Economics 302
Prof. Kelly
Problem Set 3

Answer Key

Exercise 1 *Open Economy Equilibrium*

Suppose in the country of Kelly, which is initially closed to the outside world (ie, does not trade with other nations), output is produced in each period using just two factors, capital (K) and labor (L) under the following production function:

$$Y = F(K, L) = 6K^{.5}L^{.5}$$

Kelly has a population of one million people; every individual gets an equal share of output; and each individual consumes output according to the following Keynesian consumption function, where y is the output allocated to an individual and t is the tax paid by each individual:

$$c = .8(y - t)$$

Every individual in Kelly inelastically supplies one unit of labor and four units of capital for production. We then have that k is the amount of capital provided by each individual and K is the total amount of capital provided for production. Similarly for l and L. The government of Kelly runs a balanced budget and spends two million units of output per period. Every individual faces the same tax.

a) What is total ouput in the economy?

$$\begin{aligned} Y &= 6\left(4 * 1,000,000\right)^{.5}\left(1,000,000\right)^{.5} \\ &= 6,000,000\left(4\right)^{.5}\left(1\right)^{.5} \\ &= 12,000,000 \end{aligned}$$

b) What is the wage rate in the economy

$$\begin{aligned} w &= MPL = \frac{(1-\alpha)Y}{L} \\ &= \frac{.5}{1,000,000}(12,000,000) \\ &= 6 \end{aligned}$$

c) How much does an individual consume in the economy?

$$\begin{aligned} c &= .8(12 - 2) \\ &= 8 \end{aligned}$$

d) What is total investment (I) in the economy

$$\begin{aligned} I &= S \\ &= Y - G - C \\ &= 1,000,000(12 - 2 - 8) \\ &= 2,000,000 \end{aligned}$$

Now, suppose that we have the following investment function for each individual:

$$i = \frac{.08}{r}$$

e) Determine what the interest rate is in this economy.

$$\begin{aligned} i &= 2 = \frac{.08}{r} \Rightarrow \\ 2r &= .08 \\ r &= .04 \end{aligned}$$

f) What is the interest rate if government spending increases to 3 million units and the government maintains a balanced budget? What if government spending decreases to 1 million units?

G=3,000,000 which implies T=3,000,000 so consumption must be recalculated.

$$\begin{aligned} c &= .8(12 - 3) \\ &= .8(9) \\ &= 7.2 \end{aligned}$$

$$\begin{aligned} I &= S \\ &= Y - G - C \\ &= 1,000,000(12 - 3 - 7.2) \\ &= 1,800,000 \end{aligned}$$

$$\begin{aligned} i &= 1.8 \Rightarrow \\ 1.8r &= .08 \\ r &= \frac{.08}{1.8} \\ r &= .044 \end{aligned}$$

G=1,000,000

$$\begin{aligned} c &= .8(12-1) \\ &= .8(11) \\ &= 8.8 \end{aligned}$$

$$\begin{aligned} I &= S \\ &= Y - G - C \\ &= 1,000,000(12 - 1 - 8.8) \\ &= 2,200,000 \end{aligned}$$

$$\begin{aligned} i &= 2.2 \Rightarrow \\ 2.2r &= .08 \\ r &= \frac{.08}{2.2} \\ r &= .036 \end{aligned}$$

g) What happens to the interest rate if government spending increases to 3 million units but does not change the tax to maintain a balanced budget? What happens if government spending decreases to 1 million units.

G=3,000,000 (c does not change since t does not change)

$$\begin{aligned} I &= S \\ &= Y - G - C \\ &= 1,000,000(12 - 3 - 8) \\ &= 1,000,000 \end{aligned}$$

$$\begin{aligned} i &= 1 \Rightarrow \\ r &= .08 \end{aligned}$$

G=1,000,000

$$\begin{aligned} I &= S \\ &= Y - G - C \\ &= 1,000,000(12 - 1 - 8) \\ &= 3,000,000 \end{aligned}$$

$$\begin{aligned} i &= 3 \Rightarrow \\ 3r &= .08 \\ r &= \frac{.08}{3} \\ r &= .0267 \end{aligned}$$

Next, assume that the economy of Kelly opens up to world trade and the world interest rate, r^w, is .05.

h) What is total investment in the economy?

$$\begin{aligned} I &= i(1,000,000) = 1,000,000\left(\frac{.08}{.05}\right) \\ &= 1,000,000\,(1.6) \\ &= 1,600,000 \end{aligned}$$

i) What are the net exports in the economy? Is Kelly a net exporter or a net importer? Provide intuition as to why this is the case.

$$\begin{aligned} NX &= S - I \\ &= Y - C - G - I \\ &= 2,000,000 - 1,600,000 \\ &= 400,000 \end{aligned}$$

Kelly is a net exporter. Since the world offers a higher interest rate than Kelly would have if it were closed (.05 vs .04), and investment is decreasing in the interest rate, Kelly will now save more than it invests.

Sample Answer for Question 2.
(the numerical results come from Katie Bruegger, the commentary and explanations are mine and in red.)

GDP in constant 1995 US dollars

Year	Canada	France	Ireland	United States
1995	581,664,300,000	1,553,129,799,680	66,467,782,656	7,338,399,891,456
1996	591,029,700,000	1,570,259,599,360	71,834,599,424	7,603,000,180,736
1997	616,117,400,000	1,600,164,593,664	79,682,297,856	7,942,999,900,160
1998	641,261,200,000	1,654,590,275,584	86,725,296,128	8,285,899,980,800
1999	676,296,300,000	1,707,754,127,360	96,389,455,872	8,629,099,954,176
2000	707,125,600,000	1,772,530,171,904	105,984,942,080	8,955,100,135,424
2001	717,385,800,000	1,809,675,845,632	112,010,862,592	8,977,799,708,672

Tax Revenue (%of GDP)

Year	Canada	France	Ireland	United States
1995	0.18	0.38	0.31	0.18
1996	0.19	0.38	0.31	0.18
1997	0.2	0.39	0.31	0.19
1998	0.2	0.38	0.31	0.19
1999	0.2	0.38	0.31	0.19
2000	0.2	0.38	0.31	0.2
2001	0.19	0.38	0.31	0.19

*these data were not available so I made the assumption that the % remained relatively constant

One way to calculate the total tax revenue is to multiply the GDP by the percentages above.
When we subtract this figure from GDP, we get disposable income.
A sample formula call would be: "=B3-(B3*B13)" for Canada 1995

A) Disposable Income (Y-T)

Year	Canada	France	Ireland	United States
1995	4.77E+11	9.63E+11	4.59E+10	6.02E+12
1996	4.79E+11	9.74E+11	4.96E+10	6.23E+12
1997	4.93E+11	9.76E+11	5.50E+10	6.43E+12
1998	5.13E+11	1.03E+12	5.98E+10	6.71E+12
1999	5.41E+11	1.06E+12	6.65E+10	6.99E+12
2000	5.66E+11	1.10E+12	7.31E+10	7.16E+12
2001	5.81E+11	1.12E+12	7.73E+10	7.27E+12

GDP in constant 1995 US dollars

Year	Canada	France	Ireland	United States
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Gross Domestic Savings as % of GDP

Year	Canada	France	Ireland	United States
1995	0.19	0.19	0.23	0.16
1996	0.19	0.19	0.26	0.17
1997	0.2	0.2	0.27	0.18
1998	0.19	0.21	0.27	0.18
1999	0.21	0.22	0.26	0.18
2000	0.24	0.22	0.26	0.18
2001	0.23	0.21	0.24	0.16

MFP % growth based on 'harmonised' price indices for ICT capit

Year	Canada	France	Ireland	United States
1995	0.9	1.4	4.8	-0.1
1996	-0.9	0.0	3.6	1.7
1997	1.8	1.2	8.3	1.1
1998	0.7	2.1	2.2	1.0
1999	1.6	1.1	5.5	1.3
2000	2.3	3.0	4.1	1.2
2001	0.1	0.9	2.8	0.7

Labor Force Statistics Data

Year	Canada	France	Ireland	United States
1995	14,750,100	24,842,600	1,449,600	132,304,000
1996	14,899,500	25,111,000	1,498,100	133,943,000
1997	15,153,000	25,285,300	1,530,700	136,297,000
1998	15,417,700	25,590,100	1,612,800	137,673,000
1999	15,721,200	25,898,000	1,680,100	139,368,000
2000	15,999,000	26,212,400	1,738,800	142,583,000
2001	16,246,000	26,443,900	1,775,000	143,734,000

We have savings as a percentage of GDP, but that is not the same as the mps, since mps is the percent of disposable income saved. We can use the percentages above to calculate the total savings in the same way we computed the total tax revenue, and then divide this number by the disposable income we calculated above: Savings=mps(Y-T) so mps=Savings/(Y-T)

In the dataset, the labor force was given in 1,000s so you had to multiply the numbers they gave you by 1,000. If the US has a pop of 300 million, a labor force of 143 thousand just doesn't make ser

B) Average Savings Rate

Year	Canada	France	Ireland	United States
1995	0.23	0.31	0.33	0.20
1996	0.23	0.31	0.38	0.21
1997	0.25	0.33	0.39	0.22
1998	0.24	0.34	0.39	0.22
1999	0.26	0.35	0.38	0.22
2000	0.30	0.35	0.38	0.23
2001	0.28	0.34	0.35	0.20
AVERAGE	**25.72%**	**33.26%**	**37.06%**	**21.31%**

When we use these averages later, it's important to remember that a mps of 20% means that the mps=.2. If you input mps=20 into your equations, everything can get messed.

Average MFP

Year	Canada	France	Ireland	United States
1995	0.9	1.4	4.8	-0.1
1996	-0.9	0.0	3.6	1.7
1997	1.8	1.2	8.3	1.1
1998	0.7	2.1	2.2	1.0
1999	1.6	1.1	5.5	1.3
2000	2.3	3.0	4.1	1.2
2001	0.1	0.9	2.8	0.7
AVERAGE	**0.94%**	**1.39%**	**4.46%**	**0.99%**

C)Average Growth Rate of Output

Year	Canada	France	Ireland	United States
1995	2.782975	1.670115	9.864214	2.699597
1996	1.610094	1.102924	8.074312	3.605695
1997	4.24475	1.904461	10.92468	4.471915
1998	4.080999	3.401252	8.838854	4.31701
1999	5.463479	3.213116	11.14341	4.141976
2000	4.558552	3.793053	9.954907	3.777915
2001	1.450959	2.095632	5.685641	0.2534824
AVERAGE	**3.46%**	**2.45%**	**9.21%**	**3.32%**

Some of you inadvertently used the indices for MFP instead of the percent changes. Like a CPI you could have calculated the percent change as the difference in the index but the % change was provided in the same workbook, but in a different sheet so you didn't have to make these calculations

A's

Year	Canada	France	Ireland	United States
1995	1230	1230	1230	1230
1996	1241.536802	1247.156264	1284.900402	1242.236417
1997	1253.181813	1264.551827	1342.251255	1254.594565
1998	1264.936048	1282.190026	1402.161932	1267.075657
1999	1276.800533	1300.074246	1464.746691	1279.680913
2000	1288.776301	1318.207918	1530.124888	1292.411571
2001	1300.864396	1336.594522	1598.421207	1305.268878

All countries were assumed to have A=1230 in 1995, but remember, A changes over time, and the growth rate of A is different from country to country. To get A for the US in 1996, you have to multiply: 1230*US growth rate. To get A for Ireland in 1999, you have to multiply 1230*(Irish growth rate)^4, since we are 4 years removed from 1995.

D) Average Population Growth Rate

Year	Canada	France	Ireland	United States
1995	0.8278363	0.3168697	0.8643578	1.287712
1996	1.077502	0.3141454	0.8294214	1.283272
1997	1.056011	0.3131616	0.9863093	1.278792
1998	0.8666112	0.325884	1.192426	1.274639
1999	0.8263829	0.3794293	1.337992	1.27008
2000	0.8846293	0.4646303	1.346555	1.265844
2001	1.008868	0.5047265	1.35454	1.090327
Average	**0.94%**	**0.37%**	**1.13%**	**1.25%**

E) Equilibrium Capitalt to Labor Ratio

Year	Canada	France	Ireland	United States
1995	137,512	204,900	158,050	99,031
1996	139,451	209,202	168,749	100,512
1997	141,418	213,594	180,172	102,016
1998	143,412	218,079	192,368	103,542
1999	145,435	222,657	205,390	105,091
2000	147,486	227,332	219,294	106,663
2001	149,565	232,105	234,139	108,259

For steady state, we know: sf(k)=(delta+n+g)k.

f(k)=Y/L=(A*K^(1/3)*L^(2/3))/L=A*(K/L)^(1/3)*(L/L)^(2/3)

f(k)=A*k^(1/3)

We thus have: sAk^(1/3)=(delta+n+g)k.

Solve for k by dividing both sides by k^(1/3) and (delta+n+g):

sA/(delta+n+g)=k^(2/3)

k=(sA/(delta+n+g))^(3/2)

F) Actual K

Year	Canada	France	Ireland	United States
1995	32,955	131,316	51,805	91,700
1996	32,616	126,055	51,972	95,407
1997	34,155	125,337	58,333	100,227
1998	35,550	128,233	56,403	107,168
1999	38,246	130,489	60,089	113,267
2000	40,335	134,992	63,213	114,764
2001	39,113	134,223	61,534	109,580

To get actual k, just use the result from above: f(k)=A*k^(1/3) and f(k)=Y/L=GDP/Labor force. Plugging in and solving for k yields k=(Y/AL)^3

Differences between steady state and actual k

Year	Canada	France	Ireland	United States
1995	104,557	73,584	106,244	7,331
1996	106,835	83,147	116,777	5,105
1997	107,263	88,257	121,839	1,789
1998	107,862	89,846	135,965	-3,626
1999	107,189	92,169	145,301	-8,176
2000	107,151	92,340	156,081	-8,101
2001	110,452	97,882	172,605	-1,321

The data matches fairly well for the U.S., but not the other countries. This is because we made the assumption that technology is the same in all of the countries, and used the level of technology in the U.S.

Notice also, that k* is increasing in A (it's in the numerator) while k is decreasing in A (it's in the denominator). For Canada, we have that k*>k, or in other words, we need k* to go down and k to go up. One way for this to happen is to decrease A. So the wildly different numbers for Canada, France, and Ireland can be partly explained by technology differences between them and the US.

If we look at the US data, notice that for 1995-1997, k is below k* and so we would expect that k would become closer to k* as time passed. Indeed, this occurs as we can see in the last table. Many people believed that because of the dot-com bubble, the US economy become over-capitalized. We might be seeing this in 1998, 1999, and 2000. In 2001, we see the capital stock fall back towards its steady state level.

Pretty amazing that our simple model seems to capture so much of what occurred in the US! And that you all had the tools to maturely analyze this!

I should admit, I finally settled on a value for A so that the US would be near steady state in 2001. What impact that has, I don't really know.