# *Chapter 11: Applying the IS / LM Model*

Consider the IS / LM model.
Consumption function:

$$C = a + b(Y - T) \tag{1}$$

Investment function:

$$I = c - dr \tag{2}$$

Real money demand:

$$L(r, Y) = l_1 Y - l_2 r \tag{3}$$

Parameters:
$a > 0$, $0 < b < 1$, $c > 0$, $d > 0$, $l_1 > 0$, $l_2 > 0$

Given the information above, please answer the following questions:

a) Given equations (1) and (2), solve for $Y$ as a function of $r$, $G$, $T$, and parameters (IS curve).

$$\begin{aligned} Y &= C + I + G \\ Y &= (a + b(Y - T)) + (c - dr) + G \\ Y(1 - b) &= a - bT + c - dr + G \\ Y &= \frac{1}{1-b}[a - bT + c - dr + G] \end{aligned}$$

b) How does the slope of the IS curve depend on $d$, the interest rate sensitivity of investment?

$$\begin{aligned} m_{IS} &\equiv \frac{\partial Y}{\partial r} = \frac{-d}{1-b} \\ \frac{\partial}{\partial d} m_{IS} &= \frac{-1}{1-b} < 0 \end{aligned}$$

c) Which will cause a larger horizontal shift in the IS curve, a \$100 tax cut or a \$100 increase in government spending?

$$\begin{aligned} |\frac{\partial Y}{\partial T}| &= |\frac{-b}{1-b}| = \frac{b}{1-b} \\ |\frac{\partial Y}{\partial G}| &= |\frac{1}{1-b}| = \frac{1}{1-b} \\ 0 < b < 1 \;\Rightarrow\; |\frac{\partial Y}{\partial G}| &> |\frac{\partial Y}{\partial T}| \end{aligned}$$

*Provided that $\Delta G = -\Delta T$, $\Delta G$ shifts the IS curve more than $\Delta T$.*

d) Given equation (3), solve for $r$ as a function of $Y$, $M$, $P$, and parameters (LM curve).

$$(\frac{M}{P})^d = L(r,Y) = l_1 Y - l_2 r$$
$$(\frac{M}{P})^s = \frac{M}{P}$$
$$(\frac{M}{P})^d = (\frac{M}{P})^s \Rightarrow l_1 Y - l_2 r = \frac{M}{P}$$
$$r = \frac{1}{l_2}[l_1 Y - \frac{M}{P}]$$

e) Using your answer from the previous part, how does the slope of the LM curve depend on $l_2$, the interest rate sensitivity of real money demand?

$$m_{LM} \equiv \frac{\partial r}{\partial Y} = \frac{l_1}{l_2}$$
$$\frac{\partial}{\partial l_2} m_{LM} = -\frac{l_1}{(l_2)^2} < 0$$

$l_2 \uparrow \Rightarrow m_{LM} \downarrow$ *(flatter)*
$l_2 \uparrow \Rightarrow$ *money demand more sensitive to real interest rates* $\Rightarrow r$ *responds less strongly to changes in income to achieve money market equilibrium; with some* $\Delta Y$, *smaller* $\Delta r$ *needed to return to equilibrium*

f) How does the size of the shift in the LM curve resulting from a \$100 increase in $M$ depend on $l_1$? What about $l_2$?

$$\mu \equiv \frac{\partial r}{\partial M} = -\frac{1}{P l_2}$$
$$\frac{\partial}{\partial l_1}\mu = 0$$
$$\frac{\partial}{\partial l_2}\mu = \frac{1}{P(l_2)^2} > 0$$

g) Use your answers from parts (a) and (d) to derive an expression for the aggregate demand curve. You should solve for $Y$ as a function of $P$, $M$, $G$, $T$, and parameters; the resulting expression should not depend on $r$.

From part (a):

$$Y = \frac{1}{1-b}[a - bT + c - dr + G]$$

From part (d):

$$r = \frac{1}{l_2}[l_1 Y - \frac{M}{P}]$$

AD curve (substitute LM curve into IS curve):

$$Y = \frac{1}{1-b}[a - bT + c - \frac{d}{l_2}(l_1 Y - \frac{M}{P}) + G]$$

$$Y(1 + \frac{dl_1}{l_2} - b) = a - bT + c + \frac{dM}{Pl_2} + G$$

$$Y = \frac{1}{1 + \frac{dl_1}{l_2} - b}[a - bT + c + \frac{dM}{Pl_2} + G]$$

h) Using your answer from the previous part, show that the aggregate demand curve is downward-sloping (negative slope).

$$\frac{\partial Y}{\partial P} = -\frac{dM}{(1 + \frac{dl_1}{l_2} - b)P^2 l_2} < 0$$

$\frac{\partial Y}{\partial P} < 0 \Rightarrow$ *aggregate demand curve is downward-sloping*

i) Use your answer from part (g) to show that increases in $G$ and $M$, and decreases in $T$, shift the aggregate demand curve to the right. How does this result change if parameter $l_2 = 0$ (real money demand does not depend on the real interest rate)?

Case $l_2 \neq 0$:

$$\frac{\partial Y}{\partial M} = \frac{d}{(1 + \frac{dl_1}{l_2} - b)Pl_2} = \frac{d}{P((1-b)l_2 + dl_1)} = \frac{1}{P(l_1 + \frac{1-b}{d}l_2)} > 0$$

$$\frac{\partial Y}{\partial G} = \frac{1}{1 + \frac{dl_1}{l_2} - b} = \frac{l_2}{dl_1 + (1-b)l_2} > 0$$

$$\frac{\partial Y}{\partial T} = \frac{-b}{1 + \frac{dl_1}{l_2} - b} = \frac{-bl_2}{dl_1 + (1-b)l_2} < 0$$

Case $l_2 \to 0$:

$$\frac{\partial Y}{\partial M} \to \frac{1}{Pl_1} > 0$$

$$\frac{\partial Y}{\partial G} \to 0$$

$$\frac{\partial Y}{\partial T} \to 0$$