Write clear, fully explained answers -- do not just assume the examiner will know what you mean. If you need to make additional assumptions, do so, but state them clearly.

Answer FOUR questions. (There are SIX questions.)

1. Consider an economy with n agents and two goods: a private good, x and a public good, g. Consumer i has an endowment of $?_i$ units of the private good, and there is a technology that transforms the private good into the public good.

   a. Suppose there are m firms that have access to the public good technology, and each consumer owns equal shares of each firm. How would you define a "Walrasian" (competitive) equilibrium for this two-good economy?

   b. Now suppose the public good technology has constant returns to scale, at a rate of two units of the private good per unit of the public good.

      i. What is the Walrasian equilibrium price ratio?
      ii. Are the Walrasian allocations Pareto efficient? Explain.
      iii. Relate your answer to the First Welfare Theorem.

2. Environmental economists looking at cross-national data have noticed that both very poor and very rich countries have the least pollution, while countries with modest development have the most pollution. Plotting per capita pollution against per capita national income they find an inverted-U relationship, that is, pollution rises with income at first, then decreases with income. This has been called the "environmental Kuznets curve."

   Let's explore the environmental Kuznets curve in an economy with one representative person, Robinson. Robinson cares about consumption, C, and about the pollution his consumption creates, P. Hence U=U(C,P), where $\partial U/\partial C > 0$ and $\partial U/\partial P < 0$. Suppose pollution is created from the act of consuming, but it can also be abated by employing environmental effort, E. Hence, suppose pollution is generated by the technology $P = C - C^a E^b$, where $A(C,E) = C^a E^b$ is the abatement technology. Suppose Robinson has a finite amount of resources M that can be devoted to either consumption or abatement: M = C+E.

   Suppose Robinson's Utility function is U(C,P)=C-P.

   a. Find Robinson's demand for pollution as a function of M.
   b. What must be true about the abatement technology (i.e. a and b) in order to observe an inverted-U relation between P and M?
   c. Interpret your finding in (b).

3. Let p denote price per unit and q denote number of units and let the aggregate demand curve for oil for each of two periods be $p = D(q) = Aq^{-a}$; $0 < a < 4$, $A > 0$. Let extraction cost be c dollars per unit and assume extraction cost is constant. Also there is a setup cost, i.e. a fixed cost, of F which must be borne to extract anything at all.

   a. Let a monopoly control the total reserve of oil of $E > 0$ units. The revenue function of the monopoly is $R(q) = Aq^{1-a}$. If there are only two periods in which to extract the oil before the government takes over the reserve, how should the monopoly extract the reserve in order to maximize present value of profits over the two periods assuming the discount factor on the second period profits is b? For simplicity assume that profits are zero after the government takes over the reserve and b = 1. Hence from the point of view of the monopolist, the government destroys anything left over after two periods.

      i. Are there any restrictions you must impose on the demand curve in order that a finite solution exist for the monopolist's maximization problem?
      ii. What restrictions must you impose on the vector (E,F,A,a,c) so it is optimal to extract a positive amount of oil each period?
      iii. Is there any oil left in the ground when the government takes over?
      iv. How does the answer change if demand is strengthened by increasing A and decreasing a in the demand curve? How does the answer change when you replace "two periods" by "N periods" for positive integers N? How much oil does the monopolist extract each period if he extracts any at all?
      Place a revenue tax of T per dollar revenue in each period. How do the answers change for the two period case?

   b. Let now a social planner take over this oil field but let all other conditions on demand and cost be the same. Assume that the world ends in two periods and any oil left in the ground after two periods is worthless. Assume taxes are zero. The social planner decides whether to pay the fixed cost F to open the reserve and optimizes the sum of net benefits (the discount is b=1 as above) over the two periods. Here surplus is given by

   $$\int_0^q D(x)dx - cq$$

   For the two period case, compare the social planner's solution with the monopolist solution. What is the difference?

4. Consider production of two products with separable demands $D_1(q_1)$, $D_2(q_2)$ with total cost function $C(q_1,q_2) = F + c_1q_1 + c_2q_2$ where c_i is constant marginal cost of product i = 1,2 and F > 0 is fixed cost which must be borne if a positive amount of either good is produced. Define consumer benefit, $B_i(q_i)$ to be the area under the demand curve from zero to q_i. Let $R_i(q_i) = D_i(q_i)q_i$ denote revenue from sales of product i. Examine three problems:

(1) (Social Welfare Maximum)	Max $B_1(q_1) + B_2(q_2)-C(q_1,q_2)$
(2) (Second Best)	Max $B_1(q_1) + B_2(q_2) - C(q_1,q_2)$, s.t. $R_1(q_1) + R_2(q_2) > C(q_1,q_2)$
(3) (Monopoly)	Max $R_1(q_1) + R_2(q_2) - C(q_1,q_2)$.

a. Use the Kuhn-Tucker Theorem to write out the first order necessary conditions for all three problems. Is fixed cost covered by the solution to (1)? Why or why not?

b. "Ramsey" numbers for product i=1,2 are defined by $r_i = \dfrac{p_i - c_i}{p_i} e_i$, where ei is elasticity of demand. Compare Ramsey numbers for problems (1)-(3), assuming interior solutions for all products in all three cases.

c. Assume linear demands $D_i(q_i) = A_i - M_i q_i$. Locate sufficient conditions for shut down to be socially optimal. Can there be a case where shut down is not socially optimal but it is Second Best optimal to shut down? How can you "fix" this social problem if it is possible?

d. Assume linear demands as in (c). Draw a pair of diagrams side by side with the demand curve for each product and the constant marginal cost line on each. On each demand curve tick off the vertical intercept, call it A_i, tick off the price, call it p_i, and tick off the marginal cost, call it c_i. Show for problem (2), for interior optima, that the ratio of the line segment, $A_i - p_i$, to the line segment, $A_i - c_i$, is equated across the two goods i=1,2.

5. There are two firms, 1 and 2. Firm 1 is an incumbent and firm 2 is a potential entrant. Firm 1 moves first by deciding whether or not to erect an entry barrier. Next, firm 2 decides whether to enter. The payoff consequences of four possible events are described in the following payoff matrix.

Firm 2 / **Firm 1**	**No Entry**	**Entry**
Barrier	3, 0	0, -1
No Barrier	4, 0	2, 2

(Note that this matrix is not a normal form representation of the game, which is a sequential move game.)

a. (1 point) Find a subgame perfect equilibrium, assuming that firm 2 perfectly observes firm 1's action.

b. (2 points) Find a subgame perfect equilibrium, assuming that firm 2 cannot observe firm 1's action.

For the next two questions, assume that firm 2 does not observe firm 1's action but that it observes a signal, s 0 {b,n\}, about 1's action. The signal is informative in that
Prob {s=b | 1 picks "Barrier"} = Prob{s=n| 1 picks "No Barrier"} = s 0 (½,1).

c. (3 points) Find a *pure strategy* perfect Bayesian equilibrium of this game. Does this equilibrium approach the one found in (a), as s approaches 1?

d. (4 points) Find a *mixed strategy* perfect Bayesian equilibrium of this game. Does this equilibrium approach the one found in (a), as s approaches 1? (Hint: Look for an equilibrium in which 1 randomizes in its action; and firm 2 randomizes in its action only when observing n.)

6. There are two individuals, 1 and 2, involved in a team production. When the individuals choose efforts e_1 and e_2 respectively, their joint team revenue (expressed in the monetary unit) is $e_1 + e_2 + e_1e_2$. Individual i (=1,2) has utility $U(w_i,e_i)$ ' w_i & $\frac{e_i^2}{?_i}$ when he receives (monetary) income of w_i and makes effort e_i \$ 0. For questions (a) through (c), assume that $?_1 = ?_2 = 1$.

a. (2 points) Prove that a Pareto efficient effort pair maximizes $e_1 + e_2 + e_1e_2 - e_1^2 - e_2^2$. Compute the Pareto efficient pair.

b. (1 point) Suppose that the individuals choose their efforts simultaneously and that they split the team revenue equally after making their efforts. Find the Nash equilibrium effort pair.

c. (3 points) Now the game described in (b) is repeated infinitely many times. Assume that both individuals have a common discount factor, d 0 (0,1). How high should d be for the Pareto efficient pair to be sustained in a subgame perfect equilibrium? Construct the subgame perfect equilibrium strategy profile and justify it.

d. (4 points) Consider again the one shot problem described in (b). Now, before making an effort, e_i, individual i draws $?_i$ uniformly from (0,1]. $?_i$ is private information to individual i, while individual j … i only knows its distribution. (As before, the individuals make their efforts simultaneously and split their team revenue.) Find the symmetric Bayesian Nash equilibrium of this game.