Economics 302
Spring 2007
Homework #4
Due Tuesday, March 27

Homework will be graded for content as well as neatness. Sloppy or illegible work will not receive full credit.

1. You live in country "I-love-spring-and-gosh-I-wish-I-could-go-outside-now" (ILSAG, for short). ILSAG has a Cobb Douglass production function $Y=K^{1/2}L^{1/2}$. The hard-working citizens of ILSAG save 20% of their incomes and the depreciation rate in the economy is 10%.

a. According to the Solow model, what should be the steady-state level of capital per worker?
$k^{ss} = 4$.

b. What would be the "golden rule" level of capital per worker?
*$y=k^{1/2}$, so MPK=1/(2*k^1/2), as in the book, (see pg203). Setting MPK=δ, gives k_{gold}=25.*

c. What savings rate would give rise to the golden rule level of capital?
The golden rule steady state level of capital would be achieved if citizens saved 50% of their income.

d. Suppose you are the almighty ruler of ILSAG and your dutiful citizens do everything you tell them. If you care only about your citizens long-run happiness (read: consumption) what fraction of their savings should you have your citizens save?
The gold rule level! So, 50%

e. Given your last answer, suppose that you tell your people to switch their initial savings rate from what it was initially to the rate that would make them happiest in the long-run. What will be the dynamics of the capital stock in the short run?
The capital stock will increase in the short-run, eventually converging to a higher steady-state

f. What will be the dynamics of the capital stock in the long run?
In the long run, the capital stock converges to the golden rule level of capital.

g. How will the consumption level of a typical citizen evolve over time?
The consumption level initially falls, but then grows gradually over time, eventually converge to a higher level that it was to start.

h. Finally, suppose that while your citizens like you, they don't like you *that* much. Could you see any problem in implementing the policy just suggested?
Yes. Since citizens will have their consumption fall in the short run, they might not want this Thus, they might either disobey the ruler, or, if the ruler can somehow force them to save more then they want, the citizens might decide to revolt!

2. Suppose you are given the production function $Y = F(K, L) = AK^{1/4}L^{3/4}$, where Y is real GDP, K capital, L labor and A technology.

a. Suppose K and L are both doubled in this economy. Does this production function exhibit constant returns to scale? Provide a proof of your answer.

If this production function exhibits constant returns to scale then $F(zK, zL) = zF(K, L) = zY$. So, we look at the question, what happens to Y if K and L are increased by the same proportion? Let's define a new level of K, $K' = 3K$, and a new level of L, $L' = 3L$ and consider what happens to our level of output, Y'.

$Y' = AK'^{1/4}L'^{3/4} = A(2K)^{1/4}(2L)^{3/4}$
$Y' = A\,K^{1/4}(2)^{1/4}L^{3/4}(2)^{3/4}$
$Y' = A(2)\,K^{1/4}L^{3/4}$
$Y' = 2Y$, *since* $Y = AK^{1/4}L^{3/4}$

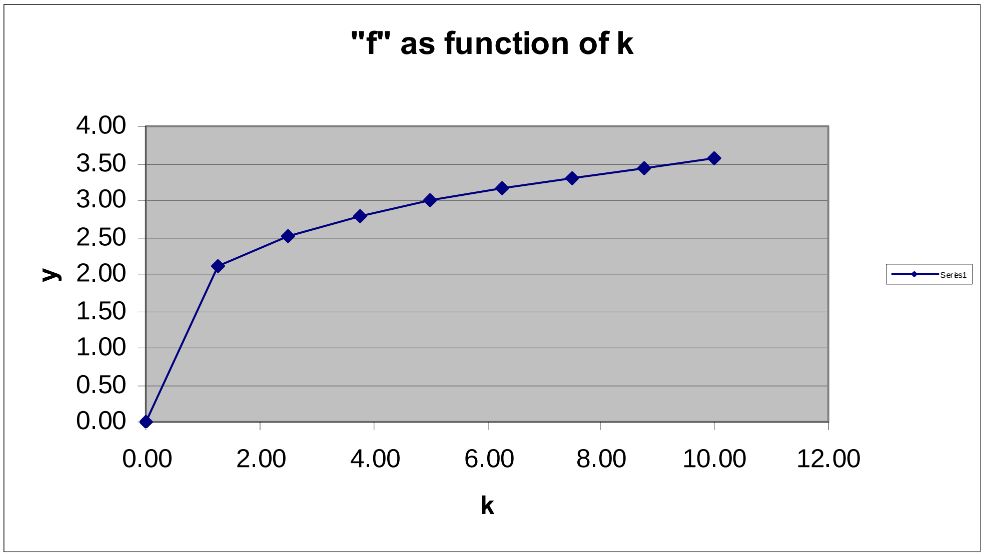
b. Rewrite the above production function in terms of output per worker, y. That is, normalize the function so that it can be written as y = f(k) where k is capital per worker. Provide a verbal interpretation of your mathematical result.

$Y = F(K, L) = AK^{1/4}L^{3/4}$
$Y/L = y$
$y = (1/L)(AK^{1/4}L^{3/4}) = AK^{1/4}L^{-1/4} = A(K/L)^{1/4} = Ak^{1/4} = f(k)$
Output per worker, y, depends upon the amount of capital per worker (k) and the level of technology (A).

c. Using the expression you found in part (b) and assuming labor is equal to 16 units, fill in the following table. Recall that k, the capital to labor ratio (K/L), measures capital intensity: that is, it provides a measure of how much capital each unit of labor has to work with in the economy. Assume that A equals 2 in this problem. Hint: this might be a good time to pull out your Excel spreadsheet program.

L	K	k = K/L	y = Y/L	Y	MPk
16	0	0.00	0.00	0	infinity
16	20	1.25	2.11	33.8	0.42
16	40	2.50	2.51	40.2	0.25
16	60	3.75	2.78	44.5	0.19
16	80	5.00	2.99	47.9	0.15
16	100	6.25	3.16	50.6	0.13
16	120	7.50	3.31	53.0	0.11
16	140	8.75	3.44	55.0	0.10
16	160	10.00	3.56	56.9	0.09

d. Use Excel to graph k and y, placing k on the horizontal axis and y on the vertical axis.



e. Let's define s as the saving rate per worker, i as the amount of investment per worker, and c as consumption per worker. Suppose that you are told that the saving rate in this economy is constant and equal to .25. Use this saving rate and the values you found in part (c) to fill in the following table. Note that i equals sy and y = c + i.

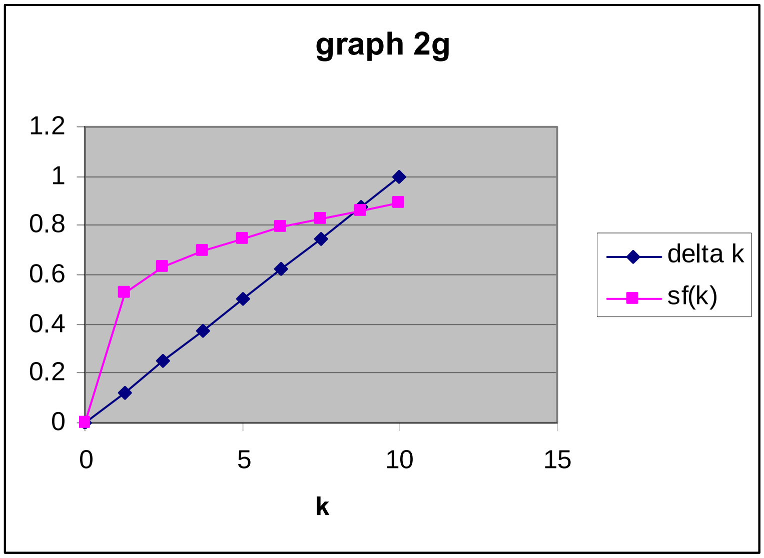
L	K	k = K/L	y = Y/L	s	i = sf(k)	c
16	0	0.00	0.00	0.25	0.00	0.00
16	20	1.25	2.11	0.25	0.53	1.59
16	40	2.50	2.51	0.25	0.63	1.89
16	60	3.75	2.78	0.25	0.70	2.09
16	80	5.00	2.99	0.25	0.75	2.24
16	100	6.25	3.16	0.25	0.79	2.37
16	120	7.50	3.31	0.25	0.83	2.48
16	140	8.75	3.44	0.25	0.86	2.58
16	160	10.00	3.56	0.25	0.89	2.67

f. Suppose you are also told that depreciation of the capital stock, δ, is equal to 10% each year. Using this information, plus the information you have previously been given or have calculated compute the change in the capital intensity, Δk, in this economy. Find Δk for each level of K in the economy by filling in the table below. Recall that the change in the capital intensity is equal to the difference between the level of investment per worker and the amount of capital per worker that depreciates during the given time period.

L	K	k = K/L	y = Y/L	s	i = sf(k)	c	δk	Δk
16	0	0.00	0.00	0.25	0.00	0.00	0.00	0.00
16	20	1.25	2.11	0.25	0.53	1.59	0.13	0.40
16	40	2.50	2.51	0.25	0.63	1.89	0.25	0.38
16	60	3.75	2.78	0.25	0.70	2.09	0.38	0.32
16	80	5.00	2.99	0.25	0.75	2.24	0.50	0.25
16	100	6.25	3.16	0.25	0.79	2.37	0.63	0.17
16	120	7.50	3.31	0.25	0.83	2.48	0.75	0.08
16	140	8.75	3.44	0.25	0.86	2.58	0.88	-0.02

16	160	10.00	3.56	0.25	0.89	2.67	1.00	-0.11

g. Now, graph the amount of depreciation, δk; the level of investment per worker, i (which is also equal to sf(k)); and the capital intensity, k. Place k on the horizontal axis and δk and sf(k) on the vertical axis.



h. Suppose we define k* as the steady-state level of capital intensity or the level of capital per worker where the amount of capital depreciation per worker equals the level of investment per worker in each period. At k* the capital intensity is constant over time. Calculate the value of k* for your economy. Does your calculation agree with your findings in part (g)?

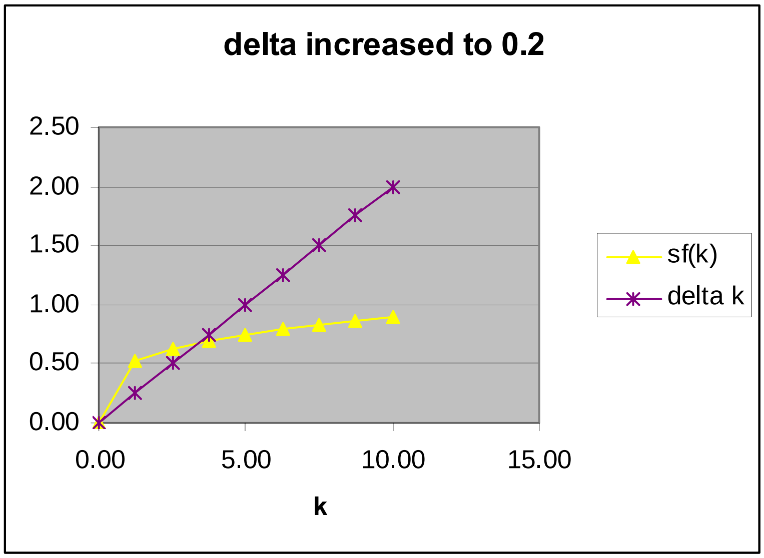
8.5, which agrees with the graph.

i. What would happen to k* if s increased, holding everything else constant? Draw a sketch illustrating the initial situation and the effect of an increase in s. [Hint: why not run your Excel program again but this time with a higher value for s?]

The sf(k) curve shifts up, implying a higher k*.

j. What would happen to k* if δ increased, holding everything else constant? Draw a sketch illustrating the initial situation and the effect of an increase in δ. *[Hint: why not run your Excel program again but this time with a higher value for δ?]*

Increasing delta increases the slope of the delta k line, thus decreasing steady state capital.


delta increased to 0.2
2.50
2.00
1.50
1.00
0.50
0.00
0.00
5.00
10.00
15.00
k
sf(k)
delta k