## **Rate of Return**

If the interest rate is r per period, then \$1 now becomes (1+r) after 1 period, and after one more period,  $(1+r)^2$ , and more generally,  $(1+r)^t$  after t periods. This means that \$1 next period is worth  $\frac{1}{1+r}$  now, and the present value of \$1 t periods from now is  $\frac{1}{(1+r)^t}$ .

An investment project involves cost,  $C_t$  and returns,  $Y_t$ , in period t, for  $t = 0, 1, 2 \dots T$  (where T might be infinite, with the interpretation that the project might last for a very long time, and it is convenient to act as if it lasts forever). Generally the costs are larger than the returns in the early periods, and the returns are larger in later periods.

#### **Present Value of Costs**

$$PV(C) = \sum_{t=0}^{T} \frac{C_t}{(1+r)^t}$$

This represents a sum of money that would be equivalent to the stream of costs if the interest rate is r, meaning that if PV(C) is put into a bank account that pays interest at the rate r, the money in the account would be just sufficient to pay all of the costs as they come due, with nothing left over at the end.

### **Present Value of Returns**

$$PV(R) = \sum_{t=0}^{T} \frac{R_t}{(1+r)^t}$$

As with the present value of costs, PV(R) is equivalent to the stream of returns.

If PV(R) > PV(C), then the investment is profitable. For example, if PV(C) is borrowed at the interest rate r, and put in an account that is used to pay the costs as they come due, and if the returns are put into this account as they are realized, then the account will have a positive balance after T periods.

### **Internal Rate of Return**

$$\sum_{t=0}^{T} \frac{R_t - C_t}{(1+\rho)^t} = 0$$

Typically, costs are higher in the beginning, and returns are higher in later periods. In this case PV(R,r) is more sensitive to the interest rate than PV(C,r). Then NPV is positive if  $\rho > r$  and negative if  $\rho < r$ .

# **Two-Period Consumption Choices**

 $\delta$  time preference

$$\max_{C_{1},C_{2}} u(C_{1}) + \frac{1}{1+\delta} u(C_{2})$$

The (intertemporal) budget constraint is

$$C_1 + \frac{1}{1+r}C_2 = W$$

Equating marginal utility per dollar gives

$$u'(C_1) = \frac{1+r}{1+\delta}u'(C_2)$$

So if the interest rate is higher than the time preference rate, then marginal utility now is higher than marginal utility later, and that means consumption is lower now.

The market interest rate reflects the time preference rates of other consumers, and also the marginal product of capital.

### **Real and Nominal Interest Rates**

$$1 + i = (1 + r)(1 + \pi)$$

\$1 in period 0 buys  $\frac{1}{p_0}$  units of the consumption good.

\$1 in period 1 buys  $\frac{1}{p_1}$  units of the consumption good.

\$1 in period 0 buys (1 + i) in period 1, and this buys  $\frac{1+i}{p_1}$  units of the consumption good.

So  $\frac{1}{p_0}$  units of the consumption good in period 0 buys  $\frac{1+i}{p_1}$  units of the consumption good in period

1

The inflation rate  $\pi$  is defined by

$$1 + \pi = \frac{p_1}{p_0}$$

So  $\frac{1}{p_0}$  units of the consumption good in period 0 buys  $\frac{1+i}{p_0(1+\pi)}$  units of the consumption good in period 1

And 1 unit of the consumption good in period 0 buys  $\frac{1+i}{1+\pi}$  units of the consumption good in period 1

Thus the rate of interest, measured in terms of real goods, is given by

$$1+r = \frac{1+i}{1+\pi}$$

Then

$$(1+r)(1+\pi) = (1+i)$$

and

$$1+r+\pi+r\pi=1+i$$

 $\mathbf{S0}$ 

and normally  $r\pi$  is small enough to be ignored, so the real interest rate is approximately the nominal rate minus the inflation rate.

 $r \approx i - \pi$ 

If the cost stream is just  $C_0$ , and the revenue stream is just  $R_1$ , then the rate of return is given by

 $C_0 = \frac{R_1}{1+\rho}$ 

 $\mathbf{S0}$ 

If  $R_1$  is in nominal dollars, the nominal rate of return is

$$1+i = \frac{R_1}{C_0}$$

$$1 + \rho = \frac{R_1}{C_0}$$

 $r = i - \pi - r\pi$ 

Converting  $R_1$  to real dollars gives the real rate of return

$$1 + r = \frac{1}{1 + \pi} \frac{R_1}{C_0}$$