Economics 750
Assignment 1
Due: September 18, 2023 (Canvas [preferred] or email; pdf or TeX or LyX files only)

1. Housing in San Francisco is much more expensive than in Madison. Suppose wages and the prices of all consumer goods (other than housing) are the same in the two places, and compare two workers with identical preferences and equal nonlabor incomes, one living in each place. Analyze the following assertion:
   "After paying for housing the San Francisco worker has less money to spend on leisure and other goods than the Madison worker, but both face the same relative prices for all other goods. So, aside from housing, the comparison boils down to the effect of a parallel shift in the budget constraint. This means that the San Francisco worker will supply more labor than the Madison worker."
   Is the above statement generally true? If so, prove it. If not, can you find a restriction on the consumers' preferences which would make the statement true?

2. Suppose a worker chooses consumption, $C$, and leisure, $\ell$, to maximize the utility function

$$U(C,\ell) = \alpha\frac{(C-c_0)^{\rho}-1}{\rho} + \frac{(\ell-\ell_0)^{\tau}-1}{\tau}$$

   subject to the budget constraint $C = w(T-\ell)+\mu$, where $T$ is the time endowment, $w$ is the real wage and $\mu$ is real nonlabor income. It is assumed that $\alpha > 0$, $\rho < 1$, $\tau < 1$, $\mu > c_0$ and $T > \ell_0$.

   (a) For which wages does the worker supply positive hours?
   (b) Suppose $\rho = 0$. Derive the Marshallian labor supply curve (the supply price of labor as a function of hours worked).
   (c) Suppose (instead) that $\rho = -2$, with $\tau = \frac{1}{2}$, $\alpha = 6$, $\ell_0 = 96$, $T = 168$, $\mu = 0$ and $c_0 = 50$. Plot the labor supply curve, and comment on whether it has any chance of fitting long-run trends in real wages and hours worked.

3. Consider a two-person household $\{a,b\}$ that maximizes the following utility function:

$$U = \alpha\frac{(C-c_0)^{\rho}-1}{\rho} + (1-\alpha)\frac{(\ell-\ell_0)^{\tau}-1}{\tau},\ \rho<1, \tau<1, c_0\geq 0, \ell_0 \geq 0, \alpha\in(0,1)$$

   where $C$ is household consumption, and $\ell$ is a leisure composite defined by

$$\ell^{\kappa} = \beta\ell_a^{\kappa} + (1-\beta)\,\ell_b^{\kappa}\ \kappa \leq 1, \beta\in(0,1)$$

   The household's budget constraint is

$$C \leq \mu + w_a H_a + w_b H_b$$

   where $w_a$ is $a$'s market wage and $H_a$ is hours worked, and similarly for $b$, and $\mu$ is nonlabor income.

   (a) Suppose $\beta = \frac{1}{2}$ (so that the leisure of each household member is treated equally). Is it then true that the person with the higher wage supplies more labor to the market?
   (b) Can you find conditions such that nonparticipation is optimal (i.e. $H = 0$) for one of the household members?