John Kennan **Microeconomics Questions** March 1998

1. Suppose the market supply and demand curves for wheat are as follows (prices are in dollars, quantities in millions of bushels):

$$P^S = .02\ Q^S\ ,\ \ P^D = 3 - .01\ Q^D$$

The government is considering two possible price support policies, A or B.

A: The government buys enough wheat so that a market price of \$2.20 is maintained. Wheat bought by the government is stored, destroyed, or given away abroad.
How much wheat does the government buy, how much is domestically consumed, and what is the cost to the government of this policy?

B: The government subsidizes wheat by \$x per bushel and buys no wheat itself. Calculate the subsidy needed if farmers are to receive \$2.20 per bushel in the new equilibrium. Under this policy how much wheat will consumers buy? How much will the government have to pay out?

2. An inventor has discovered a new method of producing a precious stone, using spring water found only in Venice and Tipton. The process is patented and manufacturing plants are set up in both places. The product is sold only in Europe and America. Trade laws are such that the price must be uniform within Europe and America, but the European and American prices may differ. Transport costs are negligible, and there is no second-hand market in the stones because of the risk of forgeries. From the production and marketing data given below, determine the profit-maximizing production and sales plans. In particular, determine the output in Venice and Tipton, sales in America and Europe, quantity shipped from America to Europe or vice versa, and prices in America and Europe.

Demand: America, $p = 1500 - \frac{1}{2}Q$; Europe, $p = 1000 - Q$

Average Cost: Tipton, $AC = 150 + .375\ Q$; Venice, $AC = 100 + \frac{1}{2}Q$

3. Beef and cowhides are produced in fixed proportions (say 1 ton of beef per cowhide), but sold separately. Suppose that the beef/cowhide industry is perfectly competitive, with identical firms and perfectly elastic input supplies. Let the minimum average cost of producing hides and beef be \$2,000 for a one-ton animal. Suppose the market demand curves are:

$$P_B = 2200 - 1000\ Q_B\ ,\ P_H = 600 - 1000\ Q_H$$

where Q_B is measured in millions of tons and Q_H in millions of hides.

(a) What will be the equilibrium price of beef and cowhides?
(b) If the demand for beef rises because of rising consumers' income, while the demand for cowhides remains unchanged, will this cause a permanent excess supply of cowhides? Explain.

4. The Phoenix Moons, a pro football team, have a stadium which seats 30,000 people. All seats are identical. The optimal ticket price is \$5, yet this results in an average attendance of only 20,000.

(a) Explain how it can be profitable to leave 10,000 seats empty.
(b) Next week the Moons play the Tucson Turkeys, who have offered to buy an unlimited number of tickets at \$4 each, to be resold only in Tucson. How many tickets should be sold to Tucson to maximize profits -- 10,000? more than 10,000?
(c) Given your answer to (b), what price should the Moons charge their own fans -- \$4? \$5? more than \$5?

5. The long-run total cost curve of a typical firm in a constant cost competitive industry is given by the formula:

$$C = 1200\,Q - 60\,Q^2 + Q^3$$

The industry demand curve is:

$$P^D = 375 - .025\,Q^D$$

An entrepreneur buys each firm in the industry, and then acts as a monopolist. Suppose the monopolist can operate each firm's plant at exactly the same cost as before, or else close down some plants and produce less output. (The monopolist can effectively prevent entry by other firms.)

(a) How many plants should be closed down, to maximize profit?
(b) Compare the monopoly price with the price in the original competitive equilibrium.

6. When people eat dinner together at a restaurant they might either agree that each person will pay his own bill (plan A), or else that each will pay an equal share of the total bill (plan B). Suppose three people with identical preferences and incomes eat at a restaurant where the price of food is $6 a pound.

(a) Draw diagrams showing the budget constraints implied by plans A and B, from the point of view of one person.
(b) Suppose each person is selfish (i.e., interested only in his own utility, not anyone else's). Use indifference curve analysis to determine whether more food will be consumed under plan A or plan B.

7. Suppose the phone company charges $20 a month for basic service, which includes 100 free calls per month. After the first 100 calls, each call costs 10 cents. Fred Bloggs makes 200 calls per month in this situation.

(a) Now suppose a new phone company offers a different payment plan, charging 15 cents for each call, with no monthly service charge, and no free calls.
Will Fred change his phone company? If he decides to change, or if he is forced to change, will he make more phone calls or less?
(b) The new phone company now raises its price to 17 cents per call. Is it obvious that Fred will choose the old phone company in this situation?

8. A firm has a monopoly in selling baseballs to major league teams. The demand curve for baseballs is $P = 10 - .1Q$, and the total cost curve is $C = 2Q + .1Q^2$.

(a) Find the profit-maximizing price and quantity.
(b) The baseball commissioner decides to regulate the price of baseballs, arguing that price is excessive and that there should be more baseballs available. A price ceiling is therefore introduced. Will the price and quantity move in the direction desired by the commissioner?

9. Consider a primitive economy in which there are two kinds of jobs, hunting and fishing. Suppose the marginal product of hunters is given by the equation $h = 200 - H$, where H is the total number of hunters in the economy, and h is their marginal product, measured in pounds of food. The marginal product of fishermen is given by the equation $f = 200 - F$, where F is the total number of fishermen in the economy, and f is their marginal product, also measured in pounds of food.

There are 210 workers in the economy, 147 whites and 63 blacks, all equally productive in both fishing and hunting. All workers prefer fishing to hunting, but preferences differ across individuals: the distribution of equalizing differences ranges evenly from 0 to 21. For example, if fishermen earned 2 pounds less than hunters, 20 people (2/21 of the total) would choose hunting, including 14 whites and 6 blacks.

(a) If the economy is competitively organized, how many people will be fishermen, and how many will be hunters? What will the wage differential be, in equilibrium?

(b) Now suppose that blacks are not allowed to hunt. How will this affect the equilibrium? What will happen to the average wages of whites and blacks?

10. The salaries of professional baseball players have risen sharply since the reserve clause was abolished in 1976, changing a monopsony into something more like a competitive labor market, in which players may become free agents, and sell their services to the highest bidder.

This winter, a number of players are eligible to become free agents, but the players' union has been complaining that the baseball owners are colluding and acting like a single monopsony employer. The owners, on the other hand, warn that if player salaries are not held down, the increase will have to be passed on to the fans, in the form of higher ticket prices.

Each baseball team is a local monopoly, and it seems safe to assume that most baseball owners are motivated by profit, rather than by a love of baseball.

Analyze the connection between player salaries and ticket prices. In particular, is it true that salary increases will be passed on as higher ticket prices, or will they be paid out of monopoly profits?

11. Thirty-six people live and work on the island of Beesare. There are 17 fishermen, 6 dairy farmers and thirteen bakery workers. The bakery is a monopoly which is owned by the fishermen: there are 170 shares of stock outstanding, and each fisherman owns 10 shares. The only economic activities on the island are the production and consumption of fish, bread and milk.

The Beesare unit of currency is the clam, and 520 clams are in circulation.

The bakery workers will supply a full day's work for 20 clams. If the wage is less than 20 clams they will not work at all.

When the bakery employs 13 workers it produces 70 pounds of bread per day, and uses 6 gallons of milk in the production process.

The bakery is regulated by the government. It is required to produce and sell 70 pounds of bread every day. Otherwise it is free to maximize profit.

Bakery workers spend 50% of their income on bread, 25% on fish and 25% on milk. Fishermen spend half of their income on bread and the other half on milk. Dairy farmers spend half on bread and half on fish.

Each fisherman catches and sells 5 pounds of fish per day. Each farmer produces and sells 7 gallons of milk per day.

Daily transactions occur as follows. Every morning the milk market opens at 8 a.m. and closes at 9 a.m. Each farmer sells 7 gallons of milk at the market price to the various buyers, and receives clams in exchange.

The bread market is open from 10 a.m. to 11 a.m. The bakery sells 70 pounds of bread at the market price, and receives clams in exchange.

The fish market is open from 4 p.m. to 5 p.m. Each fisherman sells 5 pounds of fish at the market price, and receives clams in exchange.

At 6 p.m. each bakery worker is paid 20 clams, and the bakery's profits for the day are distributed evenly to the shareholders.

Next day, everything is repeated. Each day is an exact copy of the day before.
Clams are held solely for transactions purposes. There is no saving, and no speculation.

<u>Questions</u>

(a) Analyze the demand for money. How many clams does each fisherman hold overnight? Give a complete account of money holdings, showing where all 520 clams are held overnight, and how they change hands during the day.

(b) Find the equilibrium prices of fish, bread and milk.

12. The Likelihood Ratio Test (Neyman-Pearson Lemma)

A statistician expects to obtain a vector of data generated by one of two probability distributions, p or q. The vector will lie in a finite set $X = \{\mathbf{x}^1,\mathbf{x}^2,...,\mathbf{x}^i...,\mathbf{x}^S\}$, called the sample space, and the probabilities associated with the points in this set are either $p_1,p_2,...,p_i,...,p_S$ or $q_1,q_2,...,q_i,...,q_S$, according to whether p or q is the true distribution.

The statistician wishes to design a test of the hypothesis H_0 that the true distribution is p. The test involves dividing X into two parts, R and A, rejecting H_0 if the observed data lie in R and accepting H_0 if the data lie in A. Two types of error are possible here: the test might reject H_0 when it is actually true, or it might accept H_0 when it's false.

The statistician is primarily concerned about errors of the first type. Provided that these can be held to an acceptable level it is also desirable to avoid errors of the second type. Specifically, the aim is to maximize the probability that H_0 will be rejected when it's false (this is called the <u>power</u> of the test), subject to the constraint that the probability of type I error should not exceed some number α (called the <u>size</u> of the test).

(a) Translate this problem into the language of consumer theory. (What is the utility function? What is the budget constraint?).
(b) Solve the problem.
(c) Translate the solution back into language that the statistician can understand. (Note the title of the problem).

13. State whether the following assertions are true, false or ambiguous, and explain why.

(a) A firm uses 10 units of labor and 20 units of capital to produce 10 units of output. The marginal product of labor is 0.5. If there are constant returns to scale the marginal product of capital must be 0.25.

(b) If the wage rate for skilled workers is twice the wage for unskilled workers then a cost-minimizing firm will use fewer skilled than unskilled workers to produce any given output.

(c) Short-run conditional factor demand functions do not depend on the prices of the fixed factors.

(d) The cost function $c(w_1,w_2,y) = (\sqrt{w_1}+\sqrt{w_2})y$ is an impostor (i.e. it is some other function pretending to be a cost function).

(e) A technology has decreasing returns to scale if, and only if, each factor of production has diminishing marginal product.

(f) The imposition of a price ceiling would never cause a monopolist to increase output.

(g) A new invention is discovered which for a particular industry reduces the quantity of all factors required to produce any given output by X per cent (this is Hicks-neutral technical progress). The invention is made available free of charge to all firms in the industry. The industry is competitive (there is free entry) and all firms have the same cost functions. **Labor demand will rise if and only if the price elasticity of the industry demand curve is less than -1**.

(h) The average cost curve for a firm shifts down by X%, so the new price is X% lower, and this means that each firm in the industry uses the same production plan as before, scaled down by X%. Labor demand will rise iff the industry demand curve is relatively elastic, so that industry output goes up by more than X%.

(i) In a duopoly, a firm which acts as a Stackelberg leader (with respect to output quantities) will always produce less than a monopolist facing the same industry demand curve.

(j) A rise in the wage rate implies a rise in the firm's average cost curve.

(k) A movie theater which sets admission prices in such a way that many seats remain empty cannot be maximizing profits.

(l) The opportunity cost of a military draft is the difference between wages paid to soldiers and wages paid in private business.

(m) If all prices rise by 15%, and the wage rate also rises by 15%, then the supply of labor will not change.

(n) An increase in the capital stock, ceteris paribus, will increase the total income of labor.

(o) An increase in the amount of capital per worker will increase labor's share of total output if the elasticity of substitution is less than unity.

(p) Constant relative risk aversion implies that the demand for insurance is a decreasing function of wealth.

(q) The function $\pi(w_1,w_2,p)$ defined by $\log(\pi) = \log(p)-½\log(w_1)-½\log(w_2)$ is not a valid profit function.

(r) A rise in the wage rate implies a rise in the firm's marginal cost curve.

(s) If a production technology has production isoquants defined by $y^2=K^2+L^2$, the cost function has L-shaped contours in the factor prices (i.e. the combinations of v and w which keep c(v,w,y) constant, for fixed y, follow an L-shaped isocost curve).

(t) The function $\pi(w_1,w_2,p) = \log(w_1)+\log(w_2)+\log(p)$ is not a valid profit function.

(u) In a duopoly where the two firms have constant (but possibly different) marginal costs, each firm would rather be a Stackelberg leader than a Stackelberg follower.

(v) The assumption of quasi-concavity of a utility function is sufficient to assure negatively sloped demand curves.

(w) It is possible to find a three-good utility function, in which no good is a substitute for any other.

(x) If dominated strategies are eliminated, the only possible equilibrium outcomes of noncooperative games are Pareto optimal.

(y) Constant absolute risk aversion implies that the demand price of insurance does not depend on wealth.

(z) If the marginal utility of one good is constant then no good is inferior.

(aa) If y is output, and v and w are input prices,

$$c(v,w,y) = \max\left(y[vw]^{1/2} \;,\; y[vw^2]^{1/3} \right)$$

is a valid cost function.

(bb) Starting from the Cournot equilibrium of an oligopoly with n firms, there is always an incentive for two of the firms to merge.

(cc) Suppose a consumer's income rises by $15 per week, and at the same time the price of gasoline rises by 75 cents per gallon. If this consumer previously bought 20 gallons of gas per week, then the consumer will now buy more than 20 gallons if the income effect is stronger than the substitution effect.

(dd) The expenditure function

$$e(p_1,p_2,u) = u\sqrt{p_1 p_2} + 5p_2$$

is an impostor (i.e. it is some other function pretending to be an expenditure function).

(ee) The utility function $u(x_1,x_2) = x_1^2+x_2^2$ is not quasi-concave, so the expenditure function derived from this utility function is not concave in prices.

(ff) If a state seeks to attract new industries by subsidizing production costs, it is more effective to subsidize either labor or capital costs than to allocate the same total to a mixture of labor and capital subsidies

14. Two individuals bid in a sealed bid first price auction, i.e. bids are made simultaneously, the highest bid wins the item and the winner pays his bid. Both bidders' valuations, v, are distributed identically and independently, $v \in [0,V]$ and the common distribution F(v) has a density $f(v) > 0 \ \forall \ v \in [0,V]$. Let $\beta(v)$ denote a bidding function such that $\beta(v)$ will be the bid made by a bidder with valuation v.

a. Assume first that F is the uniform distribution on [0,1]. Find a symmetric Nash equilibrium for the bidding game, i.e. a bidding strategy which solves the maximization problem

$$\pi(v) \ = \ \max_b \ (v-b) \ \left[Prob. \ of \ winning \ with \ a \ bid \ b\right]$$

b. Can you find $\beta(v)$ for a general distribution F on [0,V]?

15. [Layard and Walters] The market demand for trips along the road from A to B depends only on the time taken, according to the function p = 20 - .001x, where p is the number of hours per trip and x is the number of trips per day. When the number of trips increases, the time per trip also increases, because of congestion on the road. This relationship is represented by p = 2 + .001x. There are no other travel costs, and the value of time is $1 per hour.

(a) What is the optimal number of trips?
(b) What money tax should be levied on drivers to achieve optimal utilization of the road?
(c) It has been suggested that anyone who drives in Manhattan should be required to buy a special license costing $5 per day. Would such a policy help achieve more efficient road use? Can you suggest a better policy?

16. Consider a two-person pure exchange economy in which the initial endowments are (1,0) and (0,1), and each person has Cobb-Douglas preferences.

(a) Find the Walrasian equilibrium for this economy.
(b) Now suppose the first person's endowment increases to (2,0). One feasible allocation would be to start from the initial equilibrium and increase the first person's consumption of the first good by one unit. Find the first person's utility at this allocation, and compare this with the utility level at the new Walrasian equilibrium allocation. Explain why one of these utility levels is higher than the other.

17. Suppose that Robert needs to choose between driving a BMW and a Yugo. The BMW is more expensive than the Yugo. Robert is an expected utility maximizer, and the utility of a given income is greater for Robert if he drives the BMW than if he drives the Yugo. There is diminishing marginal utility of income with each car.

Would Robert refuse all fair gambles in this situation?

18. A monopolist produces two goods, x_1 and x_2, for constant marginal costs c_1 and c_2. The monopolist can sell the two goods separately or as a bundle where 1 unit of x_1 and k units of x_2 sell for a price p_b.

The monopolist sells to the representative consumer with the utility function $u(x_1, x_2) + y$, where y represents all other goods.

(a) Write down the monopolist's maximization problem if she sells the unbundled goods.

(b) Write down the monopolist's maximization problem if she sells the goods as a bundle.

(c) Will the monopolist choose to sell the goods separately or as a bundle?

19. Consider a duopoly model in which firm 1 can choose a technology in period 1, but firm 2's technology is fixed. Firm 1 can pay nothing in period 1 and have period 2 production costs of 2 per unit. Alternatively, it can pay $2q^*$ and have capacity q^*; production in period 2 is costless up to q^* and infinitely costly thereafter. Firm 2's production costs in period 2 are 2 per unit.

Demand in period 2 is given by $P = 10 + e - Q$, where Q is industry production and e is a stochastic demand shock. The firms play Cournot in period 2, then the game ends. The expectation of e is zero.

(a) Show that if demand is nonstochastic, (the distribution of e degenerates to a spike at 0) firm 1 will choose the fixed capacity technology.

(b) Now suppose that e is distributed as follows:

e is equal to -1 with probability a
e is equal to 1 with probability a
e is equal to 0 with probability $1 - 2a$.

Firms learn the value of e between periods 1 and 2, so that they know e when they play the Cournot game. However, firm 1 does not know e when choosing whether or not to invest in the fixed capacity technology. For what values of a will the firm choose the fixed capacity technology?

(c) Explain the economics of the trade-off driving this model.

20. Illyria has a proportional income tax. People who report income of y_R pay ty_R in taxes. The taxpayers of Illyria are audited at random. The probability $0<p<1$ of being audited is independent of the amount of income earned or reported; audits are perfectly accurate. Those who found to have cheated on their taxes are fined a fraction γ of their actual income. A citizen's utility function on disposable income is U(.), where U'(.)>0. In deciding how much income to report, the citizens of Illyria maximize the expected utility of disposable income -- the amount they have left after being taxed and possibly audited.

(a) What is the expected utility of someone with actual income y who reports income y_R?
(b) If a citizen decides to cheat on his income tax, how much income will he report?
(c) Suppose the utility function is $U(y) = \log y$, and that the probability an individual is audited is p.
i. Show that there is a penalty rate γ^* such that an individual with income y will be indifferent between reporting honestly and cheating on his income taxes.
ii. Derive an explicit formula for γ^*.

21. An exchange economy has two consumers, Frances and Francis, and two goods, fish and fowl. Each consumer must eat five fish and five fowl to survive, and, beyond this, utility is a loglinear function of extra fish and fowl consumed (the two utility functions can be different). Frances is endowed with fifty fish and fifty fowl, and Francis is endowed with fifteen fish and fifteen fowl. Find a general equilibrium for this economy.

22. Consider an economy with two firms which have the production functions

$$y_1 = 2a_1 \; ; \; y_2 = \frac{a_2}{y_1 + 1}$$

(a) Find the equation for the production frontier as a function of total input availability, $a_1 + a_2 = \bar{a}$. What is the shape of the frontier?
(b) Suppose final demand prices are equal. What production program maximizes the value of net output (for given $\bar{a}$) over the feasible set?

(c) Suppose that we now institute a tax scheme as follows: Final demand prices are as above. In addition to his payments for inputs, we impose a tax on firm 1 with the rate of the tax being determined by the marginal loss of firm 2. Show that there are two tax equilibria, one with only firm 1 in operation and one with only firm 2 in operation.

23. Consider an exchange economy with two goods, fish and cheese, and two people, A and B. A is endowed with α pounds of fish and β pounds of cheese, and B is endowed with γ pounds of fish and δ pounds of cheese. A's utility function is $u^A(f,c)=f^a c^{1-a}$, where f is the quantity of fish consumed, and c is the quantity of cheese. B's utility function is $u^B(f,c)=f^b c^{1-b}$.

a. Find the competitive equilibrium for this economy.

b. Show that the competitive equilibrium is Pareto Optimal.

24. Consider an exchange economy with two goods, fish and cheese, and two people, A and B. A is endowed with .5 pounds of fish and .5 pounds of cheese, and B is endowed with .5 pounds of fish and .5 pounds of cheese. A's utility function is $u^A(f,c)=(1-f)^{-1}(1-c)^{-1}$, where f is the quantity of fish consumed (by A), and c is the quantity of cheese consumed (by A). B's utility function is $u^B(f,c)=(1-f)^{-1}(1-c)^{-1}$

(a) Find the competitive equilibrium for this economy (if there is none, explain why).

b. Find the marginal rates of substitution of A and B at the initial allocation.

c. Is the initial allocation Pareto Optimal?

d. Find the set of Pareto Optimal allocations.

25. There are five firms in the ice cream industry, and there is no threat of further entry. Each firm manufactures a distinct flavor, and production is at constant cost of one unit of a numeraire commodity per quart.

Let x_i be a consumer's consumption of i-flavor ice cream, i = 1, 2. ..., 5, and let y be her consumption of the numeraire good. These are the only goods in the economy. Consumers are partitioned into five groups of equal size. Consumers in group i have a von Neumann-Morgenstern utility function

$$u_i(x_1, x_2, ..., x_5, y) \quad = 6 + y, \qquad \text{if } x_i \geq 1 \text{ quart,}$$
$$= y, \qquad \text{if } x_i \leq 1 \text{ quart.}$$

For simplicity we also assume that ice cream cannot be resold. Further assume that every consumer has the same income, I, which exceeds six units of the numeraire quantity.

Each firm decides on a price, and on whether to sell all of its product in labelled boxes or in plain white boxes. Market equilibrium is a Nash equilibrium in these strategies. Thus the strategy of firm i is a price p_i for its flavor and whether or not to label its boxes.

(a) Prove that there is an equilibrium in which every firm labels its boxes. What price will they charge?

(b) Prove that the situation in which no firm labels its boxes cannot be an equilibrium.

(c) What would be the effect of an FTC ruling that boxes be unlabelled? Discuss the welfare implications of such a regulation.

26. (Jevons, 1871) Your ship is overdue in port and the beer is running out. The remaining supplies are divided up and you get 22.5 fluid ounces. The ship will not reach port before tomorrow morning, and there is a 60% chance that it will arrive then. You can't take beer with you when you leave the ship, so you could drink it all today, to make sure it isn't wasted. On the other hand, there is a 40% chance that you will still be afloat all day tomorrow, and a 10% chance that you will be afloat the day after that. You could save some beer in case you need it for the second day, or the third. It is certain that you will reach port before the fourth day.

You are an expected utility maximizer, and your utility function is 6000 B - 250 B^2, where B is daily beer consumption. How much beer should you drink today?

27. The prices of consumer goods in Los Angeles and Miami are different: some things are cheaper in Miami, while others are cheaper in Los Angeles. Suppose the price of every consumer good in Houston is exactly halfway between the Miami price of that good and the Los Angeles price. A market research firm surveys 1000 consumers, who have different preferences over consumption bundles, and different incomes. The consumers are asked to rank these three cities in terms of the consumption bundles that they could afford in each place. The result of the survey is that 450 consumers rank Los Angeles first, 350 rank Houston first and 200 rank Miami first. Is this consistent with the standard theory of consumer behavior? Prove any claims that you make.

28. A factor of production is called **inferior** if the conditional demand for that factor falls as output is increased while factor prices are held constant.

(a) Draw an isoquant map showing a technology with an inferior factor of production.
(b) Do you think it is possible that an increase in the price of some factor of production might cause a profit maximizing firm to **increase** output? Explain.
(c) Analyze the following argument:
"In consumer theory it can happen that an increase in the price of some good leads to an increase in the quantity demanded (if the good is inferior and the income effect of the price increase is stronger than the substitution effect). Similarly it can happen that an increase in the price of a factor of production leads to an increase in the quantity of that factor demanded by a profit-maximizing firm. This can happen if the higher factor price leads to a reduction in output, and if the factor is inferior, so that the reduction in output leads to an increase in the use of this factor."

29. Consider the function

$$\pi(p,v,w) = p - v - w + 2\alpha[pw]^{1/2} + 2\beta[pv]^{1/2}$$

where p is the price of output, v is the price of capital, and w is the wage rate. Is this a profit function? If so, which values of the parameters α and β are allowed, and what is the production function? If not, why not?

30. Consider the following argument:

"If half of the houses in California were destroyed by an earthquake, the price of a house would rise so high that the remaining half would be more valuable in economic terms than was the original total. Thus an economist would advise us not to wait for the earthquake, but to destroy half the houses on purpose, which shows that economists give absurd advice."

(a) If half of the houses were destroyed would the value of the remainder actually exceed the value of the original total? Explain your answer. (you will get no credit for saying "it depends," unless you explain what it depends on.)
(b) Would a good economist advise that half of the houses be destroyed on purpose?
(c) If you personally owned all of the houses, would you destroy half of them?

31. Suppose the gasoline market is competitive, and is initially in equilibrium at a price of 48 cents per gallon, and a quantity of 240 billion gallons bought and sold per year. Suppose the quantity supplied is proportional to price (e.g., 120 billion gallons are supplied at 24 cents, etc.). Suppose there are 200 million consumers, and that the average consumer spends five percent of income on gas so that if the price of gas doubles, quantity demanded Will be halved.

(a) Draw the market supply and demand curves for gas, if the average consumer has an annual income of $11,520. (The total quantity demanded in the market is the demand for the average driver multiplied by 200 million.)
(b) The government levies a gas tax of 40 cents per gallon on consumers (e.g., by requiring that auto mileage for the year be reported on income tax forms, so that the appropriate tax can be calculated). Now if the gas station price is 48 cents, the consumer pays a total of 88 cents per gallon. Draw the new demand curve for gas, and calculate (exactly) the new equilibrium gas station price and the equilibrium quantity bought and sold.
(c) The government now decides to compensate drivers for the gas tax. The average driver has been using 1200 gallons per year, so everyone gets a rebate of $480 per year (a tax credit of $480, for example). This rebate is a lump sum, which is not affected by the number of miles actually driven, so that a consumer who reduces gas consumption to 500 gallons will pay $200 in gas taxes, and get a $480 rebate. Draw the new market demand curve for gas. Determine

(approximately) the new equilibrium price and quantity.

(d) Using indifference curve analysis, determine whether the average driver is made better off or worse off by the tax/rebate scheme described above.

(e) Does the Treasury gain or lose money on this scheme?

32. Suppose that the long-run total cost curve of a typical firm in a competitive industry is given by the formula:

$$c = 1200Q - 60Q^2 + Q^3.$$

(a) Show that when the industry reaches long-run equilibrium the price will be \$300 per unit, and each firm will produce 30 units, if the industry demand curve is $P^D = 375 - .025Q$. Show that there will be 100 firms in long-run equilibrium.

(b) The government wishes to raise \$45,000 by taxing this industry. Two different kinds of taxes are under consideration:

i. A sales tax of \$15 per unit;
ii. A lump-sum tax of \$450 per firm, regardless of how much the firm produces.

Will either of these taxes generate \$45,000 in revenue for the government? Will the new long-run equilibrium price be the same for both tax schemes? If not, will the price increase be greater under the sales tax or the lump-sum tax? Draw a diagram summarizing your answer.

33. Consider an isolated economy which produces just one crop, corn, using a fixed quantity of land and variable quantities of labor and tractors. Labor is also used to produce the tractors.

(a) Which of the following conditions are needed for Pareto optimality:

i. The marginal rate of technical substitution of tractors for labor in the production of corn equals the marginal product of labor in the production of tractors.
ii. The marginal utility of leisure equals the marginal product of labor in corn multiplied by the marginal utility of corn, for each individual.
iii. The marginal rate of substitution of leisure for corn is equal for all consumers.

Explain your answers in detail.

(b) Which of the above conditions would be satisfied in a competitive market economy? How?

34. At a certain university the number of students wishing to attend substantially exceeds the available number of places. The most important criteria used to allocate places are money (high tuition fees) and intellectual ability (high-school records, admission test scores). A philanthropist offers a sum of money to the university, the yield on which, at the market rate of interest, will equal total annual tuition revenue, if the university agrees to eliminate money as a criterion for admission.

Assume that, at a tuition price of zero, 100,000 students would apply for 1,300 places. The university then considers three schemes for admitting students (assume initially that admission rights are not transferable):

i. Admission based strictly on test scores;
ii. Admission based on a lottery;
iii. Potential students must form a line in front of the Admissions Office. At 9:00 a.m. on April l, each year, the first 1,300 students are admitted.

(a) Would you recommend i or iii over ii on grounds of economic efficiency? Explain your answer carefully.

(b) Now assume admission rights can be transferred. In this way a student who has won a place can sell it to the highest bidder. How would this change your answer to part a)?

35. A profit-maximizing firm uses two inputs, energy and labor, to make a single product. The firm faces perfectly elastic supply curves for labor and energy, and a perfectly elastic demand curve for its product--it takes all prices as given.

(a) Suppose that all prices rise by 20% (including the wage rate, the price of energy, and the price of the firm's product). How will the firm's demand for labor be affected?
(b) If the wage rate did not change, while both the price of energy and the product price increased by 20%, how would the firm's demand for labor be affected?

36. Suppose there are two consumption goods with production functions

$$Q_1 = \sqrt{K_1 L_1} \ , \quad Q_2 = \frac{1}{\frac{1}{K_2} + \frac{1}{L_2}}$$

where K_1 is the amount of capital used in the production of good 1, etc. The economy is competitive, with prices p_1 and p_2 for the consumption goods, and v and w for capital and labor.

(a) Find the relationship between the factor price ratio w/v and the efficient capital-labor ratio in each industry.

(b) Find the relationship between the price ratio of the consumption goods (p_1/p_2) and the factor price ratio.

(c) Suppose now that there are two countries, A and B, each having the technologies described above. A is endowed with 2 million units of K and 1 million units of L, and B is endowed with 1 million units of K and 2 million units of L. Initially the two economies are isolated from each other, and the price ratio p_1/p_2 is 1/4 in A and 1/3 in B. Then trade in the consumer goods is permitted, but **not** in the factors of production. Describe the pattern of trade, and the changes in the factor prices in the two countries.

37. There is a town with 100 families who do not have cable tv. The cost of supplying cable tv would be $20 per month for each family connected to the system, plus $2000 per month in overhead costs which do not depend on the number of families connected.

50 families live in houses, and the other 50 live in apartments. The demand price of a cable connection varies from one family to the next, in such a way that the demand curve for all those who live in houses is p = 100 - 2q, where p is the price charged per month, and q is the number of houses buying connections. The demand curve for those who live in apartments is also p = 100 - 2q.

The town has granted the right to supply cable tv to a profit-maximizing monopoly firm. How will this firm set prices? Will the outcome be efficient? If not, what advice would you give the town as to the possibility of achieving a better outcome?

38. Consider an exchange economy with two goods, fish and cheese, and two people, One and Two. One is endowed with .5 pounds of fish and .5 pounds of cheese, and Two has the same endowment. One's utility function is u(f,c) = fc, and Two has lexicographic preferences.

(a) Can you find a competitive equilibrium for this economy? If so, is it Pareto optimal? If not, explain which assumptions of the First Welfare Theorem are violated.

(b) What is the set of Pareto optimal allocations in this economy? Can all these allocations be supported as competitive equilibria? Explain.

39. Consider an exchange economy with two goods, fish and cheese, and two people, One and Two. One is endowed with .3 pounds of fish and .6 pounds of cheese, and Two is endowed with .7 pounds of fish and .4 pounds of cheese. One's utility function, u(f,c) is strictly quasiconcave, and has the additional property u(f,c) = u(c,f), for all f,c. Two's utility function, v(f,c) is also strictly quasiconcave, and v(f,c) = v(c,f), for all f,c.

(a) Can you find a competitive equilibrium for this economy? If so, is it Pareto optimal? If not, explain which assumptions of the First Welfare Theorem are violated.
(b) What is the set of Pareto optimal allocations in this economy? Can all these allocations be supported as competitive equilibria? Explain.

40. Consider an economy with two goods, meat and rice, and many people. Meat and rice are both produced by processes which use labor as the only input, and the production functions are

$$M = \sqrt{L_M}, \quad R = \sqrt{L_R}$$

where M is the total quantity of meat produced in the economy, L_M is the total amount of labor used in the production of meat, and similarly for rice.

Each person is endowed with one unit of labor, and has preferences described by the utility function $U(m,r) = 1 - (1-m)(1-r)$, where m and r denote this person's consumption of meat and rice. No one puts any value on leisure.

(a) Find a competitive equilibrium for this economy, assuming that each person owns one share in the meat-producing firm, and one share in the rice firm. Assume also that each firm takes prices as given, and maximizes profits.

(b) Determine whether the competitive equilibrium is Pareto optimal. Discuss the economic relevance of the consumers' marginal rates of substitution between meat and rice, and of the marginal rate of transformation in production.

41. Suppose there are just two consumption goods and you are given one element from the Slutsky matrix of a utility-maximizing consumer. What additional information would you need in order to calculate the other three elements of the Slutsky matrix, and how would you make the calculation? [Recall that the Slutsky matrix shows the rate of change of each good with respect to a change in each price, while utility is held constant.]

42. Housing in San Francisco is much more expensive than in Iowa City. Suppose the prices of all other consumer goods are the same in the two places, and compare two consumers with identical preferences and equal incomes, one living in each place. Analyze the following assertion:

> "After paying for housing the SF consumer has less money to spend on other goods than the IC consumer, but both consumers face the same relative prices for all other goods. So, aside from housing, the comparison boils down to the effect of a parallel shift in the budget constraint. In the case of inferior goods, the SF consumer will consumer more than the IC consumer, but for all other goods the SF consumer will consume less."

Is the above statement generally true? If so, prove it. If not, can you find a restriction on the consumers' preferences which would make the statement true?

43. Assume homogeneous labor that can move freely between two places, A and B. A single product is produced in each place, and can be shipped freely from one place to the other. The production process uses only land and labor and has constant returns to scale. Land is in fixed supply in each place. There is an amenity (such as weather or clean air) that may affect both the utility of consumers and the cost of producing the product. Each person owns one acre of land in **both** places. All persons have identical indirect utility functions that depend on their real wage rates, the real rental cost of land, and the amenity (the price of the product is the numeraire).

(a) If the amenity raises utility in A, but does not affect the cost of production, how would equilibrium wages and rents compare in the two places?

(b) If the amenity raises both costs and utility in A, how would equilibrium wages and rents compare in the two places?

(c) If the amenity is clean air, which is more healthy but which raises the cost of production, would the difference in wages measure the extent to which people value their health?

44. Suppose an empirical study estimates a cost function for an industry of similar firms that produce a homogeneous good, using two factors of production. The aim is to use the estimated model to describe the production technology for the good. The cost function is denoted $C(W_1,W_2,Y)$ where C is cost, W_1 and W_2 are input prices and Y is output. It is assumed that the cost function is a translog, as follows:

$$c = a_0y + a_1w_1 + a_2w_2 + b_{00}y^2 + b_{11}w_1^2 + b_{22}w_2^2 + b_{01}w_1y + b_{02}w_2y + b_{12}w_1w_2$$

where c,y,w_1,w_2 denote the **LOGS** of C,Y,W_1,W_2.

(a) What restrictions on the estimated parameter values should hold if the estimated function is to satisfy the theoretical requirements of a cost function?

(b) It is found that the data do not reject the hypothesis that the parameters b_{01} and b_{02} are both zero. What restriction on the production function is implied by this hypothesis?

(c) What production technology is consistent with the hypothesis that all of the b_{ij} parameters are zero?

45. Consider an economy in which many identical people live for two periods. In the first period each person is endowed with one unit of a nonstorable consumption good. In period two there are no endowments, but it is possible to exchange some of the first-period endowment for an asset which will pay off in period two.

There are two types of asset, labeled A and B, and they can each be purchased from abroad at given prices p_a and p_b (where prices are stated in units of the consumption good. There are two possible states of nature in period two, labeled α and β, and these states have probabilities p and 1-p. In state α the A asset will be worth $d_a(\alpha)$ units of the consumption good, and the B asset will be worth $d_b(\alpha)$, with $d_a(\alpha) > d_b(\alpha)$. The corresponding payoffs in state β satisfy $d_a(\beta) < d_b(\beta)$.

Each person maximizes the function $u(c_1) + \delta E[u(c_2)]$, where c_t means consumption in period t, δ is a discount factor, $u(\cdot)$ is a single-period utility function, and $E[\cdot]$ is the expectation operator.

(a) Suppose you are given all of the above data (including a specific utility function), and you are told that a person purchased A_0 units of asset A, and B_0 units of asset B. How would you determine whether these choices are optimal? Would you need a complete specification of the utility function, or would it be enough to know the marginal utility of consumption at one or a few consumption levels?

(b) Show that there is a combination of A and B which provides risk-free consumption in the second period, and calculate the price of this risk-free asset.

(c) Suppose the utility function is $u(c) = c - kc^2$, where k is a parameter in the interval [0,.25]. Find the optimal choice of consumption and asset holdings. Give special attention to the case where k=0.

46. [Tirole p146]
A monopolist faces two types of consumers, with demand curves given by $p = \theta_1(1-q)$ and $p = \theta_2(1-q)$. The marginal cost of production is a constant, c, which is less than θ_1 and θ_2. The monoplist has been selling the product at a single price, p_0, because it is not possible to charge different prices for the two types of consumer.

In this situation, is it generally true that there is a two-part tariff which increases the monopolist's profit, and which makes one type of consumer better off, without hurting the other type? Explain why or why not.

47. (a) A profit maximizing monopoly firm sells to many identical buyers. The marginal cost of production is a constant, c, with $c < .5$. Each buyer's demand function is $p = 1 - q$. The firm decides to charge a two-part tariff, so that a consumer who buys a quantity q must pay the amount $y = a + bq$. How would the firm choose a and b?

(b) Now suppose a second firm enters the industry. This firm has the same production cost as the first firm. Can you find a Nash equilibrium in which each firm chooses a two-part tariff which is optimal, given the other firm's two-part tariff?

48. A market has 10 potential buyers, and 10 potential sellers. Each seller has one unit. The first seller has a supply price of 1, the second, 2, and so on: the 10^{th} seller has a supply price of 10. The first buyer has a demand price of 1.5, the second, 2.5, and so on: the 10^{th} buyer has a demand price of 10.5 (each buyer is interested in buying only one unit).

(a) Find a competitive equilibrium price for this market. What is the quantity traded?

(b) Suppose trade takes place in a decentralized way, so that exchanges might occur at different prices in different locations (for example seller #3 might meet buyer #6, and trade at some mutually agreeable price). What is the maximal quantity which could be traded in this situation?

(c) It is true under both (a) and (b) above that trade is voluntary, so that each exchange must make both the buyer and the seller better off. Does this mean that the outcome is Pareto optimal in both cases? Explain why or why not.

49. There are four kinds of people in the universe, and they live only to munch fish and cheese. In addition to the well-known A-types and B-types, it can now be revealed that there are also C-types and D-types. The preferences of each type are given by the following utility functions:

A-types: $u(f,c) =$	fc	
B-types: $u(f,c) =$	$1-(1-f)(1-c)$	
C-types: $u(f,c) =$	$[fc]^{.5}$	if $f \geq c$
	$1-[(1-f)(1-c)]^{.5}$	if $f \leq c$
D-types: $u(f,c) =$	$1-[(1-f)(1-c)]^{.5}$	if $f \geq c$
	$[fc]^{.5}$	if $f \leq c$

Consider exchange economies with two goods, fish and cheese, and two people, where each person is one of the types described above (the two may be of the same type). In each economy, each person is endowed with .5 pounds of fish and .5 pounds of cheese.

(a) In constructing an economy of this kind, there are 4 ways to pick the first person, and 4 ways to pick the second person, so it looks as if you will have to analyze 16 possible economies. How many of these economies are really different?

(b) For every different type of economy answer these questions.
i. Find the competitive equilibrium (if there is none, explain why).
ii. Find the marginal rates of substitution at the initial allocation.
iii. Find the set of Pareto Optimal allocations.
iv. Find the core.

50. Consider a one-period pure exchange economy with three agents (denoted 1,2 and 3) and three goods (denoted x,y and z). Agent i's consumption vector is (x_i,y_i,z_i). Agent 1,2 and 3's endowment vectors are $(x^e,0,0)$, $(0,y^e,0)$ and $(0,0,z^e)$ respectively (each agent is endowed with only one type of good). Their preferences can be represented by utility functions, as follows:

$$U_1(x_1,y_1,z_1) = x_1(y_1+z_1)$$
$$U_2(x_2,y_2,z_2) = x_2y_2$$
$$U_3(x_3,y_3,z_3) = x_3z_3.$$

(a) Compute a Walrasian equilibrium for this economy under the condition that good z cannot be traded.

(b) Assume now that there exists a complete set of markets in all three goods. Compute a Walrasian equilibrium for this case.

(c) Can you Pareto-rank the equilibria which you found? If so, in which order? Explain your findings.

51. Norman Normal is risk neutral; Richard Righteous is risk averse. In the economy they live in there are only two states of the world. In state A, Richard has an accident and his income is less than in state B in which he does not have an accident. Norman offers to insure Richard. We will say he provides complete insurance if after the transfers that make up insurance have taken place Richard's income is the same in each state.

(a) Assume that Richard and Norman agree about the relative likelihood of the occurrence of state A. At a Pareto optimal insurance allocation, does Richard get complete insurance? If not, can you say whether his income is greater in state A than in state B?
(b) Does your answer change if Richard thinks that state A is more likely to occur than does Norman? If so how?

52. Is the following set of observations of price-quantity data consistent with utility maximization. Why or why not?

$p^1 = (3,4,1)$ $x^1 = (5,1,3)$
$p^2 = (2,3,2)$ $x^2 = (3,3,3)$
$p^3 = (5,3,1)$ $x^3 = (4,2,2)$

(a) There are two commodities. A consumer is observed making purchases at many different price and income pairs and it is noted that the consumer always chooses the midpoint of the budget line. Is this behavior rational? (Explain what you mean by rational consumer behavior and how you would know whether the above consumer is rational or not.)

53. [Robert Lucas] Consider land prices in a rectangular country. The northern edge of the country is a desirable seacoast. which is 100 indivisible lots wide. From north to south the country is 1000 lots deep. There are 10,050 families, each of which can occupy just one lot. Each family's utility function is $u(c,x) = c^{\alpha} x^{-\alpha}$, where c is consumption of a single consumption good, α is positive, and x is distance from the coast, in lots (x=1,2,...,1000). Each family is endowed with y units of the consumption good, and owns one share in each lot.

Find the competitive allocation of families over lots, and the price of each lot.

54. Suppose a union and an employer start to negotiate on January 1, 1994 over wages to be paid for the year 1994. There will be 50 paid weeks in the year (the rest being unpaid vacation time). No work will be done until they reach an agreement.

The employer's net revenue, after paying all costs other than wages (including a normal return on capital), is $500 per worker per week. The workers can earn $240 per week if they leave this employer and go to work elsewhere.

While negotiations continue workers can collect $130 per week in unemployment benefits (this is a straight subsidy that does not have to be repaid). The employer has retained an expert negotiator who charges $200 per week per worker.

The negotiations proceed as follows. At the beginning of each week the employer's and the union's negotiators meet, and one side proposes a wage for the rest of the year. If this proposal is accepted work begins immediately. If not, the workers collect unemployment benefits for the week, the employer's negotiator is paid for the week, and nothing happens for the rest of the week; next week there is a new meeting and a new proposal.

At the first meeting a coin is tossed to determine which side makes the proposal for that week. In subsequent meetings they take turns: first one side makes a proposal, then the following week the other side makes a proposal, and so on.

How long will these negotiations take? What wage agreement will be reached?

55. State whether the following assertion is true, false or ambiguous, and **explain why**.

"There are many firms which produce wooden chairs, and many firms which produce wooden tables. If these firms are all separate, and if they all maximize profits, taking prices as given, then the equilibrium cannot be efficient. This is because when the chair firms increase output they bid up the price of wood, which reduces the profits of the table-producing firms. But the chair firms ignore the effect of their output decisions on the profits of the table firms. An efficient equilibrium would be achieved if all of the firms produced both tables and chairs."

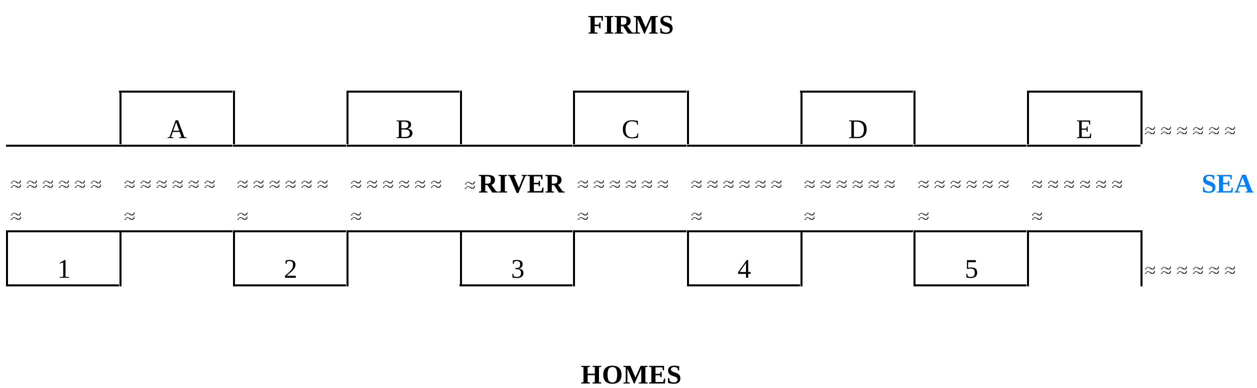
56. Suppose a consumer has the following utility function

$$U(x_1,x_2,\ldots,x_k) = \alpha x_1 + u_2(x_2) + u_3(x_3) + \ldots + u_k(x_k)$$

where α is a positive parameter, and the functions $u_i(\cdot)$ are all concave.

(a) Is it true that x_1 absorbs all income effects: the income elasticity of demand for each of the other goods is zero? Explain.
(b) Is it true that the cross price elasticities of demand are all zero? Explain.
c. Under what conditions would $x_1=0$ be optimal?

57. Below is a map showing the location of five firms and five homes. Each firm dumps 12 gallons of crap in the river each week, and this flows down to the sea. The homeowners like to swim in the river, and each homeowner would pay $1 per week to reduce the amount of crap in his section of the river by 1 gallon. Each firm could stop polluting at a cost of $25 per week. Each homeowner could put in a private swimming pool at a cost of $40 per week, and then would not care about the river.



a. If the firms are not liable for pollution, what transactions would you expect to see (would the firms install pollution controls? Would anyone build a swimming pool? Who would pay?)

b. If you were the mayor of this town, what would you do?

58. There is a fixed total X of some good to be divided between two jealous individuals. If person 1 gets X_1 and 2 gets X_2, the utilities are

$$U_1 = X_1 - kX_2^2 \text{ and } U_2 = X_2 - kX_1^2$$

where k is a positive constant.

a. How would you judge the efficiency of alternative allocations (X_1,X_2)?

b. Suppose a social planner's objective is to maximize the sum of utilities. What is the optimal allocation? Interpret your result.

59. The Hicks-Marshall Rules

There is an old piece of economic analysis, going back at least to Marshall, of what determines the elasticity of demand for a factor of production. When the real wage rate rises, for example, producers are more inclined to use a capital-intensive technology, and consumers also tend to substitute away from goods (such as services) which are labor intensive. The following quaint summary is from Pigou:

i. "The demand for anything is likely to be more elastic, the more readily substitutes for the thing can be obtained."
ii. "The demand for anything is likely to be less elastic, the less important is the part played by the cost of that thing in the total cost of some other thing, in the production of which it is employed."
iii. "The demand for anything is likely to be more elastic, the more elastic is the supply of co-operant agents of production."
iv. "The demand for anything is likely to be more elastic, the more elastic is the demand for any further thing which it contributes to produce.

Consider a good which is produced using labor and capital and a CES technology, with constant returns to scale. Everyone has free access to this technology, and so in equilibrium the good must sell at average cost, and each firm which produces it just breaks even, as long as they minimize the cost of production. The total quantity produced is determined by the market demand curve (i.e. it is the total quantity demanded when price equals average cost).

Suppose the demand curve in the product market has constant elasticity, $\boldsymbol{\eta}$. Suppose there is an upward sloping market supply curve of capital, with constant elasticity, **e**, so that when the industry demand curve for capital shifts out because of a wage increase, the price of capital must rise.

a. Derive a formula showing how the elasticity of labor demand, $\boldsymbol{\lambda}$, depends on labor share, **s**, the elasticity of substitution, $\boldsymbol{\sigma}$, and the demand elasticity $\boldsymbol{\eta}$, assuming that the supply curve of capital is infinitely elastic.

b. Use your formula to determine whether the rules given above are correct, ignoring rule iii.

c. ("Optional") Extend your formula to cover an arbitrary value for **e**, and re-check all of the rules, including iii.

60. Is it possible to find a two-good utility function such that the two goods are complements? If so, find one; if not, explain why none can be found.

61. Consider an exchange economy with two goods, fish and cheese, and two people, One and Two. One is endowed with .5 pounds of fish and .5 pounds of cheese, and Two has the same endowment. One's utility function is u(f,c) = fc, and Two's utility function is $u(f,c) = f^2+c^2$.

(a) Can you find a competitive equilibrium for this economy? If so, is it Pareto optimal? If not, explain which assumptions of the First Welfare Theorem are violated.
(b) What is the set of Pareto optimal allocations in this economy? Can all these allocations be supported as competitive equilibria? Explain.

62. (a) State and prove Hotelling's Lemma.
(b) Show that a factor of production is inferior if and only if an increase in the price of that factor reduces marginal cost.

63. (a) Suppose that all workers in the US economy had the same innate abilities, and the same capacities to acquire skills through training. Suppose also that everyone could borrow and lend money at the same interest rate. Would grave-diggers then earn higher wages than doctors?

(b) Suppose that at any given wage rate the market demand for workers is greater in Hawaii than in Alaska. Suppose also that all workers have identical preferences and abilities. Will the equilibrium wage in Alaska then be higher than in Hawaii? Explain.

64. "If all or most of the returns to education were in fact due to ability, there would be no substantial tendency for high-ability people to invest in education. But in fact high-ability people stay in school longer than average. So schooling must have a direct influence on productivity." Evaluate this statement.

65. Suppose there are N firms in the market for some product, and entry of new firms is impossible. Suppose the market demand curve is linear, and the marginal cost of production is constant. Find the market equilibrium when each firm behaves as a Cournot oligopolist with respect to the quantity of output produced. Is this equilibrium similar to the competitive equilibrium if N is large?

66. Suppose the market demand curve for mineral water is

$$P = 30 - .25\,Q$$

where Q is the annual quantity sold, and P is the price in dollars. There are just two firms which produce mineral water, and the production cost is zero (the water comes from springs).

(a) Find the Cournot equilibrium.
(b) Suppose the Cournot equilibrium has been established, and that the market interest rate is 10% per annum. Now the owner of one of the firms offers to sell out to the other at a cash price of $5200. Will this offer be accepted? Explain.

67. Consider an exchange economy with two consumers, California and Wisconsin. California has a stock of 1 gallon of wine, and Wisconsin has 10 gallons of beer.

California's utility u equals twice the product of wine consumption x_w and beer consumption x_b: $u = 2x_wx_b$. Wisconsin's utility function is the same. The income of each consumer is the revenue derived from selling wine or beer, so it depends on the prices of wine or beer.

(a) Draw indifference curves representing utility levels of 5, 10 and 20.

(b) Let the price of beer be fixed at 50 cents per gallon. Draw budget constraints for California when the price of wine is (i) $5 per gallon, (ii) $10, (iii) $20.

(c) Find California's optimal consumption plan for each of the three budget constraints.

(d) Repeat parts (a), (b), (c) for Wisconsin.

(e) Identify the competitive equilibrium price list.

(f) If the price of beer had been fixed at $1 instead of 50 cents how would the competitive equilibrium be affected?

68. Suppose two players are involved in a game with the following payoff matrix:

		Player II: C	Player II: D
Player I	C	1, 1	-1,1.5
	D	1.5,-1	0,0

The first element in each pair of numbers is the payoff to Player I and the second element is the payoff to Player II.

(a) First, consider the single shot, simultaneous play game between I and II. Find all Nash equilibria for this game.

(b) Now consider the repeated game in which I and II play the above game each period forever. Assume that each player chooses a strategy for the repeated game which maximizes the net present value of the payoff, using a discount factor δ ($0<\delta<1$). Consider the following strategy:

Play C in the first period and then play C each period thereafter unless the other player has ever played D. In that case, play D.

Find a number α such that if $\delta \geq \alpha$ then there is a Nash equilibrium of the repeated game in which I and II both play the above strategy.

69. A monopolist faces the demand curve

$$P = 60 - .4Q$$

where Q is the annual quantity sold, and P is measured in dollars. Labor is the only input, and the labor supply curve is perfectly elastic at a wage of \$2/hr. The production function is

$$Q = [10\,L\text{-}5000]^{1/2}$$

where L is hours worked.

(a) Find the profit-maximizing price and quantity.
(b) Suppose a price ceiling of \$35 is imposed. What is the new profit maximizing plan, and how much profit is made?

70. Players A and B play the following one-shot game infinitely often:

	B		
A	4,4	0,3	0,3
	7,7	2,8	0,7
	10,3	1,0	1,10

A chooses rows, B chooses columns in the one-shot game. Each cell in the matrix is to be read: A's payoff, B's payoff. Stage game payoffs are discounted with a discount factor $0 < \delta < 1$.

a. Compute a subgame-perfect equilibrium for the infinitely repeated game.
b. What is the lowest discount factor for which (4,4 'forever') can be supported as the outcome of a subgame-perfect equilibrium?
c. Suppose there is a social convention to avoid punishments which are unnecessarily severe. In particular let this convention rule out a subgame-perfect equilibrium σ if there is an alternative subgame-perfect equilibrium τ all of whose continuation payoffs exceed the lowest continuation payoff of σ. Show that this convention rules out the equilibrium you found under b.

71. Consider a simple economy in which there are just two occupations, coal mining and auto repair. The mining and auto repair industries are perfectly competitive, and they happen to have identical labor demand curves, given by $w = 400 - L$, where w is the daily wage (net of any training costs borne by workers), and L is the number of workers employed in the industry.

There are 420 workers in the economy, 294 men and 126 women, all equally productive in both jobs. All workers prefer auto repair work to coal mining, but the extent of this preference varies from one worker to another. The distribution of equalizing differences over workers is uniform between 0 and \$42. Sex and occupational preferences are independently distributed.

a. Find the equilibrium wage differential and occupational distribution for this economy.

b. Suppose Fred is an "average" worker, who considers the equalizing differential to be \$21 a day. Does Fred gain or lose from the diversity of preferences in the economy? That is, would Fred be better or worse off if everyone else in the economy had the same preferences as he does?

c. Suppose that women are excluded from coal mining jobs. How will this affect the equilibrium? What will happen to the average wages of men and women? Who will gain under this restriction, and who will lose?

d. Suppose that employers who have excluded women are found liable for damages. How would you compute the damages?

e. Does it make sense to interpret the exclusion of women from mining jobs as rational exploitation of the minority by the majority?

72. Design a contract to maximize the expected profits received by a risk-neutral principal who will hire a risk-averse agent. The agent's utility function is $u = \log(w) - e$, where e is effort (high or low), and w is the wage payment. The agent has an outside option that is a sure thing worth -½. The low effort level is zero, and the high effort level is ½. Gross revenue depends on the agent's effort level. If effort is high, revenue R is uniformly distributed on the interval [0,1]. If effort is low, R is also distributed on [0,1], with density $f(R) = 2(1-R)$. The principal cannot observe the agent's effort.
Analyze how the optimal contract changes as the cost of effort decreases.

73. Consider an economy in which there are equal numbers of two kinds of workers, a and b, and two kinds of jobs, good and bad. Some workers are qualified for the good job, and some are not. Employers believe that the proportion of a-workers who are qualified is 2/3 and the proportion of b-workers who are qualified is 1/3. If a qualified worker is assigned to the good job the employer gains \$1000, and if an unqualified worker is assigned to the good job the employer loses \$1000. When any worker is assigned to the bad job, the employer breaks even.

Workers who apply for jobs are tested and assigned to the good job if they do well on the test. Test scores range from 0 to 100. The probability that a qualified worker will have a test score less than t is t. The probability that an unqualified worker will have a test score less than t is $t(2-t)$. Employers are subject to a rule that requires the proportion of a-workers assigned to the good job to be the same as the proportion of b-workers. Otherwise employers maximize expected profits.

Find the profit-maximizing policy for an employer.

Test your policy as follows. If you are told that a worker has just barely passed the test (and you are not told whether the worker is an a-type or a b-type), what is the probability that the worker is qualified? Is it the case that such a worker is a fair bet from the employer's point of view? If not, should the policy be changed?

74. Let F and G be distribution functions defined on the real interval [a,b], with density functions f and g.

a. Can you find an example in which F and G are ordered in the sense of first-order stochastic dominance, but the likelihood ratio is not monotonic? If you can't find such an example, can you prove that no one else can find one either?

b. Can you find an example in which the likelihood ratio is monotonic, but F and G are not ordered in the sense of first-order stochastic dominance? If you can't find such an example, can you prove that no one else can find one either?

75. A firm has a large accumulated inventory of a storable good. There are no competing sellers of this good, and there is a linear relationship between the quantity sold in each period and the price that the firm sets. Inventory holding costs are negligible, but the cost of production is higher than any buyer would ever pay. The firm can borrow and lend freely at a fixed discount rate, and acts to maximize the present discounted value of profits. How will the firm set prices?

76. Suppose a union and an employer start to negotiate on January 1, 1998 over wages to be paid for the year 1998. There will be 50 paid weeks in the year (the rest being unpaid vacation time). No work will be done until they reach an agreement.

The employer's net revenue, after paying all costs other than wages (including a normal return on capital), is $500 per worker per week. The workers can earn $240 per week if they leave this employer and go to work elsewhere.

While negotiations continue workers can collect $130 per week in unemployment benefits (this is a straight subsidy that does not have to be repaid). The employer has retained an expert negotiator who charges $200 per week per worker.

The negotiations proceed as follows. At the beginning of each week the employer's and the union's negotiators meet, and one side proposes a wage for the rest of the year. If this proposal is accepted work begins immediately. If not, the workers collect unemployment benefits for the week, the employer's negotiator is paid for the week, and nothing happens for the rest of the week; next week there is a new meeting and a new proposal. At the first meeting a coin is tossed to determine which side makes the proposal for that week. In subsequent meetings they take turns: first one side makes a proposal, then the following week the other side makes a proposal, and so on.

(a) How long will these negotiations take? What wage agreement will be reached?
(b) Find the Nash bargaining solution for this situation.

77. American Family Publishers regularly sends a mass mailing to millions of people inviting them to enter their sweepstakes, with prizes worth millions of dollars. The odds of winning depend on the number of entrants.

Resolve the following riddle.

How could anyone be so foolish as to waste time and stamps entering such a competition, given that the odds of winning are minuscule?

If everyone is smart enough to figure this out, how could anyone be so dumb that they would not enter the competition?

78. (George Mailath) Consider an economy with n agents and two goods: a private good, x and a public good, g. Consumer i has an endowment of ω_i units of the private good, and there is a technology that transforms the private good into the public good.

a. Suppose there are m firms that have access to the public good technology, and each consumer owns equal shares of each firm. How would you define a "Walrasian" (competitive) equilibrium for this two-good economy?

b. Now suppose the public good technology has constant returns to scale, at a rate of two units of the private good per unit of the public good.

i.What is the Walrasian equilibrium price ratio?
ii.Are the Walrasian allocations Pareto efficient? Explain.
iii.Relate your answer to the First Welfare Theorem.

79. There are two kinds of actors in the theater business, those who are members of Actors Equity, and those who are not. Non-Equity actors negotiate their own deal for each performance, but Equity actors are governed by a union contract.

Consider a union representing skilled workers in a competitive industry. Suppose that production requires skilled and unskilled labor, with no other factors. The market for unskilled labor is competitive, and the supply curve is upward-sloping. The market for the industry's product is also competitive, and the demand curve is downward-sloping. Wages for skilled workers in this industry are determined by collective bargaining, and once the wage is set, employers are free to choose the number of skilled workers hired.

a. If the union succeeds in getting a new contract with a higher wage for skilled workers, what will happen to the wages of unskilled workers?

b. In the U.S. economy, unions representing skilled workers have been relatively unsuccessful in recent years. Does this observation imply anything about changes in skill differentials?

80. A monopolist faces two types of consumers, with demand curves given by $p_1 = 3\text{-}q_1$ and $p_2 = 4\text{-}2q_2$. There are 100 consumers of each type. The (constant) marginal cost of production is 1. The monopolist cannot discriminate between the two consumer types, but can charge a two-part tariff: a buyer can buy q units by paying A+pq, or buy nothing and pay nothing. Does the profit-maximizing two-part tariff (A,p) involve charging a price p below marginal cost? Explain why or why not.

81. Consider a simple economy in which there are just two firms, which both produce the same peculiar variety of cheese, in different locations. One location is warm and pleasant, and the other is cold and dreary, but this difference doesn't have any direct effect on cheese production. All cheese is sold abroad, and no cheese is produced elsewhere. The demand curve for cheese is downward sloping, and price is determined so as to clear the market, given the quantity produced.

Labor is the only input in production, and the economy contains N identical workers, who can move costlessly between the two production locations. Workers value leisure at w_0 per week, and they would rather be warm, other things equal. A wage differential of d per period is just enough to induce workers to move from the warm to the cold location.

Each firm takes the quantity produced and the wage in the other location as given, and acts as if it can hire as many workers as desired at this wage, plus or minus d. Wages adjust so as to clear the labor market.

Describe the equilibrium of this economy. Do the two firms produce the same quantity? Do all workers move to the warm location?

82. Suppose an art dealer wishes to auction a painting so as to maximize expected profit. There are two potential buyers, with independent valuations of the painting which are uniformly distributed between 0 and $6,000. The dealer has no other use for the painting. The dealer is considering two alternative auction procedures: a first price auction, or a Vickrey (second-price) auction. Suddenly a third buyer appears, and makes a single take-it-or-leave-it offer of $2,200 for the painting. Will the dealer accept this offer? If not, which auction procedure will the dealer use (after the third buyer has left)?

83. Design a contract to maximize the expected profits received by a risk-neutral principal who will hire a risk-averse agent. The agent's utility function is u = log(w) - e, where e is effort (high or low), and w is the wage payment. The principal can observe gross revenue, but cannot observe the agent's effort. The agent has an outside option that is a sure thing worth -½. The low effort level is zero, and the high effort level is ½. Gross revenue depends on the agent's effort level. If effort is high, revenue R is distributed on the set {10,20,40} with probabilities P(10) = .4, P(20) = .2, P(40) = .4. If effort is low, R is distributed on the same set with probabilities P(10) = .2, P(20) = .6, P(40) = .2.