Assignment #4*

Econ 302: Intermediate Macroeconomics

November 4, 2009

1 Aggregate Demand / Aggregate Supply

Long-run aggregate supply curve (vertical):

$$Y_{LR} = 3000 \tag{1}$$

Short-run aggregate supply curve (horizontal):

$$P_{SR} = 1 \tag{2}$$

Aggregate demand curve:

$$Y = \frac{3M}{P} \tag{3}$$

Money supply:

$$M = 1000 \tag{4}$$

1.1 If the economy is initially in long-run equilibrium, what are the values of $P_{LR,1}$ and $Y_{LR,1}$?

1.2 What is the velocity of money in initial long-run equilibrium $(P_{LR,1}, Y_{LR,1})$?

1.3 Now suppose a supply shock moves the short-run aggregate supply curve to $P = 1.5$ (still horizontal). What is the new short-run equilibrium $(P_{SR,2}, Y_{SR,2})$?

1.4 If the aggregate demand and long-run aggregate supply curves are unchanged, what is the long-run equilibrium $(P_{LR,2}, Y_{LR,2})$ after the supply shock?

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1.5 Suppose that after the supply shock, the Federal Reserve wants to hold output at its long-run level. What level of the money supply M' would be required to achieve this in the short-run?

2 Growth Accounting / Solow Residual

Assume the following functional form for the aggregate production function:

$$Y = AK_p^{\alpha_1} K_h^{\alpha_2} L^{\alpha_3} \tag{5}$$

Here, K_p denotes physical capital; K_h human capital; L labor. The constant returns to scale condition $\alpha_1 + \alpha_2 + \alpha_3 = 1$ holds.

2.1 Write $\%\Delta Y$ in terms of $\%\Delta A$, $\%\Delta K_p$, $\%\Delta K_h$, and $\%\Delta L$ (hint: $\Delta \log(x) \approx \%\Delta x$).

2.2 Based on your answer to the previous part, write $\%\Delta A$ in terms of the other variables. Interpret this as the growth rate of the "Solow residual". In what sense is A a residual here?

2.3 <u>For this part only</u>, assume that $\%\Delta Y = 0.05$, $\%\Delta K_p = 0.02$, $\%\Delta K_h = 0.04$, $\%\Delta L = 0.01$, $\alpha_1 = 0.2$, $\alpha_2 = 0.6$, $\alpha_3 = 0.2$. Compute $\%\Delta A$. What is the most important source of output growth here?

2.4 Now, let's write everything in per-worker terms. Define $y = \frac{Y}{L}$; $k_p = \frac{K_p}{L}$; $k_h = \frac{K_h}{L}$. Write y as a function of k_p and k_h only.

2.5 Write $\%\Delta y$ in terms of $\%\Delta A$, $\%\Delta k_p$, and $\%\Delta k_h$.

2.6 Based on your answer to (2.5), write $\%\Delta y$ in terms of $\%\Delta A$, $\%\Delta K_p$, $\%\Delta K_h$, and $\%\Delta L$. *(hint:* $\%\Delta(\frac{A}{B}) \approx \%\Delta A - \%\Delta B$*)*

2.7 Based on your answer to (2.5), write $\%\Delta A$ in terms of $\%\Delta y$, $\%\Delta k_p$, and $\%\Delta k_h$ only. In what sense is A a residual here?

2.8 **If human capital K_h is unobservable (can't gather data for it), we set $\%\Delta k_h = 0$ and try to estimate productivity growth using the equation: $\%\Delta\hat{A} = \%\Delta y - \alpha_1 \%\Delta k_p$. Assuming that our data on y, α_1, and k_p are accurate, will our estimated value $\%\Delta\hat{A}$ be too high or too low relative to the true $\%\Delta A$? *(hint: you should have three cases based on $\%\Delta k_h$)***

2.9 **Is it possible that $\%\Delta\hat{A} < 0$? Interpret. Is this consistent with the idea that A is the level of technology available in an economy?**

3 Solow Model with Population Growth

For this problem, try to keep time subscript t on all the relevant (i.e. changing over time) variables out of steady-state. This will help you for the next problem. Assume that the following variables are constant over time: depreciation rate δ; savings rate s; population growth rate n. Because population growth is non-zero, the labor force is now a function of time.

Definitions:

$$y_t = \frac{Y_t}{L_t} \tag{6}$$

$$k_t = \frac{K_t}{L_t} \tag{7}$$

$$c_t = \frac{C_t}{L_t} \tag{8}$$

$$i_t = \frac{I_t}{L_t} \tag{9}$$

Aggregate production function in per-worker terms:

$$y_t = f(k_t) \tag{10}$$

Labor force (population) growth equation:

$$L_t = (1+n)L_{t-1} \tag{11}$$

3.1 Write c_t and i_t as functions of k_t.

3.2 Write the law of motion for k_t, $\Delta k_t \equiv k_{t+1} - k_t$.

3.3 What is the law of motion for c_t, $\Delta c_t \equiv c_{t+1} - c_t$; the law of motion for i_t, $\Delta i_t \equiv i_{t+1} - i_t$? *(hint: should be a function of k_t and constants)*

3.4 Assume a steady-state in k_t; $k_t = \bar{k} = k_{ss}$. What are Δk_t, Δc_t, and Δi_t?

3.5 Continue to assume a steady-state in k_t. Write $\frac{\Delta K_t}{K_t}$, $\frac{\Delta L_t}{L_t}$, $\frac{\Delta Y_t}{Y_t}$, $\frac{\Delta C_t}{C_t}$, and $\frac{\Delta I_t}{I_t}$ in terms of constants. *(hint: these are not per-worker quantities, so they will have non-zero growth rates)*

3.6 **Continue to assume a steady-state in k_t. What are the steady-state levels k_{ss}, y_{ss}, c_{ss}, i_{ss}? *(hint: you can leave k_{ss} in your answers)***

3.7 **Solve for the "golden rule" level of the (per-worker) capital stock, k_{gr}, provided that $f(k_t) = k_t^\alpha$. *(hint: maximize c_{ss} as a function of k_{ss} from the previous part; your function for c_{ss} shouldn't contain savings rate s; your answer should depend only on constants)***

3.8 **Solve for the savings rate s_{gr} that supports k_{gr} from the previous part. *(hint: use the law of motion for the capital stock; your answer should depend only on constants)***

3.9 **Let $\delta = 0.02$, $n = 0.01$, and $\alpha = 0.3$. Compute k_{gr} and s_{gr}.**

4 Computational Solow Model (do Q3 first)

From parts (3.1) and (3.2) of Q3, you should be able to implement the Solow model with population growth in Excel.
Parameter values and initial conditions:

$$f(k_t) = k_t^\alpha \tag{12}$$

$$\delta = 0.02 \tag{13}$$

$$n = 0.01 \tag{14}$$

$$\alpha = 0.3 \tag{15}$$

$$s = 0.15 \tag{16}$$

$$k_{t=0} = k_0 = 2 \tag{17}$$

General hints:

1. You can leave everything in per-worker terms. The $-nk_t$ term in the law of motion is all you need to include to account for L_t growing at rate n.
2. Make a table of t, k_t, y_t, c_t, and i_t in the usual way. You already know how y_0, c_0, and i_0 depend on k_0; write analogous formulas in Excel. Your formulas should refer to cells (relative references), so don't substitute in the numbers directly. Fill in k_0.
3. Use the law of motion to write k_1 in terms of k_0 and constants (again, refer to cells). Copy your row formulas for k_t, y_t, c_t, and i_t down to run the model. Now you can see convergence to the steady-state!

4.1 Graph k_t vs. time for $0 \leq t \leq 300$. What is the steady-state level of the capital stock?

4.2 Let $s = s_{gr}$, where s_{gr} is the golden rule savings rate that you found in part (3.9). Graph k_t vs. time for $0 \leq t \leq 300$. Is the steady-state level of the capital stock now the golden rule level found in (3.9)?

4.3 Continue with $s = s_{gr}$. At $t = 300$, a new production technology is introduced such that $\alpha_{t \geq 300} = 0.4$. Graph k_t, c_t, and i_t vs. time for $300 \leq t \leq 600$. What is the new steady-state level of the capital stock?

4.4 **Continue with** $s = s_{gr}$ **and** $\alpha_{t \geq 300} = 0.4$**. A natural disaster at time** $t = 600$ **occurs such that** $\delta_{t \geq 600} = 0.10$ **and** $n_{t \geq 600} = -0.01$**. Graph** k_t**,** c_t**, and** i_t **vs. time for** $600 \leq t \leq 900$**. What is the new steady-state level of the capital stock?**

4.5 **In words, describe how you would modify your Excel file to account for** s**,** n**, and** δ **varying over time. You do not have to implement this in Excel.**