

Saturday May 10, 1997, 2-4pm.

**Instructions:** Answer **four** questions. Each question is worth 25 points, unless you say otherwise: you can reallocate points subject to the constraint that each question is worth at least 20 (e.g. 40/20/20/20). Explain your answers carefully, using diagrams where appropriate. Write as if you are trying to convince a reasonably intelligent person who does not already know the answers. Be as precise as you can, but remember that an imprecise answer is better than nothing, and intuitive reasoning can sometimes be convincing. If the problem is too hard, answer a simplified version of it, and then try to sketch an argument for the more general version.

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1. A monopolist faces two types of consumers, with demand curves given by  $p_1 = 3 - q_1$  and  $p_2 = 4 - 2q_2$ . There are 100 consumers of each type. The (constant) marginal cost of production is 1. The monopolist cannot discriminate between the two consumer types, but can charge a two-part tariff: a buyer can buy  $q$  units by paying  $A + pq$ , or buy nothing and pay nothing. Does the profit-maximizing two-part tariff  $(A, p)$  involve charging a price  $p$  below marginal cost? Explain why or why not.
2. Consider an economy with two goods, meat and rice, and many people. Meat and rice are both produced by processes which use labor as the only input, and the production functions are

$$M = \sqrt{L_M}, \quad R = \sqrt{L_R}$$

where  $M$  is the total quantity of meat produced in the economy,  $L_M$  is the total amount of labor used in the production of meat, and similarly for rice.

Each person is endowed with one unit of labor, and has preferences described by the utility function

$U(m, r) = 1 + (1 + m)(1 + r)$ , where  $m$  and  $r$  denote this person's consumption of meat and rice. No one puts any value on leisure.

- a. Find a competitive equilibrium for this economy, assuming that each person owns one share in the meat-producing firm, and one share in the rice firm. Assume also that each firm takes prices as given, and maximizes profits.
- b. Determine whether the competitive equilibrium is Pareto optimal. Discuss the economic relevance of the consumers' marginal rates of substitution between meat and rice, and of the marginal rate of transformation in production.

3. Consider a simple economy in which there are just two firms, which both produce the same peculiar variety of cheese, in different locations. One location is warm and pleasant, and the other is cold and dreary, but this difference doesn't have any direct effect on cheese production. All cheese is sold abroad, and no cheese is produced elsewhere. The demand curve for cheese is downward sloping, and price is determined so as to clear the market, given the quantity produced.

Labor is the only input in production, and the economy contains  $N$  identical workers, who can move costlessly between the two production locations. Workers value leisure at  $w_0$  per week, and they would rather be warm, other things equal. A wage differential of  $d$  per period is just enough to induce workers to move from the warm to the cold location.

Each firm takes the quantity produced and the wage in the other location as given, and acts as if it can hire as many workers as desired at this wage, plus or minus  $d$ . Wages adjust so as to clear the labor market.

Describe the equilibrium of this economy. Do the two firms produce the same quantity? Do all workers move to the warm location?

4. Suppose an art dealer wishes to auction a painting so as to maximize expected profit. There are two potential buyers, with independent valuations of the painting which are uniformly distributed between 0 and \$6,000. The dealer has no other use for the painting. The dealer is considering two alternative auction procedures: a first price auction, or a Vickrey (second-price) auction. Suddenly a third buyer appears, and makes a single take-it-or-leave-it offer of \$2,200 for the painting. Will the dealer accept this offer? If not, which auction procedure will the dealer use (after the third buyer has left)?
5. Suppose there are  $I$  voters with preferences over a set  $X$  that are single-peaked with respect to a linear order  $\$$ . Prove that there is some alternative in  $X$  that cannot be defeated by any other alternative by pairwise majority voting. Give an example.
6. Design a contract to maximize the expected profits received by a risk-neutral principal who will hire a risk-averse agent. The agent's utility function is  $u = \log(w) - e$ , where  $e$  is effort (high or low), and  $w$  is the wage payment. The principal can observe gross revenue, but cannot observe the agent's effort. The agent has an outside option that is a sure thing worth  $-\frac{1}{2}$ . The low effort level is zero, and the high effort level is  $\frac{1}{2}$ . Gross revenue depends on the agent's effort level. If effort is high, revenue  $R$  is distributed on the set  $\{10, 20, 40\}$  with probabilities  $P(10) = .4$ ,  $P(20) = .2$ ,  $P(40) = .4$ . If effort is low,  $R$  is distributed on the same set with probabilities  $P(10) = .2$ ,  $P(20) = .6$ ,  $P(40) = .2$ .