Appendix A: Value Functions

The solution given in the text for the vector of value functions can be summarized as

$$V(a) = \begin{cases} \Omega_2^{-1} Y_2 & K^g \le a \\ \Omega_1^{-1} Y_1 + e^{-\Omega_1 (K^g - a)} \left[V(K^g) - \Omega_1^{-1} Y_1 \right] & K^b \le a \le K^g \\ \Omega_0^{-1} Y_0 + e^{-\Omega_0 (K^b - a)} \left[V(K^b) - \Omega_0^{-1} Y_0 \right] & t_0 \le a \le K^b \\ e^{-\Omega_{00}(t_0 - a)} \left[V(t_0) - P(t_0) \right] + \left(I - e^{-\Omega_{00}(t_0 - a)} \right) \Omega_{00}^{-1} Y_{00} & 0 \le a < t_0 \end{cases}$$

The components of this expression are defined as follows

$$\Omega_2 = \begin{bmatrix} r+\lambda_L+\lambda_b+\lambda_0+\delta^b & -\lambda_b & -\lambda_L & 0 & 0 & 0 & 0 & 0 \\ -\lambda_g & r+\lambda_L+\lambda_g+\lambda_0+\delta^g & 0 & -\lambda_L & 0 & 0 & 0 & 0 \\ -\lambda_H & 0 & r+\lambda_H+\lambda_b+\lambda_0+\delta^b & -\lambda_b & 0 & 0 & 0 & 0 \\ 0 & -\lambda_H & -\lambda_g & r+\lambda_H+\lambda_g+\lambda_0+\delta^g & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & r+\lambda_L+\lambda_b+\lambda_0+\delta^b & -\lambda_b & -\lambda_L & 0 \\ 0 & 0 & 0 & 0 & -\lambda_g & r+\lambda_L+\lambda_g+\lambda_0+\delta^g & 0 & -\lambda_L \\ 0 & 0 & -\lambda_0 & 0 & -\lambda_H & 0 & r+\lambda_H+\lambda_b+\delta^b & -\lambda_b \\ 0 & 0 & 0 & -\lambda_0 & 0 & -\lambda_H & -\lambda_g & r+\lambda_H+\lambda_g+\delta^g \end{bmatrix}$$

$$\Omega_1 = \begin{bmatrix} r+\lambda_L+\lambda_b+\lambda_0+\delta^b & -\lambda_b & -\lambda_L & 0 & 0 & 0 & 0 & 0 \\ -\lambda_g & r+\lambda_L+\lambda_g+\lambda_0+\delta^g & 0 & -\lambda_L & 0 & 0 & 0 & 0 \\ -\lambda_H & 0 & r+\lambda_H+\lambda_b+\lambda_0+\delta^b & -\lambda_b & 0 & 0 & 0 & 0 \\ 0 & \lambda_0-\lambda_H & -\lambda_g & r+\lambda_H+\lambda_g+\delta^g & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & r+\lambda_L+\lambda_b+\lambda_0+\delta^b & -\lambda_b & -\lambda_L & 0 \\ 0 & -\lambda_0 & 0 & 0 & -\lambda_g & r+\lambda_L+\lambda_g+\delta^g & 0 & -\lambda_L \\ 0 & 0 & 0 & 0 & -\lambda_H & 0 & r+\lambda_H+\lambda_b+\lambda_0+\delta^b & -\lambda_b \\ 0 & -\lambda_0 & 0 & 0 & 0 & -\lambda_H & -\lambda_g & r+\lambda_H+\lambda_g+\delta^g \end{bmatrix}$$

$$\Omega_0 = \begin{bmatrix} r+\lambda_L+\lambda_b+\lambda_0+\delta^b & -\lambda_b & -\lambda_L & 0 & 0 & 0 & 0 & 0 \\ -\lambda_g & r+\lambda_L+\lambda_g+\lambda_0+\delta^g & 0 & -\lambda_L & 0 & 0 & 0 & 0 \\ \lambda_0-\lambda_H & 0 & r+\lambda_H+\lambda_b+\delta^b & -\lambda_b & 0 & 0 & 0 & 0 \\ 0 & \lambda_0-\lambda_H & -\lambda_g & r+\lambda_H+\lambda_g+\delta^g & 0 & 0 & 0 & 0 \\ -\lambda_0 & 0 & 0 & 0 & r+\lambda_L+\lambda_b+\delta^b & -\lambda_b & -\lambda_L & 0 \\ 0 & -\lambda_0 & 0 & 0 & -\lambda_g & r+\lambda_L+\lambda_g+\delta^g & 0 & -\lambda_L \\ -\lambda_0 & 0 & 0 & 0 & -\lambda_H & 0 & r+\lambda_H+\lambda_b+\delta^b & -\lambda_b \\ 0 & -\lambda_0 & 0 & 0 & 0 & -\lambda_H & -\lambda_g & r+\lambda_H+\lambda_g+\delta^g \end{bmatrix}$$

$$\Omega_{00} = \begin{bmatrix} r+\lambda_L+\lambda_b+\delta^b & -\lambda_b & -\lambda_L & 0 & 0 & 0 & 0 & 0 \\ -\lambda_g & r+\lambda_L+\lambda_g+\delta^g & 0 & -\lambda_L & 0 & 0 & 0 & 0 \\ -\lambda_H & 0 & r+\lambda_H+\lambda_b+\delta^b & -\lambda_b & 0 & 0 & 0 & 0 \\ 0 & -\lambda_H & -\lambda_g & r+\lambda_H+\lambda_g+\delta^g & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & r+\lambda_L+\lambda_b+\delta^b & -\lambda_b & -\lambda_L & 0 \\ 0 & 0 & 0 & 0 & -\lambda_g & r+\lambda_L+\lambda_g+\delta^g & 0 & -\lambda_L \\ 0 & 0 & 0 & 0 & -\lambda_H & 0 & r+\lambda_H+\lambda_b+\delta^b & -\lambda_b \\ 0 & 0 & 0 & 0 & 0 & -\lambda_H & -\lambda_g & r+\lambda_H+\lambda_g+\delta^g \end{bmatrix}$$

$$Y_2 = \begin{bmatrix} y_L^b + \lambda_0 V_L^b(0) \\ y_L^g + \lambda_0 V_L^g(0) \\ y_H^b + \lambda_0 V_H^b(0) \\ y_H^g + \lambda_0 V_L^g(0) \\ \delta^b U_0^b + \lambda_0 U_L^b(0) \\ \delta^g U_0^g + \lambda_0 U_L^g(0) \\ \delta^b U_0^b - \lambda_0 V_H^b(0) \\ \delta^g U_0^g - \lambda_0 V_H^g(0) \end{bmatrix}, \quad Y_1 = \begin{bmatrix} y_L^b + \lambda_0 V_L^b(0) \\ y_L^g + \lambda_0 V_L^g(0) \\ y_H^b + \lambda_0 V_H^b(0) \\ y_H^g + \lambda_0 V_L^g(0) \\ \delta^b U_0^b + \lambda_0 U_L^b(0) \\ \delta^g U_0^g - \lambda_0 V_L^g(0) \\ \delta^b U_0^b + \lambda_0 \left[V_H^b(K^g) - V_H^b(0) + U_H^b(K^g) \right] \\ \delta^g U_0^g - \lambda_0 V_L^g(0) \end{bmatrix}, \quad Y_0 = \begin{bmatrix} y_L^b + \lambda_0 V_L^b(0) \\ y_L^g + \lambda_0 V_L^g(0) \\ y_H^b + \lambda_0 V_L^b(0) \\ y_H^g + \lambda_0 V_L^g(0) \\ \delta^b U_0^b - \lambda_0 V_L^b(0) \\ \delta^g U_0^g - \lambda_0 V_L^g(0) \\ \delta^b U_0^b - \lambda_0 V_L^b(0) \\ \delta^g U_0^g - \lambda_0 V_L^g(0) \end{bmatrix}$$

$$Y_{00}' = \begin{bmatrix} 0 & 0 & 0 & 0 & \delta^b U_0^b & \delta^g U_0^g & \delta^b U_0^b & \delta^g U_0^g \end{bmatrix}$$