Public Affairs 818
Professor Wallace
Fall 2010

Midterm #1 Review Sheet

You will not be provided with any formulas

- **Probability Rules**
  - $P(A \cap B) = P(A \mid B) \cdot P(B) = P(B \mid A) \cdot P(A)$
  - $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
  - $$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cup B) - P(A \cup C) - P(B \cup C) + P(A \cap B \cap C)$$

- **Bayes' Theorem**

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B \mid A)P(A) + P(B \mid A^c)P(A^c)}$$

- **Counting Rules form Combinations and Permutations**
  - $_nC_k = \binom{n}{k} = \frac{n!}{k!(n-k!)}$
  - $_nP_k = \frac{n!}{(n-k)!}$

- **Discrete Random Variables**
  - The distinction between discrete and continuous RVs
  - The probability function $f(x) = P(X = x)$
  - The cumulative distribution function $F(x) = P(X \le x)$
  - How to calculate the expected value of a discrete RV
    $$E(x) = \mu_x = \sum x_i f(x_i)$$
  - How to calculate the variance of a discrete RV
    $$Var(x) = \sum (x_i - \mu)^2 f(x_i)$$

- **Properties of Expectation and Variance**
    - $E(a \cdot x) = a \cdot E(x)$
    - $E(a \cdot x + b) = a \cdot E(x) + b$
    - $E(x + y) = E(x) + E(y)$
    - $Var(x) = E(x^2) - \left[E(x)\right]^2 = E(x^2) - \mu^2$
    - $Var(a \cdot x) = a^2 \cdot Var(x)$
    - $Var(a \cdot x + b) = a^2 \cdot Var(x)$
    - $Var(x + y) = Var(x) + Var(y) + 2 \cdot Cov(x, y)$
    - $Var(x - y) = Var(x) + Var(y) - 2 \cdot Cov(x, y)$

- **Binomial Experiments**
    - Be able to identify
    - Be able to calculate the probabilities of outcomes of binomial experiments. You ***will*** need to remember the binomial probability function, but you will need to remember how to use it.

$$f(x) = \binom{n}{x} p^x \cdot (1 - p)^{n-x}$$

    - The mean and variance of binomial RVs

$$E(x) = np$$

$$Var(x) = np(1 - p)$$

- **The Hypergeometric Distribution**
    - Be able to identify hypergeometric RVs – Make sure that you understand the distinction between hypergeometric RVs and Binomial RVs.
    - Be able to calculate the probabilities associated with hypergeometric random variables.
    - You ***will*** need to remember the hypergeometric probability function

$$\frac{\binom{r}{x}\binom{N-r}{n-x}}{\binom{N}{n}}.$$

- **Continuous RVs**
  - Be able to distinguish continuous RVs from discrete RVs.
  - Know that $P(X = x) = 0$ for all X
  - The area under the probability density function over a certain interval $\left[x_0, x_1\right]$ equals $P(x_0 < X < x_1)$
  - The cumulative distribution function $F(x) = P(X \le x)$

- **Uniform Distribution**
  1. Be able to the probability that a uniformly distributed RV falls in a specified range.
  2. You will need to know the uniform pdf
  $$f(x) = \frac{1}{b-a} \qquad for\ a \le x \le b$$
  3. The expected value of a uniform RV
  $$E(x) = \frac{\left(b+a\right)}{2}$$

- **The Normal Distribution**
  - Important Properties of
    - Symmetry
    - Highest Point of the normal pdf is the mean and median
    - There are infinitely many normal distributions differentiated by the values of μ and σ^2.
  - Important Properties of Normal Random Variables
    - A liner transformation of a normal of a normal RV is a normal RV
    - The sum of two normal random variables is normal
  - How to turn any normal RV into a standard normal. If $X \sim N(\mu_X, \sigma_X^2)$ then $Z = \dfrac{X - \mu_X}{\sigma_X} \sim N(0,1)$
  - Be comfortable using the normal table that is on the second page of the text.

- **Chapter 7 - Sampling Theory**
  - Understand the implications and power of the Central Limit Theorem.
  - Know the sampling distributions of $\bar{x}$ and $\bar{p}$ when the assumptions of the Central Limit Theorem are met
  $$\bar{x} \sim N\left(\mu, \frac{\sigma_x^2}{n}\right)$$
  $$\bar{p} \sim N\left(p, \frac{p(1-p)}{n}\right)$$

- At this point you should understand that drawing a random sample of size n to obtain a sample proportion is a binomial experiment with n trials. The outcome of this experiment is the number of success obtained. The number of successes divided by the number of trials is the sample proportion $\overline{p}$.
- Understand why knowing the distributions of $\overline{x}$ and $\overline{p}$ are important. You should be able to reason through the exercises that we did in class were we assumed a specific population mean (proportion) and then determined if the sample mean (proportion) we obtained was likely given the population mean (proportion) that we assumed.

- **Chapter 8 - Interval Estimation**
  - Be able to construct 90, 95, and 99 percent confidence intervals in the cases where the Central Limit Theorem is applicable.
  - Understand how to construct 90, 95, and 99 percent confidence intervals when the Central Limit Theorem is not applicable
  - If the population is normal you can construct confidence intervals using the t-table (if σ is unknown and must be estimated by s) or the standard normal table (if σ is known)).
  - Be able to find the sample size needed to insure a specific margin of error for a specified confidence level.

- **Chapter 9 – Hypothesis Testing**
  - The sampling distributions of $\overline{x}$ and $\overline{p}$
  - The distinction between Type I and Type II errors.
  - One and two tailed hypothesis test for population means and proportions.
  - Be able to describe and provide intuition for the hypothesis testing procedure in words.
  - Understand power and be able to do power calculations.