Ch. 8 Exercise: Solow Model (Population Growth, Technological Change)

Model:

Consider the Solow growth model with population growth and technological change. Time is discrete and is indexed by subscript t.

a) Rewrite aggregate production function $Y_t = 20K_t^{\frac{1}{2}}(L_tE_t)^{\frac{1}{2}}$ in effective units.

$$\frac{Y_t}{E_tL_t} = 20K_t^{\frac{1}{2}}(E_tL_t)^{-\frac{1}{2}} \Rightarrow y_t = 20k_t^{\frac{1}{2}}$$

b) Let $K_0 = L_0 = 100$; $E_0 = 1$. What is k_0? *(note: $t = 0$ here)*

$$k_0 = \frac{K_0}{E_0L_0} = \frac{100}{1(100)} = 1$$

c) Using your answer from the previous part, solve for the initial level of GDP Y_0.

$$Y_t = 20K_t^{\frac{1}{2}}(E_tL_t)^{\frac{1}{2}} \Rightarrow Y_0 = 20K_0^{\frac{1}{2}}(E_0L_0)^{\frac{1}{2}} = (20)(10)(10) = 2000$$

d) Provided that $\delta = 0.2$ and $n = g = 0.15$, what savings rate is necessary to sustain k_0 as a steady-state? *(hint: set $k_0 = k_{ss}$ in the steady-state condition and solve for s)*

$$\Delta k_0 = s(20k_0^{\frac{1}{2}}) - (n + g + \delta)(k_0) = 0 \Rightarrow 20s = 0.5 \Rightarrow s = \frac{1}{40} = 0.025$$

e) Using your answer from the previous part, solve for c_{ss} and i_{ss}.

$$c_{ss} = 20k_{ss}^{\frac{1}{2}} - (n+g+\delta)k_{ss} = 20 - 0.5 = 19.5 \Rightarrow i_{ss} = y_{ss} - c_{ss} = 20 - 19.5 = 0.5$$

f) What is Y_1, the level of real GDP next year? *(note: $t = 1$ here)*

$$Y_1 = (1+n+g)Y_0 = 1.3(2000) = 2600$$

g) What is $\frac{K_1}{L_1}$, the capital-labor ratio next year?

$$\frac{K_1}{L_1} = (1+g)\frac{K_0}{L_0} = 1.15(1) = 1.15$$

h) What is the "golden rule" level of k for this economy? Recall that the golden rule level of the capital stock per effective worker k_{gr} maximizes consumption per effective worker in steady-state.

$$c_{ss} = 20k_{ss}^{\frac{1}{2}} - (n+g+\delta)k_{ss}$$

$$\frac{\partial}{\partial k_{ss}}(c_{ss}) = 10k_{ss}^{-\frac{1}{2}} - (n+g+\delta) = 0 \Rightarrow k_{gr} = [\frac{10}{(n+g+\delta)}]^2 = [\frac{10}{0.5}]^2 = 400$$