

restart

$$Demand[1] := p[1] = \theta_1 \cdot (1 - q[1])$$

$$Demand_1 := p_1 = \theta_1 (1 - q_1) \quad (1)$$

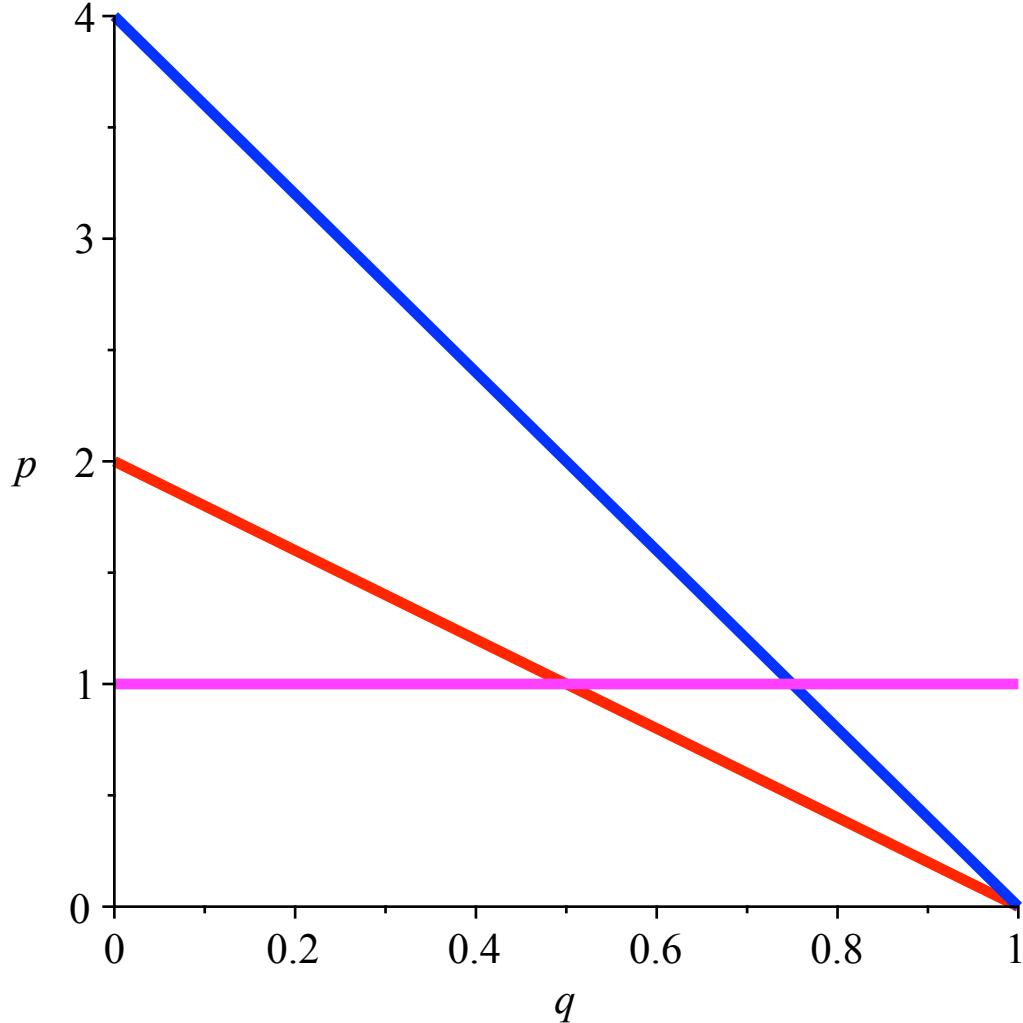
$$Demand[2] := p[2] = \theta_2 \cdot (1 - q[2])$$

$$Demand_2 := p_2 = \theta_2 (1 - q_2) \quad (2)$$

$$xmp := \{\theta_1 = 2, \theta_2 = 4, c = 1, N = 32\}$$

$$xmp := \{N = 32, c = 1, \theta_1 = 2, \theta_2 = 4\} \quad (3)$$

$$\begin{aligned} plots[implicitplot]\left([subs(p[1]=p, q[1]=q, xmp, Demand[1]), subs(p[2]=p, q[2]=q, xmp, \right. \\ \left. Demand[2]), subs(xmp, p=c)], q = 0 .. 1, p = 0 .. subs(xmp, \max(\theta_1, \theta_2)), color = [red, blue, \right. \\ \left. magenta], thickness = 4 \right) \end{aligned}$$



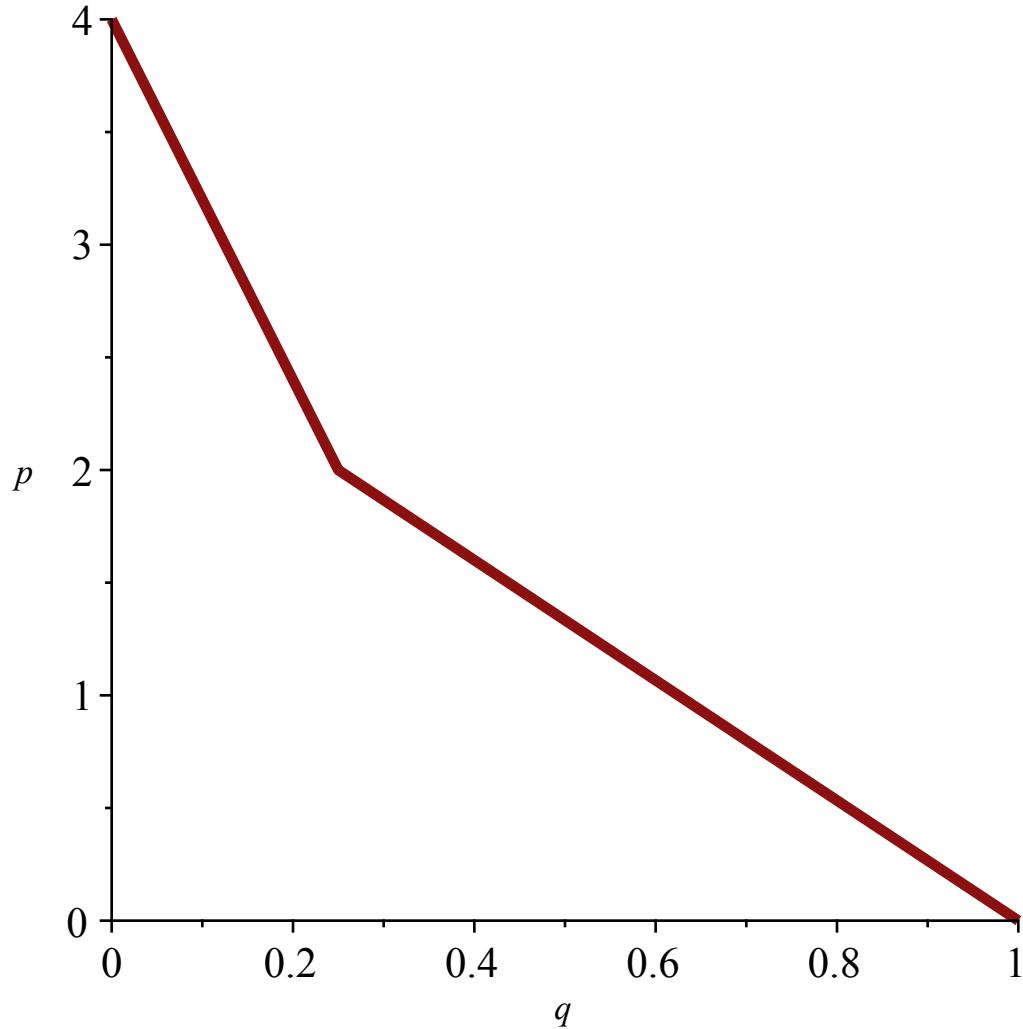
$$plots[implicitplot]\left(q = \frac{1}{2} \cdot subs(xmp, \max(solve(subs(p[1]=p, Demand[1]), q[1])), 0) \right)$$

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+ solve(subs(p[2]=p, Demand[2]), q[2])), q=0..1, p=0..subs(xmp, max(theta_1, theta_2)),  

thickness = 4, numpoints = 10000)

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Consumer Surplus

$$CS[1] := \text{subs}\left(p = p[1], \frac{(\theta_1 - p) \cdot \text{solve}(\text{eval}(\text{subs}(p[1]=p, \text{Demand}[1])), q[1])}{2}\right)$$

$$CS_1 := -\frac{(\theta_1 - p_1)(-\theta_1 + p_1)}{2\theta_1} \quad (4)$$

$$CS[2] := \text{subs}(p[1]=p[2], \theta[1]=\theta[2], CS[1])$$

$$CS_2 := -\frac{(\theta_2 - p_2)(-\theta_2 + p_2)}{2\theta_2} \quad (5)$$

$$cs := \text{unapply}(CS[2], \theta[2], p[2])$$

$$cs := (\theta_2, p_2) \mapsto -\frac{(\theta_2 - p_2)(-\theta_2 + p_2)}{2\theta_2} \quad (6)$$

$$\begin{aligned} pi &:= N \cdot \left(cs(\theta[1], p) + \frac{1}{2} \cdot (p - c) \cdot (solve(subs(p[1]=p, Demand[1]), q[1])) \right. \\ &\quad \left. + solve(subs(p[2]=p, Demand[2]), q[2])) \right) \\ \pi &:= N \left(-\frac{(\theta_1 - p)(-\theta_1 + p)}{2\theta_1} + \frac{(p - c) \left(-\frac{-\theta_1 + p}{\theta_1} - \frac{-\theta_2 + p}{\theta_2} \right)}{2} \right) \end{aligned} \quad (7)$$

$$diff(pi, p) = N \left(-\frac{\theta_1 - p}{2\theta_1} - \frac{-\theta_2 + p}{2\theta_2} + \frac{(p - c) \left(-\frac{1}{\theta_1} - \frac{1}{\theta_2} \right)}{2} \right) \quad (8)$$

$$pstar := solve(\%, \{p\})$$

$$pstar := \left\{ p = \frac{c(\theta_2 + \theta_1)}{2\theta_1} \right\} \quad (9)$$

$$eval(subs(xmp, pstar))$$

$$\left\{ p = \frac{3}{2} \right\} \quad (10)$$

$$eval(subs(\%, xmp, pi))$$

$$9 \quad (11)$$

This is the maximal profit, using a two-part tariff

$$eval(subs(p = c, xmp, pi))$$

$$8 \quad (12)$$

This is the profit using a two-part tariff when the price is just marginal cost

$$\begin{aligned} pi_m &:= N \cdot \left(\frac{1}{2} \cdot (p - c) \cdot (solve(subs(p[1]=p, Demand[1]), q[1])) + solve(subs(p[2]=p, \right. \\ &\quad \left. Demand[2]), q[2])) \right) \\ pi_m &:= \frac{N(p - c) \left(-\frac{-\theta_1 + p}{\theta_1} - \frac{-\theta_2 + p}{\theta_2} \right)}{2} \end{aligned} \quad (13)$$

$$diff(pi_m, p) = \frac{N \left(-\frac{-\theta_1 + p}{\theta_1} - \frac{-\theta_2 + p}{\theta_2} \right)}{2} + \frac{N(p - c) \left(-\frac{1}{\theta_1} - \frac{1}{\theta_2} \right)}{2} \quad (14)$$

$$pstarm := solve(\%, \{p\})$$

$$pstarm := \left\{ p = \frac{c\theta_1 + c\theta_2 + 2\theta_1\theta_2}{2(\theta_2 + \theta_1)} \right\} \quad (15)$$

$$collect(\%, c, factor)$$

$$\left\{ p = \frac{c}{2} + \frac{\theta_1\theta_2}{\theta_2 + \theta_1} \right\} \quad (16)$$

$$eval(subs(xmp, \%))$$

$$\left\{ p = \frac{11}{6} \right\} \quad (17)$$

$$eval(subs(\%, xmp, pi_m))$$

$$\frac{25}{3} \quad (18)$$

This is the standard monopoly profit: no entry fee, MR=MC