

# ECON 468: Industrial Organization

## Problem Set 4

Due date: May 8th 2008

May 2, 2008

1. Consider the problem of a coffee-shop who wants to set the price and the optimal size of cup of coffee. The market is composed to two types of consumers  $i \in \{L, H\}$  with the following utility from buying a cup of size  $q$ :

$$u_i(q) = \theta_i q - p(q),$$

where  $\theta_L < \theta_H$  and  $p(q)$  is the price of cup of size  $q$ . The proportion of consumers of type  $H$  in the population is  $1/2$  (the other half is of type  $L$ ). Moreover the cost of producing and selling a cup of size  $q$  is given by:

$$C(q) = \frac{1}{2}q^2.$$

- (a) Assume that the store's owner is constrained to offer only one type of cup (i.e.  $q^u$ ), and is not able to price discriminate between consumers. What is the optimal cup size and price such that both types of consumers are buying the good? (hint: solve for the maximum price such that both participation constraints are satisfied and then maximize the supplier's profits to find the optimal cup size).
- (b) Assume now that the owner is perfectly able to discriminate across consumers. What is the optimal cup size and price offered to **each** consumer type?
- (c) Finally, consider a more realistic situation in which the owner cannot perfectly discriminate between consumers but is allowed to offer two categories of cups (i.e.  $q_H > q_L$ ). Write down the constraint optimization problem face by the owner (i.e. the profit maximization problem subject to the incentive compatibility and the participation constraints of both types). What are the optimal cup sizes and prices that solve this problem? You will find useful to use the following properties of the price discrimination solution:
  - The size of the largest cup  $q_H$  is optimal (i.e.  $C'(q_H) = q_H = \theta_H$ ).
  - The incentive constraint of the type  $H$  consumers is binding.
  - The participation constraint of the type  $L$  consumers is binding.
- (d) What is the informational rent that high-type consumers receive because of the inability of the coffee-shop owner to perfectly price discriminate? To get this, compare the utility of the high-type consumer under second-degree price discrimination and perfect discrimination.

2. Consider the problem of a mutual funds manager who wants to induce effort from its employees. By exerting effort (i.e.  $e = 1$ ) the employees can raise the probability that the fund will be successful and generate a return equal to  $x_s$ . If the fund is not successful the return is equal to  $x_f < x_s$ . The probability of success conditional of the effort level of employees is given by:

$$\Pr(x_s|e = 1) = p_1 > \Pr(x_s|e = 0) = p_0.$$

In order to provide enough incentives to its employees, the manager decided to offer following bonus scheme to its employees:

$$w(x) = \begin{cases} \bar{w} + b & \text{if } x = x_f \\ \bar{w} & \text{else} \end{cases} \quad (1)$$

Employees are risk-averse and choose their level of effort  $e = \{0, 1\}$  by maximising the following expected utility function:

$$EU(e) = p_e \sqrt{\bar{x} + b} + (1 - p_e) \sqrt{\bar{x}} - e.$$

Moreover, if their expected utility is too low, employees can choose to work for another company and receive utility  $\bar{u}$ .

- (a) Write down the incentive compatibility and the participation constraints of the employees such that (i) they find it beneficial to exert effort, and (ii) they choose to accept the contract.
- (b) Assume that it is profitable for the manager to set  $b$  and  $\bar{w}$  such that these two constraints are satisfied with equality. What is the level of the bonus  $b$  and the base wage  $\bar{w}$  which satisfy these conditions?
- (c) What is the expected profit of the fund-manager if this contract is accepted by the employees?
- (d) If the manager instead finds it more profitable **not** to induce effort from his workers, what is the optimal base wage  $\bar{w}$ ? What will be the value of the bonus in this case?