Public Affairs 818
Professor Wallace
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Midterm Examination #2

Instructions: You have 2 hours to complete the examination and there are 4 questions worth a total of 120 possible points. You may use a calculator, but you may not use pre-programmed formulas.* The standard normal "Z" table from the second page of your book has been provided to you. Partial credit is possible on all questions so be sure to write clearly and show all of your work. If you have questions regarding the wording of a particular question feel free to ask for clarifications. Good luck!

1. **Only Four Years to 2012 (30 pts.)** Now that the 2008 election has ended, we can all begin obsessing over polls for the 2012 election. The first poll for the 2012 Republican Primary was done by Newsweek last week. The following results come from 257 registered Republican voters in their sample.

Newsweek 2012 Republican Presidential Nomination

Mitt Romney	35%
Mike Huckabee	26%
Sarah Palin	20%

a. (5 pts) Describe the distribution of the sample proportion of Americans who would vote for Mitt Romney. A complete answer describes the distribution, mean and variance in terms of population parameters.

$$\bar{p}_{mr} \sim N\left(p_{mr}, \frac{p_{mr}(1-p_{mr})}{257}\right)$$

b. (5 pts) Using the results of the CNN/Opinion Research Corporation poll, construct a 95% confidence interval around the proportion of Republicans who support Mitt Romney.

$$0.35 \pm 1.96\sqrt{\frac{0.35 \cdot 0.65}{257}} \Rightarrow 0.35 \pm 0.059$$

* In the interest of fairness I expect those of you with fancy graphing calculators that can compute probabilities associated with the normal distribution not to use this feature.

c. (5 pts) Suppose that Rachel Maddow claims that more than 40% of Republicans support Mitt Romney. Write down a null and alternative hypothesis in terms of population parameters that will allow you to assess whether there is strong evidence against Rachel Maddow's claim.

$$H_0 : p_{mr} \geq 0.40$$
$$H_A : p_{mr} < 0.40$$

d. (5 pts) What is the p-value associated with this hypothesis test? Can we reject Maddow's claim?

$$p-value = P\left(\overline{p}_{mr} < 0.35 \text{ given } p_{mr} = 0.40\right) = P\left(Z < \frac{0.35-0.40}{\sqrt{\frac{0.40\cdot 0.60}{257}}} = -1.63\right) \approx 0.052$$

Thus, we reject the null (and conclude that Maddow was wrong) at $\alpha = 0.10$, but fail to reject at lower significance levels.

e. (5 pts) Recalculate your p-value from part (d), but instead assume that n=500.

$$p-value = P\left(\overline{p}_{mr} < 0.35 \text{ given } p_{mr} = 0.40\right) = P\left(Z < \frac{0.35-0.40}{\sqrt{\frac{0.40\cdot 0.60}{500}}} = -2.28\right) = 0.011$$

Thus, we would be able to reject the null at any significance level above 0.01 if the sample size was 500

f. (5 pts) Why did your p-value shift from part (d) to part (e)?

The standard error of p-bar got substantially smaller due to the larger sample size.

2. **Government Spending and Growth (30 pts.)** The Penn World Table from the University of Pennsylvania's economics department contains an array of data relating growth, trade, population, and government expenditure. The following data come from the 82 countries in 2004 that had available data.

   You are interested here in the relation between gross domestic product per capita (*gdp*) and government expenditure as a share of gross domestic product (*gov*). You separate *gdp* based on whether government expenditure is high (gdp^{HI}) and whether government expenditure is low (gdp^{LO}).

   Assume that you know that the true distribution of *gdp* comes from a normal distribution with mean μ=15,800. You observe a sample standard error of s=3000. Assume α=.05.

   a. (5 pts) What is the assumed distribution of the sample mean of *gdp*? A complete answer describes the distribution, mean and variance in terms of population parameters.

   $$\overline{X} \sim N\left(15,800, \frac{\sigma_{gdp}^2}{82}\right)$$

   b. (5 pts) You observe a value of gdp^{LO}=16,850, with n=41. State the relevant null and alternative hypotheses.

   $$H_0 : \mu_{gdp}^{Lo} = 15,800$$
   $$H_A : \mu_{gdp}^{Lo} \neq 15,800$$

   c. (5 pts) Give a general definition of a p-value. (5 pts)

   The p-value is the probability of observing a sample mean at least as likely as the one that was observed. In this context we observed 16,850 so the p-value would be the probability of drawing a value of $\overline{X} > 16,850$ ***or a value of*** $\overline{X} < 14,750$ ***given n=41 and*** $\mu_{gdp}^{Lo} = 15,800$**.**

   d. (5 pts) What is the p-value for this particular hypothesis test? (Give a specific number.) Do we reject the null?

$$p - value = P\left(\overline{X} > 16{,}850 \text{ or } \overline{X} > 14{,}750 \text{ given } n = 41 \text{ and } \mu_{gdp}^{Lo} = 15{,}800\right)$$

$$= P\left(Z > \frac{16850 - 15{,}800}{\frac{3000}{\sqrt{41}}} = 2.24 \text{ or } Z < \frac{14{,}750 - 15{,}800}{\frac{3000}{\sqrt{41}}} = -2.24 \right) \approx 0.025$$

We would reject the null with α >0.025 and fail to reject otherwise.

e. (5 pts) What does the p-value for this particular hypothesis tell us in simple language?

The probability of drawing a sample mean at least as extreme as the one was drawn is 0.025.

f. (5 pts) Explain in simple language how the size of the p-value helps us to determine whether or not to reject the null hypothesis.

If the probability of drawing a sample mean at least as likely as the one that was draw is small, then (i) a fairly rare event happened or (ii) the true distribution of the sample mean is not centered on 15,800. In the hypothesis testing framework we always choose (ii) and reject the null

3. **Is Our Children Learning? (40 pts.)** One area of policy research involves whether students using vouchers to attend private schools perform better on standardized tests than those attending public schools. Research by Prof. John Witte has explored this question at length.

A pilot program in Milwaukee from 1990-1995 was called the Public Choice Program. Parent had the opportunity to use a voucher and apply for their child's acceptance to a private school, and the private school then selected applications at random. The following question concerns 1991 scores of students who were accepted to private schools in Milwaukee, the set of all Milwaukee Public School low-income students, and a "control" group of students who applied but were not accepted. The data is below:

	Enrolled Choice		Milwaukee Public School Low-Income		Milwaukee Public School Control	
	Reading	Math	Reading	Math	Reading	Math
1991 Mean Test Scores	39.8	39.0	38.0	41.5	40.0	43.4
Standard Deviation	17.0	18.6	15.1	17.3	16.6	18.4
N	192	198	911	895	1173	1148

a. (5 pts) Comparisons of tests scores between charter schools and public school students ignore a crucial selection bias. Explain.

True. The students that attend charter and choice schools are likely to differ according on both observable and non-observable dimensions from public school children. If these observable and unobservable characteristics are associated (correlated) with test performance then a comparison of means will lead to biased estimates of the impact of attending a charter or choice school on test performance.

b. (5 pts) Consider students enrolled in the choice program. Construct a 95% confidence interval for the sample mean of observed math scores.

$$39 \pm 1.96\left(\frac{18.6}{\sqrt{198}}\right) \Rightarrow 39 \pm 2.59$$

c. (5 pts) Would you expect wider or shorter confidence intervals for the control group? Why?

A much shorter (narrower) confidence interval for the control group because the sample standard deviation is similar, but the sample size is much, much larger.

The Milwaukee City Council will only institute the choice program if there is strong evidence to suggest that students enrolling in the program will have a mean math test score above 42. *This is now your alternative hypothesis*.

d. (5 pts) Clearly state the relevant null and alternative hypotheses. Define Type II error and power. What does Type II error mean in this situation, i.e. what effect will a Type II error committed by the legislature have on Milwaukee students?

$$H_0 : \mu_c \leq 42$$
$$H_a : \mu_c > 42$$

Type II Error – failure to correctly reject the null.

Power=1-P(Type II Error) – the probability of correctly rejecting the null.

In this context a Type II Error would involve not instituting a choice program despite the fact that mean test scores for the program are above 42

e. (5 pts) If we take α= 0.05 and do a one-tailed test, what is the cut-off value between accepting and rejecting the null?

Note the distribution of $\bar{x}$ ***under the null is*** $\bar{X} \sim N\left(42, \frac{\sigma^2}{198}\right)$

Thus, the null will be rejected anytime

$$\frac{\bar{x} - 42}{\left(\frac{18.6}{\sqrt{198}}\right)} > 1.645 \Rightarrow \bar{x}_c = 44.17$$

Anytime the mean is greater than 44.17 the null will be rejected.

f. (5 pts) What is the probability of a Type II error, with α= 0.05 and your alternative hypothesis from part (d)?

Assume $\mu_c = 45$***. Then to calculate the power we need to have calculate the probability of drawing a value of*** $P\left(\overline{X}_c > 42.15 \text{ given } \mu_c = 45\right)$***.***

$$P\left(\overline{X}_c > 44.17 \text{ given } \mu_c = 45\right) = P\left(Z > \frac{44.17 - 45}{\left(\frac{18.6}{\sqrt{198}}\right)} = -0.62 \right) \approx 0.73$$

With the true mean test square equal to 45 (we could have selected a higher value or lower value), the power of the test is 0.73. Thus, the probability of a Type II Error is zero.

g. (5 pts) If the scientists used α= 0.01 instead, how would that change the previous two answers? How are α and power related?

$$\frac{\overline{x} - 42}{\left(\frac{18.6}{\sqrt{198}}\right)} > 2.33 \Rightarrow \overline{x}_c = 45.07$$

$$power = P\left(\overline{X}_c > 45.07 \text{ given } \mu_c = 45\right) = P\left(Z > \frac{45.07 - 45}{\left(\frac{18.6}{\sqrt{198}}\right)} \approx 0.05 \right) = 0.48$$

$$\Rightarrow P\left(Type\ II\ \ Error\right) = 1 - 0.48 = 0.52$$

4. **Super Batteries, Inc. (20 points)** The current state of the art cell phone battery last an average of 5 hours in continuous talk mode before it needs to be recharged. Super Batteries Inc. believes it has developed a new cell phone battery that last longer than an average of 5 hours in continuous talk mode. Super Batteries Inc. decides to test a sample of batteries in continuous talk mode and record how long each batter last. Suppose you know that σ = 2, and you decide to fix the probability of a Type I error at 5%.
   a. (5 points) State the relevant null and alternative hypotheses if Super Batteries, Inc would like to claim that there is strong evidence that their new battery last longer than the current state of the art battery.

$$H_0 : \mu \leq 5$$
$$H_A : \mu > 5$$

   b. (15 points) Super Batteries commissions you to conduct a study to evaluate the hypothesis test in part (a). Determine the sample size that will allow you to correctly conclude that Super Batteries Inc's new battery lasts longer than the current state of the art battery with probability 0.95 if the true average life for their new battery is 5.5 hours.

Step 1: Find the values of $\overline{X}$ that will lead to the rejection of the null in part

$$\frac{\overline{X}_c - 5}{\frac{2}{n}} = 1.645 \Rightarrow \overline{X}_c = 5 + \frac{3.29}{\sqrt{n}} = \frac{5\sqrt{n} + 3.29}{\sqrt{n}}$$

Step 2: We need the probability of drawing a value of $\overline{X}_c > \frac{5\sqrt{n}+3.29}{\sqrt{n}}$ to be equal to 0.95 if the true battery life is 5.5. Thus, we need

$$\overline{X}_c > \frac{\frac{5\sqrt{n}+3.29}{\sqrt{n}} - 5.5}{\left(\frac{2}{\sqrt{n}}\right)} = -1.645 \Rightarrow \frac{5\sqrt{n} + 3.29 - 5.5\sqrt{n}}{2} = -1.645$$

$$\Rightarrow -0.5\sqrt{n} + 3.29 = -3.29 \Rightarrow \sqrt{n} = 13.16 \Rightarrow n \approx 174$$

An n around 174 should ensure power of 0.95.