Economics 302
Prof. Kelly
Problem Set 5
Due: Wednesday, April 27

ANSWER KEY

Exercise 1 *Comparing Classical and Keynesian-Sticky Price Models*

Consider the market for wheat. In Econ 101, you analyze this market by finding the equilibrium price and quantity as the point where the supply curve intersects the demand curve. If we throw out the graph and handed our functions for supply and demand to a mathematician, she would see two equations (an equation that gave the quantity supplied as a function of price and an equation that gave the quantity demanded as a function of price) and two unknowns (price and quantity) [to be technical, there is a third equation in the system: that quantity demanded equals quantity supplied equals quantity]. This system of equations produces an equilibrium because we have the same number of equations as unknowns.

Now, consider the whole economy. We have 3 unknowns: output, the real interest rate, and the price level. Since we have 3 unknowns, to solve for these 3 unknowns, we better have 3 equations to describe what is going on.

a) Before reading the rest of the question, can you think which 3 equations we are going to use (Hint: what equations/relations connect r, Y, and P)? Try and explain why you are selecting these 3 equations. Don't worry about getting this wrong or right, just think.

The rest of this question proceeds through a series of steps, building the necessary equations one-by-one

Step 1 Loanable Funds Market:

The marginal propensity to consume is mpc. Full-employment output (the output of the economy when workers and capital are employed at their normal levels and intensities) is $\bar{Y} = F(\bar{K}, \bar{L})$. Government spending is G and the tax level is T. The investment function is :

$$I = \frac{.1}{r}$$

b) Define and derive the IS curve in this closed economy (it will have many, many letters hanging around).

The IS curve is the combination of Y and r that are equilibria in the Loanable Funds Market when T, G, and mpc are held constant.

$$\begin{aligned} Y &= C + I + G \\ Y &= mpc(Y - T) + \frac{.1}{r} + G \\ \frac{.1}{r} &= (1 - mpc)Y + mpcT - G \\ r(Y) &= \frac{.1}{(1 - mpc)Y + mpcT - G} \end{aligned}$$

Step 2 Money Market:

The nominal money supply is M, the price level is P, and the demand for real money balances is:

$$L(Y,r) = \frac{Y}{r}$$

c) Is "money" a normal good? Why? (Hint: this is the perfect time to reference an equation in your answer: "If you look at equation ...., we know that .... because ....)

Money is a normal good since $\frac{\partial L(Y,r)}{Y} = \frac{1}{r} > 0$. That is, when income increases, the demand for real money balances increases so long as the price of holding money is positive.

d) Define and derive the LM curve in this economy.

The LM curve is the combination of Y and r that are equilibria in the money market when P and M are held constant.

$$\begin{aligned} r(Y) &= Y/L(Y,r) \\ &= \frac{Y}{M/P} \\ &= \frac{PY}{M} \end{aligned}$$

Step 3 Combining markets:

The IS gives the equilibrium combinations of r and Y in the Loanable Funds Market. The LM curve gives the equilibrium combinations of r and Y in the Money Market. We need both markets to be in equilibrium at the same time.

e) Use your expressions for the IS and LM curves to derive the AD curve.

The AD curve gives P as a function of Y. Equating the IS and LM curves yields:

$$\begin{aligned} \frac{PY}{M} &= \frac{.1}{(1-mpc)Y + mpcT - G} \Rightarrow \\ P(Y) &= \frac{M}{Y\left[(1-mpc)Y + mpcT - G\right]} \end{aligned}$$

If you took the derivative with respect to Y you would find:

$$\begin{aligned} \frac{\partial P(Y)}{\partial Y} &= \frac{-M\left[2Y(1-mpc) + mpcT - G\right]}{\left(Y\left[(1-mpc)Y + mpcT - G\right]\right)^2} \\ &= \frac{-M\left[Y(1-mpc) + S\right]}{\left(Y\left[(1-mpc)Y + mpcT - G\right]\right)^2} \end{aligned}$$

If saving is positive, then the AD curve is downward sloping, as we would expect.

Hopefully you see that the IS and LM curves give you 2 of the 3 necessary equations to solve the system.

f) Is the AD curve the 3rd equation?

No. Once you have the IS and LM curves, equating them to get rid of r still leaves you with 2 unknowns and no additional equations to use. We need another equation that somehow connects P to Y.

The third equation is actually the AS curve and it is what determines whether we are using a Keynesian or a Classical model.

g) What assumption does the Classical Model make about output?

In the Classical Model, it is assumed that Y is fixed, and thus does not change as P changes.

h) What assumption does the Keynesian Model make about the price level?

In the Keynesian Model, it is assumed that P is fixed, and thus does not change as Y changes.

These assumptions provide the third equation, basically they tell us what the AS curve looks like.

i) What is the equation for the AS curve in the Classical Model?

$$Y(P) = \bar{Y}$$

j) What is the equation for the AS curve in the Keynesian Model?

$$P(Y) = \bar{P}$$

Now, let's put in some numbers. Let full-employment output, $\bar{Y}$, equal 12, $T = 2$, $G = 2$, $mpc = .5$, $M = 6000$.

k) Assuming that in equilibrium the economy is producing at full-employment output, how could you use the Loanable Funds and Money Markets to solve for r and P? What is the equilibrium level of r and P?

First, use the Loanable Funds market to solve for r :

$$\begin{aligned} r(Y) &= \frac{.1}{(1 - mpc)Y + mpcT - G} \\ &= \frac{.1}{(1 - .5)12 + .5(2) - 2} \\ &= \frac{.1}{5} = .02 \end{aligned}$$

Now that we have r, use the Money Market to solve for P :

$$\begin{aligned} r(Y) &= \frac{PY}{M} \\ P &= Mr(Y)/Y \\ &= 6000(.02)/12 \\ &= 10 \end{aligned}$$

l) Suppose you don't know the full-employment output, but know that $P = 50$ and the other values (obviously, excluding Y) from above are the same. What are the equilibrium levels of r and Y?

Since we don't know Y or r, we have to use the AD to solve for Y :

$$\begin{aligned} P(Y) &= \frac{.1M}{Y\left[(1-mpc)Y + mpcT - G\right]} \\ 50 &= \frac{600}{.5Y^2 + (.5)(2)Y - 2Y} \\ .5Y^2 - Y &= \frac{600}{50} = 12 \\ Y^2 - 2Y - 24 &= 0 \end{aligned}$$

Now, use the quadratic equation to solve for Y :

$$\begin{aligned} Y &= \frac{2 \pm \sqrt{4 - 4(24)}}{2} \\ &= \frac{2 \pm \sqrt{4 + 96}}{2} \\ &= \frac{2 \pm 10}{2} \end{aligned}$$

Y must be positive, so the answer is $Y = 6$. Now, return to the Money Market to solve for r :

$$r = \frac{50(6)}{6000} = .02$$

Suppose we have the following equilibrium: $Y = 10, G = T = 2, r = .025, P = 20, M = 8000, mpc = .5$. The fed decides to increase M to $10,000$.

m) Show that when $M = 8000$, the economy is in equilibrium.

First, look at the Loanable Funds Market:

$$\begin{aligned} r(Y) &= \frac{.1}{(1-mpc)Y + mpcT - G} \\ r(Y) &= \frac{.1}{.5(10) + .5(2) - 2} \\ &= \frac{.1}{4} = .025 \end{aligned}$$

Thus, the Loanable Funds market clears. Now, consider the Money Market:

$$\begin{aligned} r(Y) &= \frac{PY}{M} \\ &= \frac{20(10)}{8000} \\ &= \frac{200}{8000} = .025 \end{aligned}$$

Thus, the Money Market also clears, and we have equilibrium in the economy.

n) Pretend you are a Classical economist. What happens in the economy? What is the new equilibrium?

We assume the $Y = 10$, so P must adjust. Since Y is the same, the Loanable Funds market hasn't changed and thus the equilibrium interest rate hasn't changed. We can thus look to the Money Market to discover what must happen to P :

$$\begin{aligned} .025 &= \frac{P(10)}{10000} \\ P &= 25 \end{aligned}$$

Notice, we could rewrite money demand as:

$$rM = PY$$

If we assume r is fixed, then we have the quantity theory of money with velocity equaling the interest rate. Because we are using the Classical framework in which Y is fixed, we don't have to assume r is fixed, as it is a result that follows from the Loanable Funds Market. Remember, with the quantity theory of money, an x% increase in the money supply results in an x% increase in the price level. In this example, the money supply went up 25%, and it is no surprise then that P when from 20 to 25. Neat, huh?

o) Pretend instead that you are a Keynesian economist. What happens in the economy? What is the new equilibrium?

Now, P is fixed, and so we need to use the AD curve to solve for Y :

$$\begin{aligned} 20 &= \frac{1000}{.5Y^2 + (.5)(2)Y - 2Y} \\ 50 &= .5Y^2 - Y \\ 0 &= Y^2 - 2Y - 100 \end{aligned}$$

Once again, use the quadratic:

$$\begin{aligned} Y &= \frac{2 \pm \sqrt{4 - 4(-100)}}{2} \\ &= \frac{2 \pm \sqrt{404}}{2} \\ &= \frac{2 \pm 2\sqrt{101}}{2} = 1 \pm \sqrt{101} \end{aligned}$$

Since Y must be positive, we have:

$$Y = 1 + \sqrt{101} \approx 11$$

Plugging into the LM curve, we get:

$$r(Y) = \frac{20(11)}{10000} \approx .022$$

Exercise 2 *Friedman and Schwartz*

A sample response (I'll leave the graphs to your imagination):

In September of 1931, in response to a run on pounds sterling, Britain abandoned the gold standard (pg 315). Within the following year, 25 other nations followed. Across Europe, both central banks and private holders of US dollars expected the United States to similarly abandon gold precipitating a substantial conversion of US dollars for gold. As gold holdings are a primary determinant of the size of the money stock, the net result was a decrease in the money supply.

Looking to the money market, a decrease in the money supply with everything else held constant results in a higher equilibrium interest rate at any level of income implying that the LM curve shifts upward. Assuming that prices are perfectly sticky in the short run, this further implies a decrease in the equilibrium level of output. Were prices perfectly flexible, any decrease in the money supply would be matched by a decrease in the price level and there would be no impact on output. Looking to Chart 28 on page 303, we see that after Britain's departure from the gold standard, there is a fall in both the price level and personal income, implying that neither perfectly sticky nor flexible pricing is an appropriate assumption, i.e. the AS curve is upward sloping but not vertical.

As the Fed drew down its gold holdings, it could have maintained the money stock had it used the incoming dollars to purchase government securities via open market operations. It chose not to do.