Assignment #5*

Econ 302: Intermediate Macroeconomics

December 2, 2009

1 Keynesian Cross

Consider a closed economy.
Consumption function:

$$C = \bar{C} + MPC(Y - T) \tag{1}$$

In addition, suppose that planned investment expenditure is I, the level of taxes is T, and government spending is G. Use your knowledge of the Keynesian cross (goods market / aggregate expenditures model) as a starting point to answer the following questions.

1.1 Write equilibrium output level Y as a function of T, MPC, $\bar{C}$, I, and G. *(hint: write down the equilibrium condition for the goods market, i.e. $AE = Y$; solve for Y)*

$$AE = C + I + G = \bar{C} + MPC(Y - T) + I + G = Y$$

$$Y = \frac{1}{1 - MPC}[\bar{C} - MPC(T) + I + G]$$

1.2 Define tax multiplier $M_T \equiv \frac{\partial Y}{\partial T}$; government expenditure multiplier $M_G \equiv \frac{\partial Y}{\partial G}$. Given the equilibrium output expression from the previous part, write M_T and M_G, the fiscal policy multipliers, in terms of constants. Which multiplier is larger in magnitude?

$$M_T = -\frac{MPC}{1 - MPC}$$

$$M_G = \frac{1}{1 - MPC}$$

$$0 < MPC < 1 \Rightarrow |M_T| < |M_G| \tag{2}$$

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1.3 **From this part forward, assume the following: $\bar{C} = 200$, $MPC = \frac{2}{3}$, $I = 300$, $G = T = 300$. If $Y = 1500$, what is planned expenditure? Given your answer, would you expect equilibrium Y to be higher or lower than $Y = 1500$? Explain briefly.**

$$\textit{planned expenditure} = AE = C + I + G = 200 + \frac{2}{3}(1500 - 300) + 300 + 300 = 1600$$

Given $Y = 1500$, $AE > Y$ implies that inventories are drawn down, so firms will scale up output in order to keep inventories constant. Therefore, we'd expect that equilibrium Y is greater than $Y = 1500$.

1.4 **What is the equilibrium level of Y? Is it consistent with your guess in the previous part?**

$$Y = \frac{1}{1 - MPC}[\bar{C} - MPC(T) + I + G] = 3[200 - \frac{2}{3}(300) + 300 + 300] = 1800$$

Yes, our guess was in the correct direction; $Y = 1800 > 1500$ in equilibrium.

1.5 **Now, suppose that G is reduced to 200 units, but T remains unchanged. What is the new equilibrium output level?**

$$Y = \frac{1}{1 - MPC}[\bar{C} - MPC(T) + I + G] = 3[200 - \frac{2}{3}(300) + 300 + 200] = 1500$$

1.6 **Given your answer to the previous part, calculate $\frac{\Delta Y}{\Delta G}$. Is it consistent with the government expenditure multiplier M_G you computed in part (1.2)?**

$$\frac{\Delta Y}{\Delta G} = \frac{1500 - 1800}{200 - 300} = \frac{-300}{-100} = 3$$

$$M_G = \frac{1}{1 - MPC} = \frac{1}{1 - \frac{2}{3}} = \frac{1}{\frac{1}{3}} = 3$$

The answers are consistent; $\frac{\Delta Y}{\Delta G} = M_G$.

1.7 **Now, suppose that T is reduced to 180 units with $G = 300$. What is the new equilibrium output level?**

$$Y = \frac{1}{1 - MPC}[\bar{C} - MPC(T) + I + G] = 3[200 - \frac{2}{3}(180) + 300 + 300] = 2040$$

1.8 **Given your answer to the previous part, calculate $\frac{\Delta Y}{\Delta T}$. Is it consistent with the tax multiplier M_T you computed in part (1.2)?**

$$\frac{\Delta Y}{\Delta T} = \frac{2040 - 1800}{180 - 300} = \frac{240}{-120} = -2$$

$$M_T = -\frac{MPC}{1 - MPC} = -\frac{\frac{2}{3}}{1 - \frac{2}{3}} = -\frac{\frac{2}{3}}{\frac{1}{3}} = -2$$

Again, the answers are consistent; $\frac{\Delta Y}{\Delta T} = M_T$.

2 Dynamics of the Goods Market

Let's add an explicit dimension of time to our model of the goods market (Keynesian cross). Here, subscript t stands for time, so all variables are now indexed by t. $\bar{C}$ and MPC are still treated as constants; they don't vary over time. The economy is open to international trade and international capital flows; this is reflected in the equations. Implement the following economic model in Excel.

Aggregate expenditure:

$$Y_{t+1} = C_t + I_t + G_t + (X_t - M_t) \tag{3}$$

Consumption:

$$C_t = \bar{C} + MPC(Y_t - T_t) \tag{4}$$

Investment:

$$I_t = 0.3Y_t - 250r_t \tag{5}$$

Government expenditure:

$$G_t = 50 + 0.2Y_t - 100r_t \tag{6}$$

Taxes:

$$T_t = 50 + 0.15Y_t + 50r_t \tag{7}$$

Exports:

$$X_t = 0.1Y_t - 10\epsilon_t - 0.1(X_{t-1} - M_{t-1}) \tag{8}$$

Imports:

$$M_t = 0.1Y_t + 200r_t + 20\epsilon_t + 0.1(X_{t-1} - M_{t-1}) \tag{9}$$

Monetary policy:

$$r_{t+1} = 0.0005Y_t + 0.0001\epsilon_t + 0.0002G_t \tag{10}$$

Exchange rates:

$$\epsilon_{t+1} = \epsilon_t + 0.0001(X_t - M_t) + 0.5(r_{t+1} - r_t) \tag{11}$$

Initial conditions (at time $t = 0$):

$$Y_0 = 500 \tag{12}$$

$$X_0 = 30 \tag{13}$$

$$M_0 = 100 \tag{14}$$

$$r_0 = 0.05 \tag{15}$$

$$\epsilon_0 = 2 \tag{16}$$

Constants:

$$\bar{C} = 50 \tag{17}$$

$$MPC = 0.8 \tag{18}$$

You should have the following columns in your Excel file: t , Y_t, C_t, I_t, G_t, X_t, M_t, T_t, ϵ_t, and r_t. You do not have to print out your tables, only your graphs (when asked to do so).

2.1 Translate the model formulas above into Excel formulas, and then run the model for $t = 0, 1, \ldots, 125$. Write out the first two periods, $t = 0$ and $t = 1$, and then copy the $t = 1$ row down until $t = 125$ to run the model. Don't get discouraged! This is really just an exercise in writing Excel formulas.

See Excel file, tab "2.1-2.5".

2.2 Graph Y_t, C_t, I_t, and $NX_t = X_t - M_t$ vs. t (time). What trends do you observe? Please print this out.

See Excel file.

2.3 Graph G_t and T_t vs. t (time). What trends do you observe?

See Excel file.

2.4 Graph ϵ_t and r_t vs. t (time). What trends do you observe?

See Excel file.

2.5 Graph $S_{private,t}$ and $S_{public,t}$ vs. t (time). What trends do you observe? Please print this out.

See Excel file.

2.6 **Copy your result from (2.1) - (2.5) into a new spreadsheet (save the old one!). Let's say that there is a preference shock at $t = 75$, so now $MPC = 0.95$ from $t = 75$ onwards; people want to consume more. To translate this into the Excel file, make a column for the marginal propensity to consume, and fill in $MPC = 0.8$ from $t = 0$ to $t = 75$; $MPC = 0.95$ from $t = 75$ to $t = 125$. Your formula for C_t should now refer to MPC_t, this new column, not $MPC = 0.8$ for all t. Reprint your graphs from parts (2.2) and (2.5). Interpret.**

See Excel file, tab "2.6".

2.7 **Copy your result from (2.6) into a new spreadsheet (save the old one!). Let's change the consumption function to: $C_t = \bar{C} + MPC(Y_t - T_t) - 125r_t$; only the $-125r_t$ term is new. Since consumers have to borrow money at rate r_t to buy expensive items like automobiles and housing, they are less likely to increase consumption by taking out a loan with high real interest rates. Reprint your graphs from parts (2.2) and (2.5). Interpret.**

See Excel file, tab "2.7".

3 IS / LM Model

Assume a closed economy.
Consumption function:

$$C = 200 + 0.75(Y - T) \tag{19}$$

Investment function:

$$I = 200 - 2500r \tag{20}$$

Fiscal policy:

$$G = T = 100 \tag{21}$$

Money demand:

$$(\frac{M}{P})^d = Y - 10000r \tag{22}$$

Money supply:

$$(\frac{M}{P})^s = \frac{1000}{P} \tag{23}$$

3.1 Derive the IS curve. *(hint: write interest rate r_{IS} as a function of Y)*

$$AE = C + I + G = Y$$

$$200 + 0.75(Y - T) + 200 - 2500r + G = Y$$

$$200 - 0.75(100) + 200 - 2500r + 100 = 425 - 2500r = 0.25Y$$

$$2500r = 425 - 0.25Y \ \Rightarrow\ r_{IS} = 0.17 - 0.0001Y$$

3.2 Let $P = 2$. Derive the LM curve. *(hint: write interest rate r_{LM} as a function of Y)*

$$(\frac{M}{P})^s = (\frac{M}{P})^d \ \Rightarrow\ \frac{1000}{P} = \frac{1000}{2} = 500 = Y - 10000r$$

$$10000r = Y - 500 \ \Rightarrow\ r_{LM} = 0.0001Y - 0.05$$

3.3 Solve for IS / LM equilibrium (r, Y).

$$r_{IS} = r_{LM} \ \Rightarrow\ 0.17 - 0.0001Y_e = 0.0001Y_e - 0.05$$

$$0.0002Y_e = 0.22 \ \Rightarrow\ Y_e = 1100$$

$$r_e = 0.17 - 0.0001(1100) = 0.17 - 0.11 = 0.06 = 6\%$$

$$(r_e, Y_e) = (0.06, 1100)$$

3.4 Suppose that government purchases are raised from $G = 100$ to $G = 150$. All else equal, describe the IS curve shift. What is the new IS / LM equilibrium (r, Y)?

The IS curve shifts to the right; no direct effect on the LM curve.
New IS curve:

$$200 + 0.75(Y - 100) + 200 - 2500r + 150 = Y$$

$$r_{IS} = 0.19 - 0.0001Y$$

LM curve:

$$r_{LM} = 0.0001Y - 0.05$$

Equilibrium:

$$r_{IS} = r_{LM} \;\Rightarrow\; 0.19 - 0.0001Y_e = 0.0001Y_e - 0.05$$

$$0.0002Y_e = 0.24 \;\Rightarrow\; Y_e = 1200$$

$$r_e = 0.19 - 0.0001(1200) = 0.19 - 0.12 = 0.07 = 7\%$$

$$(r_e, Y_e) = (0.07, 1200)$$

3.5 With $G = 100$, the money supply is increased from $(\frac{M}{P})^s = \frac{1000}{P}$ to $(\frac{M}{P})^s = \frac{1200}{P}$. All else equal, describe the LM curve shift. What is the new IS / LM equilibrium (r, Y)?

The LM curve shifts to the right; no direct effect on the IS curve.
IS curve:

$$r_{IS} = 0.17 - 0.0001Y$$

New LM curve:

$$(\frac{M}{P})^s = (\frac{M}{P})^d \;\Rightarrow\; \frac{1200}{P} = \frac{1200}{2} = 600 = Y - 10000r$$

$$10000r = Y - 600 \;\Rightarrow\; r_{LM} = 0.0001Y - 0.06$$

Equilibrium:

$$r_{IS} = r_{LM} \;\Rightarrow\; 0.17 - 0.0001Y_e = 0.0001Y_e - 0.06$$

$$0.0002Y_e = 0.23 \Rightarrow Y_e = 1150$$

$$r_e = 0.17 - 0.0001(1150) = 0.17 - 0.115 = 0.055 = 5.5\%$$

$$(r_e, Y_e) = (0.055, 1150)$$

3.6 With $G = 100$ and $(\frac{M}{P})^s = \frac{1000}{P}$, suppose that the price level rises from $P = 2$ to $P = 4$ due to an oil price shock (cost-push inflation). All else equal, describe the LM curve shift. What is the new IS / LM equilibrium (r, Y)?

The LM curve shifts to the left; no direct effect on the IS curve.
IS curve:

$$r_{IS} = 0.17 - 0.0001Y$$

New LM curve:

$$(\frac{M}{P})^s = (\frac{M}{P})^d \Rightarrow \frac{1000}{P} = \frac{1000}{4} = 250 = Y - 10000r$$

$$10000r = Y - 250 \Rightarrow r_{LM} = 0.0001Y - 0.025$$

Equilibrium:

$$r_{IS} = r_{LM} \Rightarrow 0.17 - 0.0001Y_e = 0.0001Y_e - 0.025$$

$$0.0002Y_e = 0.195 \Rightarrow Y_e = 975$$

$$r_e = 0.17 - 0.0001(975) = 0.17 - 0.0975 = 0.0725 = 7.25\%$$

$$(r_e, Y_e) = (0.0725, 975)$$

3.7 Return to $G = 100$ and $(\frac{M}{P})^s = \frac{1000}{P}$. Recall that money demand (liquidity preference) is given by the equation: $(\frac{M}{P})^d = Y - 10000r$. Leave price level P as a variable and write an equation for the aggregate demand curve. ***(hint: (3.1) gave you an equation for r_{IS} in terms of Y, and (3.2) gave you an equation for r_{LM} in terms of P and Y; set $r_{IS} = r_{LM}$ in IS / LM equilibrium and solve for P as a function of Y)***

IS curve:

$$r_{IS} = 0.17 - 0.0001Y$$

New LM curve:

$$(\frac{M}{P})^s = (\frac{M}{P})^d \Rightarrow \frac{1000}{P} = Y - 10000r$$

$$10000r = Y - \frac{1000}{P} \Rightarrow r_{LM} = 0.0001Y - \frac{1}{10P}$$

Equilibrium:

$$r_{IS} = r_{LM} \Rightarrow 0.17 - 0.0001Y = 0.0001Y - \frac{1}{10P}$$

$$\frac{1}{10P} = 0.0002Y - 0.17 \Rightarrow 10P = \frac{1}{0.0002Y - 0.17} = \frac{1}{\frac{1}{500}(0.1Y - 85)} = \frac{500}{0.1Y - 85}$$

$$P = \frac{500}{Y - 850}$$

4 Retirement Planning

This question goes through the mechanics of thinking about consumption smoothing over the course of your working and retirement years. In the problem you will go through the process three times: 1) the general model; 2) a model where you specify the various parameters; and 3) a revised model where you act as a financial advisor suggesting concrete steps you can take to result in a better consumption path for your lifetime.

4.1 First Time Process

You know the following information: Susy has a house that is worth $300,000 and a money market fund with $100,000. She also has a mortgage on her house and she currently owes $200,000 on that mortgage. Susy does not anticipate any change in her net worth or wealth position other than from saving a portion of her annual income. Susy is thirty years old, expects to live to be 80 years old, and would like to retire when she is age 60. Susy currently earns $50,000 and she expects her real income to remain constant over the course of her working life. Assume there is no inflation over the course of Susy's life (this is a simplifying assumption to make the calculations far easier).

4.1.1 What is the value of Susy's assets at this point in time?

4.1.2 What is the value of Susy's liabilities at this point in time?

4.1.3 What is Susy's net worth or wealth at this point in time?

4.1.4 What is the value of Susy's income over the course of her working life?

4.1.5 If Susy smooths out her consumption so that her annual level of consumption is constant over the rest of her life, what will be her annual level of consumption? Show your work in arriving at this level.

Answer:

1. *Assets = $400,000*
2. *Liabilities = $200,000*
3. *Net worth = Wealth = $200,000*
4. *Value of income over lifetime = (40)($50,000) = $2,000,000*
5. *Consumption = ((Wealth + (years to retirement)(annual income))/(Expected Years to Live) = ($200,000 + $2,000,000)/ (50 years) = $44,000*

4.2 Second Time Process

This time calculate your annual consumption level if you smooth your consumption over the course of the rest of your life. To do this calculation you will need to specify what your real income per year is projected to be (so, do research on a particular profession and the annual income associated with that profession-in your answer identify the profession you are using), how many years you plan to work, how many years you anticipate living, and what your current wealth position is. Be as specific as you can so that you can see what level of annual consumption you can afford given your assumptions.
Answers will vary here according to the assumptions that students make.

4.3 Third Time Process

Having done the calculation in part (b), redo the calculation but with some change in one of the variables (e.g., you might decide to work more years, or you might consider a different occupation that has a different salary). What happens to your level of annual consumption with this change? Be very specific in your example and be thoughtful about how your choices today affect your future choices. Identify the choices that you feel are the most critical for you in determining your level of annual consumption over your lifetime.
Answers will vary here according to the assumptions that students make.