THE UNIVERSITY OF WISCONSIN

Economics 711, Part II

SECOND MIDTERM EXAM

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PART 1

Directions: Answer <u>all five</u> questions. Please put answers to Part 1 in one blue-book and answers to Part 2 in a separate blue-book.

1. Define **Weak Pareto Efficiency** and **Strong Pareto Efficiency.** Add the assumptions and show that an allocation is Weakly Pareto Efficient if and only if it is also Strongly Pareto Efficient.

2. Prove the following:
   Proposition: Suppose $\mathbf{x}^*$ is Pareto efficient and that preferences are non-satiated. Then there exists a $\mathbf{p}'$ such that $(\mathbf{p}', \mathbf{x}^*)$ is a Walrasian equilibrium.

3. Suppose $(\mathbf{p}, \mathbf{x}^*)$ is a Walrasian equilibrium in pure exchange. Assume utility functions are concave. Then show that there exist welfare weights θ_i, $i = 1, \ldots, n$, and a linear social welfare function $W = \sum \theta_i u_i(x_i)$ for which $\mathbf{x}^*$ is a solution. Find the θ's you need. What interpretation can you put on θ_i's in this case?

4. In the proof of the *Second Theorem of Welfare Economics* we had to assume that all agents hold positive amounts of all goods and that preferences are strictly convex. Explain why we need to do this.

5. Bill and Al consume two goods, speeches x and votes y, and they each have identical Cobb-Douglas utility functions $U_i = x_i^{1/4} y_i^{3/4}$. Bill and Al are each endowed with one unit of labor, ℓ, which they supply perfectly inelastically. Speeches are produced using only labor: $x = 4\ell_x$. However, votes require the use of some speeches as well as labor: $y = x_y^{1/2} \ell_y^{1/2}$. Find the Competitive Equilibrium in Bill and Al's economy.

PART 2

Directions: Answer <u>both</u> questions. Please put the answers to Part 2 in a separate blue-book.

1. There are two possible states of the world tomorrow, $s = 1$ and $s = 2$. Today, n consumers trade contingent claims on $m \geq 1$ physical goods. Consumers have preferences
$$U_i(x_1^i, x_2^i) = \pi_1 u_i(x_1^i) + \pi_2 u_i(x_2^i),$$
where (π_1, π_2) is the probability distribution on states 1 and 2, and x_s^i is the consumption of consumer i in state s. Finally, let ϖ_{ks}^i be i's endowment of good k in state s. Then assume that the endowments of the physical goods satisfy $\sum_i \varpi_{k1}^i = \sum_i \varpi_{k2}^i$ for all k goods.

   (a) Show that if all the functions $u(\cdot)$ are strictly concave, then any Arrow-Debreu equilibrium allocation $\widehat{\mathbf{x}}$ satisfies $\widehat{x}_1^i = \widehat{x}_2^i$ for all i. Interpret this result.

   (b) For the case of $m = 1$ (i.e. one physical good), what are the equilibrium prices (assuming at least one consumer's utility is differentiable).

   (c) Consider the case where $m = n = 2$, $\pi_1 = \pi_2 = 1/2$, and
$$\begin{aligned} u_1(x_s^1) &= \ln x_{s1}^1 + \ln x_{s2}^1, \text{ with } \varpi_1^1 = (1,1),\ \varpi_2^1 = (3,0), \\ u_2(x_s^1) &= \ln x_{s1}^2 + \ln x_{s2}^2, \text{ with } \varpi_1^2 = (2,2),\ \varpi_2^2 = (0,3). \end{aligned}$$
   Find the Arrow-Debreu equilibrium. Interpret the result.

2. An economy has a private good x and a public good G. The private good can be converted to the public good on a 1-for-1 basis. There are N consumers. They each have preferences $U_i = x_i + \ln G$, and are endowed with money m_i, for $i = 1, \ldots, N$. Finally, assume $m_i > 1$ for all i.

   (a) Find the Pareto Efficient allocation in this economy. Is it unique?

   (b) Find the Lindahl Solution for the economy. Is it unique?

   (c) Give the definition of a *core allocation.*

   (d) Suppose that an improving coalition *cannot* be excluded from consuming the public good G that the rest of the economy produces. Identify the core.

   (e) Now suppose that an improving coalition *can* be excluded from consuming the public good G that the rest of the economy produces. Identify the core.