Ch. 7 Exercise: Solow Model

Model:
Consider the Solow growth model without population growth or technological change. The parameters of the model are given by $s = 0.2$ (savings rate) and $\delta = 0.05$ (depreciation rate). Let k denote capital per worker; y output per worker; c consumption per worker; i investment per worker.

a) Rewrite production function $Y = K^{\frac{1}{3}} L^{\frac{2}{3}}$ in per-worker terms.

Divide both sides by L to get output per worker on the left-hand side.

$$\frac{Y}{L} = \frac{K^{\frac{1}{3}} L^{\frac{2}{3}}}{L} = (\frac{K}{L})^{\frac{1}{3}} = k^{\frac{1}{3}}$$

b) Find the steady-state level of the capital stock, k_{ss}.

Write the steady-state condition for the Solow model and solve for the steady-state level of the capital stock, k_{ss}.

$$sf(k_{ss}) = \delta k_{ss}$$

$$sk_{ss}^{\frac{1}{3}} = \delta k_{ss}$$

$$k_{ss}^{\frac{2}{3}} = \frac{s}{\delta}$$

$$k_{ss} = (\frac{s}{\delta})^{\frac{3}{2}} = (\frac{0.2}{0.05})^{\frac{3}{2}} = 8$$

c) What is the "golden rule" level of k for this economy? Recall that the golden rule level of the capital stock k_{gr} maximizes consumption per worker in steady-state. Report your answer to two decimal places.

Write consumption per worker as a function of the capital stock in steady-state.

$$c(k) = f(k) - \delta k$$

We are maximizing steady-state consumption; take the first-order condition with respect to k.

$$c'(k) = f'(k) - \delta = 0 \Rightarrow f'(k_{gr}) = \delta$$

$$\frac{1}{3}k_{gr}^{-\frac{2}{3}} = \delta$$

As per the question, report your numerical answer to two decimal places.

$$k_{gr} = (\frac{1}{3\delta})^{\frac{3}{2}} = (\frac{1}{3(0.05)})^{\frac{3}{2}} = 17.21$$

d) Let's say that a benevolent social planner wishes to obtain $k = k_{gr}$ in steady-state. What is the associated savings rate s_{gr} that must be imposed by the social planner to support k_{gr}?

To find the associated savings rate s_{gr}, solve for s in the law of motion $\Delta k = 0$ (steady-state again) provided that $k = k_{gr}$.

$$\Delta k = s_{gr} f(k_{gr}) - \delta k_{gr} = 0$$

Use the exact value for k_{gr} here; you will introduce error into your answer for s_{gr} if k_{gr} is rounded or truncated.

$$s_{gr} = \frac{\delta k_{gr}}{f(k_{gr})} = \delta((\frac{1}{3\delta})^{\frac{3}{2}})^{\frac{2}{3}} = \frac{1}{3}$$

Therefore, we conclude that the welfare-maximizing social planner sets $s = \frac{1}{3}$.

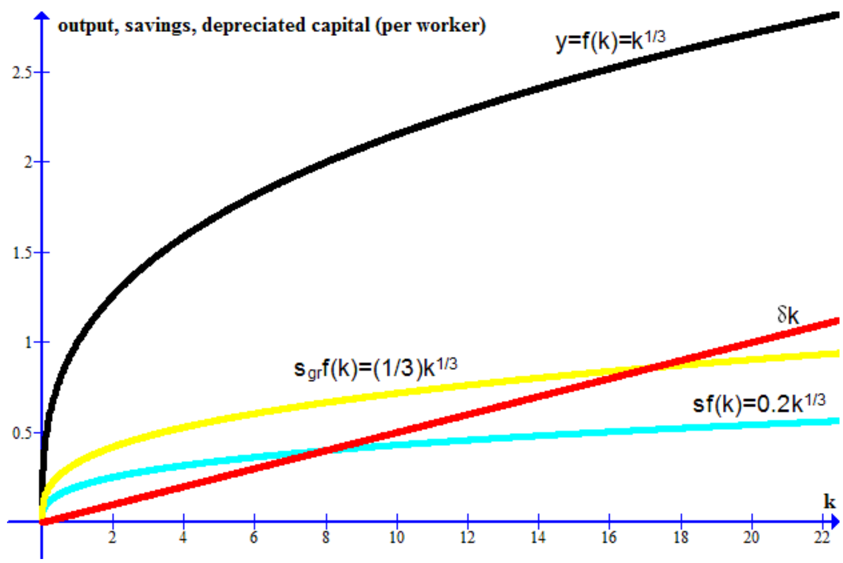
e) Compare your result in the previous part with the assumed savings rate s. To obtain k_{gr}, do citizens need to save more or less?

$$s = 0.2;\ s_{gr} = \frac{1}{3}$$

$$s_{gr} > s$$

Citizens need to save more under the "golden rule" policy regime, which implies that consumption per worker decreases in the short-run to obtain a long-run improvement in the standard of living (as measured by consumption per worker).

f) Plot the following on a single graph: $y = f(k)$, δk, $sf(k)$, and $s_{gr}f(k)$. Does the savings curve pivot up or down, relative to its initial position, when the planner's s_{gr} is implemented?



Relative to the $s = 0.2$ case, the savings curve pivots up as s_{gr} is implemented by the social planner.

g) Discuss two to three economic policies that could help the social planner implement s_{gr} in a real-world situation.

1. *Savings tax credit for consumers.*
2. *Tax on consumer goods.*
3. *Social security or any type of mandatory savings program.*
4. *Education on the long-term benefits of saving for retirement.*
5. *Contractionary monetary policy $\Rightarrow$ real interest rate increases $\Rightarrow$ increased level of savings.*