer has an unlimited number of vacancies in both kinds of jobs. Some workers are qualified for the good job, and some are not. If a qualified worker is assigned to the good job the employer gains $2,000, and if an unqualified worker is assigned to the good job the employer loses $1,000. When any worker is assigned to the bad job, the employer breaks even.

Workers who apply for jobs are tested and assigned to the good job if they do well on the test. Test scores range from 0 to 1. The probability that a qualified worker will have a test score less than t is $t^2$. The probability that an unqualified worker will have a test score less than t is t. These probabilities are the same for A-workers and B-workers.

There is a fixed wage premium of $4,000 attached to the good job. Workers can become qualified by paying an investment cost, and this cost is higher for some workers than for others: the distribution of costs is uniform between 0 and $3,000, for both A-workers and B-workers. This distribution is the same for a-workers and b-workers. Workers make investment decisions so as to maximize earnings, net of the investment cost (all of these amounts are expressed as present values).

Can you find an equilibrium in which there are more A-workers than B-workers in the good jobs?

**6.** State whether the following assertion is true, false or ambiguous, and **explain why**.

"There are many firms which produce wooden chairs, and many firms which produce wooden tables. If these firms are all separate, and if they all maximize profits, taking prices as given, then the equilibrium cannot be efficient. This is because when the chair firms increase output they bid up the price of wood, which reduces the profits of the table-producing firms. But the chair firms ignore the effect of their output decisions on the profits of the table firms. An efficient equilibrium would be achieved if all of the firms produced both tables and chairs."