Saturday December 15, 2001, 2-4pm. **Economics 711: Final Exam** John Kennan

Answer **four** questions. Each question is worth 25 points, unless you say otherwise: you can reallocate points subject to the constraint that each question is worth at least 20 (e.g. 40/20/20/20). Explain your answers carefully, using diagrams where appropriate. Write as if you are trying to convince an intelligent person who does not already know the answers. Be as precise as you can, but remember that an imprecise answer is better than nothing, and intuitive reasoning can sometimes be convincing. If the problem is too hard, answer a simplified version of it, and then try to sketch an argument for the more general version.

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1. (Scarf) An exchange economy has three consumers, labeled A, B and C, and three goods, labeled x, y and z. Initially, A has with 1 unit of x, B has 1 unit of y, and C has 1 unit of z. A's utility function is $u^A(x,y,z) = \min(x,y)$. B's utility function is $u^B(x,y,z) = \min(y,z)$. C's utility function is $u^C(x,y,z) = \min(z,x)$.

a. Find a competitive equilibrium for this economy

b. Find two allocations that are in the core of this economy.

2. Suppose there are two consumption goods with production functions $\sqrt{Q_1} = 2\sqrt{K_1} + \sqrt{L_1}$, $\sqrt{Q_2} = 2\sqrt{K_2} + 2\sqrt{L_2}$, where K_1 is the amount of capital used in the production of good 1, etc. The economy is competitive.

a. Find the relationship between the factor price ratio and the efficient capital-labor ratio in each industry.

b. Suppose now there are two small countries, A and B, each having the technologies described above. A is endowed with 3 units of K and 24 units of L, and B is endowed with 6 units of K and 36 units of L. There is free trade in the consumer goods, but the factors of production cannot move between countries. The prices of the consumer goods, established in the world market, are $p_1 = 5$ and $p_2 = 2$. Taking these prices as given, find the competitive equilibrium in each country.

c. In your equilibrium, are factor prices equal across these two countries? If so, explain why; if not, explain why not.

3. Consider an exchange economy in which each consumer has a complete, transitive and strictly convex preference ordering over a convex consumption set. Does the first welfare theorem hold for this economy? If so, prove it. If not, give a counterexample.

4. An economy contains many identical consumers, with utility functions $u(x) = \sum_{i=1}^{N} \sqrt{x_0 x_i}$. Each consumer is endowed with some quantity of good 0, and the other goods are produced using identical technologies which require 0.01 units of x_0 to get started, and 2 units of x_0 for each unit of x_i produced. Good i is produced by a single firm that maximizes profits. The number of possible goods, N, is big relative to the number of consumers. There is free entry in the production of all goods.

a. How many goods will be produced in equilibrium?

b. If there is technical progress such that the marginal cost of x_i falls from 2 to 1, what happens to the equilibrium number of firms? Give an economic interpretation of your result.

5. Consider an economy in which there are equal numbers of two kinds of workers, a and b, and two kinds of jobs, good and bad. Each employer has an unlimited number of vacancies in both kinds of jobs. Some workers are qualified for the good job, and some are not. If a qualified worker is assigned to the good job the employer gains $1,000, and if an unqualified worker is assigned to the good job the employer loses $23,000. When any worker is assigned to the bad job, the employer breaks even.

Workers who apply for jobs are tested and assigned to the good job if they do well on the test. Test scores range from 0 to 1. The probability that a qualified worker will have a test score less than t is t^3. The probability that an unqualified worker will have a test score less than t is $1-(1-t)^3$.

There is a fixed wage premium of $23,000 attached to the good job. Workers can become qualified by paying an investment cost, and this cost is higher for some workers than for others: the distribution of costs is uniform between 0 and $18,000. This distribution is the same for a-workers and b-workers. Workers make investment decisions so as to maximize earnings, net of the investment cost (all of these amounts are expressed as present values).

Is there an equilibrium in which there are more a-workers than b-workers in the good jobs? If so, say what proportion of a-workers and b-workers are in the good jobs (your answer need not be exact, but it should be close). Otherwise prove that no such equilibrium exists.

6. Suppose that two firms produce identical goods, and consumers buy from the firm that charges the lowest price (and if the prices are equal the quantity demanded by consumers is divided equally between the two firms). Each firm sets a price and sells whatever quantity consumers choose to buy at that price. The demand curve is $p = 1-Q$, where Q is the total quantity demanded by all consumers when the price is p. The cost function for each firm is $c(q) = \frac{1}{2}q^2$, where q is the quantity produced.

Find a Nash equilibrium, and determine whether it is unique