Monday December 18, 2000, 2-4pm. **Economics 711: Final Exam** John Kennan

Answer **four** questions. Each question is worth 25 points, unless you say otherwise: you can reallocate points subject to the constraint that each question is worth at least 20 (e.g. 40/20/20/20). Explain your answers carefully, using diagrams where appropriate. Write as if you are trying to convince an intelligent person who does not already know the answers. Be as precise as you can, but remember that an imprecise answer is better than nothing, and intuitive reasoning can sometimes be convincing. If the problem is too hard, answer a simplified version of it, and then try to sketch an argument for the more general version.

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1. Consider an economy with n agents and two goods: a private good, x and a public good, g. Consumer i has an endowment of ω_i units of the private good, and there is a technology that transforms the private good into the public good.

a. Suppose there are m firms that have access to the public good technology, and each consumer owns equal shares of each firm. How would you define a "Walrasian" (competitive) equilibrium for this two-good economy?

b. Now suppose the public good technology has constant returns to scale, at a rate of two units of the private good per unit of the public good.
   i. What is the Walrasian equilibrium price ratio?
   ii. Are the Walrasian allocations Pareto efficient? Explain.
   iii. Relate your answer to the First Welfare Theorem.

2. An economy contains many identical consumers, with utility functions $u(x) = \log(x_0) + \log\left(\sum_{i=1}^{N} \sqrt{x_i}\right)$. Each consumer is endowed with some quantity of good 0, and the other goods are produced using identical technologies which require a units of x_0 to get started, and c units of x_0 for each unit of x_i produced. Good i is produced by a single firm that maximizes profits. The number of possible goods, N, is big relative to the number of consumers. There is free entry in the production of all goods.
a. How many goods will be produced in equilibrium?
b. Is the equilibrium Pareto optimal?

3. a. State and prove the First Welfare Theorem.
b. State the Second Welfare Theorem. Give an example in which one of the assumptions of the theorem does not hold, and the conclusion of the theorem is false.

4. Suppose there are two consumption goods with production functions $\sqrt{Q_1} = \sqrt{K_1} + \sqrt{L_1}$, $\frac{1}{Q_2} = \frac{1}{K_2} + \frac{1}{L_2}$, where K_1 is the amount of capital used in the production of good 1, etc. There are two small countries, A and B. A is endowed with 4 units of K and 50 units of L, and B is endowed with 20 units of K and 3 units of L. Both economies are competitive, and there is free trade in the consumer goods, but the factors of production cannot move between countries. The prices of the consumer goods, established in the world market, are $p_1 = 0.8$ and $p_2 = 9$. Find the competitive equilibrium in each country.
In your equilibrium, are factor prices equal across these two countries? If so, explain why; if not, explain why not.

5. Modify the Bertrand duopoly model to allow different marginal costs for the two firms. Find an equilibrium, and determine whether it is unique.

6. Consider an economy in which there are equal numbers of two kinds of workers, a and b, and two kinds of jobs, good and bad. Each employer has an unlimited number of vacancies in both kinds of jobs. Some workers are qualified for the good job, and some are not. If a qualified worker is assigned to the good job the employer gains \$2,000, and if an unqualified worker is assigned to the good job the employer loses \$14,000. When any worker is assigned to the bad job, the employer breaks even.
Workers who apply for jobs are tested and assigned to the good job if they do well on the test. Test scores range from 0 to 1. The probability that a qualified worker will have a test score less than t is t^2. The probability that an unqualified worker will have a test score less than t is $1-(1-t)^2$.
There is a fixed wage premium of \$42,000 attached to the good job. Workers can become qualified by paying an investment cost, and this cost is higher for some workers than for others: the distribution of costs is uniform between 0 and \$20,000. This distribution is the same for a-workers and b-workers. Workers make investment decisions so as to maximize earnings, net of the investment cost (all of these amounts are expressed as present values).

a. Can you find an equilibrium in which there are more a-workers than b-workers in the good jobs?
b. Now suppose that employers are required to assign the same proportion or a-workers and b-workers to the good jobs. If there is no change in the workers' investment behavior, what standards will the employers use?

1. Mailath [cf prelim_j98.wpd, sol97a-d.pdf]

(a) Taking prices as given, agents optimize: consumers maximize utility and firms maximize profits.
Prices are such that when demand is aggregated over all consumers, it matches the aggregate amount supplied by firms.
Here there are just two goods, so one relative price is needed.

Consumer i: max $u_i(x_i,G)$ st $x_i + pg_i \leq \omega_i + \Pi/n$, where the private good is numeraire, and p is the price of the public good, and Π is aggregate profits. Here G is aggregate output of the public good, even though i only pays for g_i units. That is, i gets $\Sigma_{k\neq i} g_k$ units of the public good for free, and considers whether to add to that.

Producer j: $\pi^j = \max pg^j - x^j$ st $(x^j,g^j) \in T$, where T is the production set.

$\Sigma_i x_i + \Sigma_j x^j = \Sigma_i \omega_i$

$G = \Sigma_i g_i = \Sigma_j g^j$

$\Pi = \Sigma_j \pi_j$

(b)

(i) The Walrasian price ratio has to be 2: if less, none would be produced, and if more, producers would make infinite profits.

(ii) Pareto efficiency means among other things that it is not possible to make everyone better off. But if G is increased by a small amount ϵ, and x_i is reduced by $2\varepsilon/n$ for each i so that some producer can provide the increased output of G, all consumers gain, because the MRS is 2, and the price is 2/n.

(iii) The First Welfare Theorem assumes selfish preferences.

2. [Dixit-Stiglitz]

[this question was used in the May 98 exam, but the answer in that file is apparently wrong – it gets $x_0 = 1/3$ instead of $x_0 = 2/3$.

a. The utility function is not the one in Dixit-Stiglitz: theirs is $u(x) = \log(x_0) + 2\log\left(\sum_{i=1}^{N} \sqrt{x_i}\right)$.

Here $u(x) = \log(x_0) + \log\left(\sqrt{y}\right) = \log(x_0) + \frac{1}{2}\log(y)$, where $\sqrt{y} = \sum_{i=1}^{N} \sqrt{x_i}$

This can also be written as $u(x) = \log(x_0) + \log(Y)$ where $Y = \sum_{i=1}^{N} \sqrt{x_i}$, but then the cost function for Y is not the standard CES computation, because Y is not a linearly homogeneous function of x.

Each product is priced so that $MR_i = p_i[1-1/\epsilon_i] = c$, and the quantity is such that profit is zero.

The demand elasticity is found by setting $MU_i/p_i = \lambda$, so $\frac{1}{2}(x_i)^{-\frac{1}{2}} = \lambda p_i$, which gives $\epsilon_i = 2$.

Then $p_i = 2c$, and $x_i = a/c$ for each i.

The price of the composite good y is given by $q^{-1} = \Sigma_i (p_i)^{-1} = np^{-1}$, and the quantity is $y = n^2x = n^2a/c$

Note that this is the price of y, NOT the price of $Y = y^2$.

So $q = (1/n)p = (1/n)2c$ and $qy = 2na$

Income is I = 1, and this is divided between x_0 and y, with $qy = ⅓ = npx$

But $px = a + cx = a + \frac{1}{2}px$ so $px = 2a$ and $⅓ = n2a$

This gives $\boxed{n = \frac{1}{6a}}$

b. This depends on what is meant by Pareto optimality. The outcome is not first-best, because if lump-sum subsidies can be used, the products should be priced at marginal cost, with the fixed cost a covered by lump-sum transfers.

The MRS between good 0 and good i is MU_0/MU_i, where $MU_i = \frac{1}{Y}\frac{1}{2\sqrt{x_i}} = \frac{1}{n\sqrt{x_i}}\frac{1}{2\sqrt{x_i}}$. Thus $MRS = \frac{2nx_i}{x_0}$. But $x_i = a/c$ and $x_0 = ⅔$ so $MRS = \frac{3na}{c} = \frac{1}{2}\frac{1}{c}$. On the other hand the MRT is just marginal cost, which is 1/c in this notation. Thus the outcome is not Pareto optimal.

The constrained optimum problem is to maximize u subject to a zero-profit condition. For example, taxes can be used to set prices above marginal cost.

As above, $q = (1/n)p$, and $y = n^2x$, so $qy = npx$, with $x = a/(p-c)$, so $qy = npa/(p-c) = ⅔$

The problem is to maximize $n\sqrt{x}$, subject to $x = a/(p-c)$ and $npa/(p-c) = \frac{1}{2}$

Put $z = \sqrt{p-c}$ and maximize: the result is $p = 2c$, as in the monopolistic competition result.

3.

4. There is no equilibrium in which A specializes in good 2 with {w = 4/81, v = 625/81}. The cost function for good 2 is sqrt(c) = sqrt(v)+sqrt(w), and this gives $c = 9 = p_2$. Also, the capital-labor ratio in good 2 satisfies k_2 = sqrt(w)/(sqrt(v)) = 2/25, and this matches the endowment: $K_A = 4$ and $L_A = 50$. Finally, the cost function for good 1 is 1/c = 1/v+1/w, and this gives $c_1 = 2500/50949 < 4/5$, so it would be profitable to produce good 1.

The ranking of the factor intensities depends on whether v/w is above or below 1: thus the assumptions of the factor price equalization theorem don't hold.

Solving the zero-profit equations sqrt(9) = sqrt(v)+sqrt(w) and 5/4 = 1/v+1/w for (v,w) involves solving a quartic. The positive roots are (1,4) and (4,1), but it is not clear how to find them. The equations are symmetric, which might help.

The easiest way to do this is to plot the two curves.

5. In the standard Bertrand model, consumers buy equal amounts from both firms if the prices are equal. In that case, there is no equilibrium when the marginal costs are different.

Suppose instead that all consumers buy from the low-cost firm unless the other firm offers a strictly lower price. Then any price between c_1 and c_2 is an equilibrium, with both firms offering the same price. The high-cost firm makes zero profit, and there is no deviation that beats that. The low-cost firm has the whole market, but would have zero profit if the price was a bit higher.

6. The fair bet condition is p/(1-p) = L/W. Here $\frac{p}{1-p} = \frac{\pi}{1-\pi}\frac{t}{1-t}$, so $\frac{\pi}{1-\pi}\frac{t}{1-t} = 7$

The investment decision is based on $G(\omega[F_u(s) - F_q(s)]) = \pi$, so $\frac{42}{20}2t(1-t) = \pi$

This gives s = 1/3 with π = 14/15 or s = 5/6 with π = 7/12

a. This gives s = 1/3 with π = 14/15 or s = 5/6 with π = 7/12

b. If π_a = 14/15 and π_b = 7/12, the solution is {s[a] = .8036591838, s[b] = .7017872169}. There is no way to get this analytically (without solving a quartic equation).