Assignment #3*

Econ 302: Intermediate Macroeconomics

October 30, 2009

1 Small Open Economy

Coconut Island, a small economy, is closed to capital flows and international trade. The nation is holding a democratic election. There are two candidates for President, and each has proposed different domestic and foreign economic policies. Assume that this small open economy can be described by the set of equations provided below. Recall that r^* denotes the world equilibrium real interest rate.

$$r^* = 0.08 \tag{1}$$

$$Y = 6000 \tag{2}$$

$$C = C(r, Y - T) = 0.8(Y - T) - 10000r + 500 \tag{3}$$

$$I = I(r) = 3790 - 25000r \tag{4}$$

$$Y = C + I + G + NX \tag{5}$$

Candidate 1 believes that the economy should remain closed, the government should collect 300 in taxes and run a balanced budged.
Candidate 2 believes that the country should allow international trade and the government should collect 300 in taxes and spend 500.

1.1 For each Presidential candidate, what would be the level of public savings if their proposed policies were enacted?

Candidate 1: balanced budget implies that T=G, so $S_{public,1} = 0$.
Candidate 2: $S_{public,2} = 300 - 500 = -200$.

1.2 For each Presidential candidate, what would be the level of private savings if their proposed policies were enacted? Note: for the closed economy, leave r as a variable for now.

First, compute consumption as a function of the real interest rate r because Y and T are the same across the two policies.

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$$C = C(r, Y - T) = 0.8(Y - T) - 10000r + 500$$

$$C = 0.8(6000 - 300) - 10000r + 500 = 5060 - 10000r$$

Candidate 1: closed economy, need to use the domestic real interest rate. Let's leave it unresolved as r for now.

$$S_{private,1} = Y - T - C = 6000 - 300 - (5060 - 10000r) = 640 + 10000r$$

Candidate 2: open economy, the world real interest rate will be the equilibrium interest rate ($r_2^* = 0.08$).

$$S_{private,2} = 640 + 10000r_2^* = 640 + 10000(0.08) = 1440$$

1.3 For each Presidential candidate, what would be the level of national savings if their proposed policies were enacted?

Candidate 1: closed economy

$$S_1 = S_{public,1} + S_{private,1} = (0) + (640 + 10000r) = 640 + 10000r$$

Candidate 2: open economy

$$S_2 = S_{public,2} + S_{private,2} = (-200) + (1440) = 1240$$

1.4 For each Presidential candidate, what would be the domestic real interest rate if their proposed policies were enacted?

Candidate 1: closed economy

$$S = I(r_1^*)$$

$$640 + 10000r_1^* = 3790 - 25000r_1^*$$

$$35000r_1^* = 3150$$

$$r_1^* = 0.09 = 9\%$$

Candidate 2: open economy

$$r_2^* = 0.08 = 8\%$$

1.5 For each Presidential candidate, what would be the level of investment spending if their proposed policies were enacted?

Candidate 1: closed economy

$$I_1 = I(r_1^*) = 3790 - 25000(0.09) = 1540$$

Candidate 2: open economy

$$I_2 = I(r_2^*) = 3790 - 25000(0.08) = 1790$$

1.6 For each Presidential candidate, what would be the level of net capital outflows (NCO) if their proposed policies were enacted (recall that $NCO = S - I$)? Would this country be a net borrower or net lender on international financial markets?

Candidate 1: closed economy

$$NCO_1 = S_1 - I_1 = (640 + 10000(r_1^*)) - (1540) = (640 + 900) - (1540) = 1540 - 1540 = 0$$

This should be the case since the economy is still closed to international capital flows, so no lending either way. Alternatively, we know that savings equals investment in a closed economy, thus $S_1 = I_1$ and $NCO_1 = 0$ immediately.

Candidate 2: open economy

$$NCO_2 = S_2 - I_2 = 1240 - 1790 = -550$$

This country is a net borrower since net capital outflows are negative; capital is flowing into the country in the form of loans. This is because the world real interest rate (8%) is below the equilibrium real interest rate in autarky (9%).

1.7 You are an entrepreneur living in Coconut Island. Suppose you hold the patent to a unique technology which can produce a unique product, and you want to borrow money to finance the production of this good. *Ceteris paribus* (all else equal), which presidential candidate would you support? Why?

All else equal, you prefer the open economy (candidate 2). Your product is unique, so competition (domestic or foreign) is not an issue. After the opening of Coconut Island to international trade, your can export your product without any decrease in domestic demand. In other words, the total available demand for your good increases under the open economy scenario. Finally, under the proposal of candidate 2, the interest rate will drop from 9% (under autarky) to 8% (with international trade), decreasing the cost of borrowing money to finance the operation.

2 Foreign Exchange Market

Let's say that there are only two countries in the world, the United States and Japan. Their currencies are traded in an open (i.e. competitive) market without regulation or price controls. The nominal exchange rate (e) of Yen for US Dollars ($\frac{¥}{\$US}$) is the equilibrium price determined in this foreign exchange (currency) market. You are given the following equations for the supply/demand of US Dollars.

$$Q_s = -3750 + 125e \tag{6}$$

$$Q_d = 30000 - 100e \tag{7}$$

2.1 What is the nominal exchange rate in equilibrium (e^*)?

Let the quantity supplied of \$US equal the quantity demanded of \$US; solve for the equilibrium nominal exchange rate (price) e^*.

$$Q_s = Q_d$$

$$-3750 + 125e = 30000 - 100e$$

$$e^* = 150\,\frac{¥}{\$US}$$

In words, 150 units of Japanese currency are required to purchase one unit of US currency in the currency market, in equilibrium.

2.2 The ratio of the price levels in the two economies is $\frac{P_{US}}{P_{JPN}} = \frac{90}{1500}\frac{\$US}{¥}$. What is the real exchange rate of US Dollars for Japanese Yen?

$$\epsilon^* = e^*\frac{P_{US}}{P_{JPN}} = (150\frac{¥}{\$US})(\frac{90}{1500}\frac{\$US}{¥}) = 9$$

In words, you could exchange nine units of Japanese goods for one unit of US goods, in equilibrium. These are the real terms of trade.

2.3 Using your result from the previous part, compute the level of government spending in the US. You may assume the following macroeconomic model for the (open) US economy. Equilibrium in the world loanable funds market has been determined already, giving you r^* exogenously.

$$r^* = 0.05 \tag{8}$$

$$Y = 2000 \tag{9}$$

$$T = 50 \tag{10}$$

$$I(r) = 325 - 2000r \tag{11}$$

$$NX(\epsilon) = 200 - 2\epsilon \tag{12}$$

$$C = 25 + 0.6(Y - T) \tag{13}$$

$$Y = C + I + G + NX \tag{14}$$

From the previous part, we know that $\epsilon^* = 9$. This enters into the net exports function, $NX(\epsilon)$, for the United States.

$$NX(\epsilon) = 200 - 2(9) = 182$$

$$I(r) = 325 - 2000(0.05) = 225$$

$$C = 25 + 0.6(2000 - 50) = 1195$$

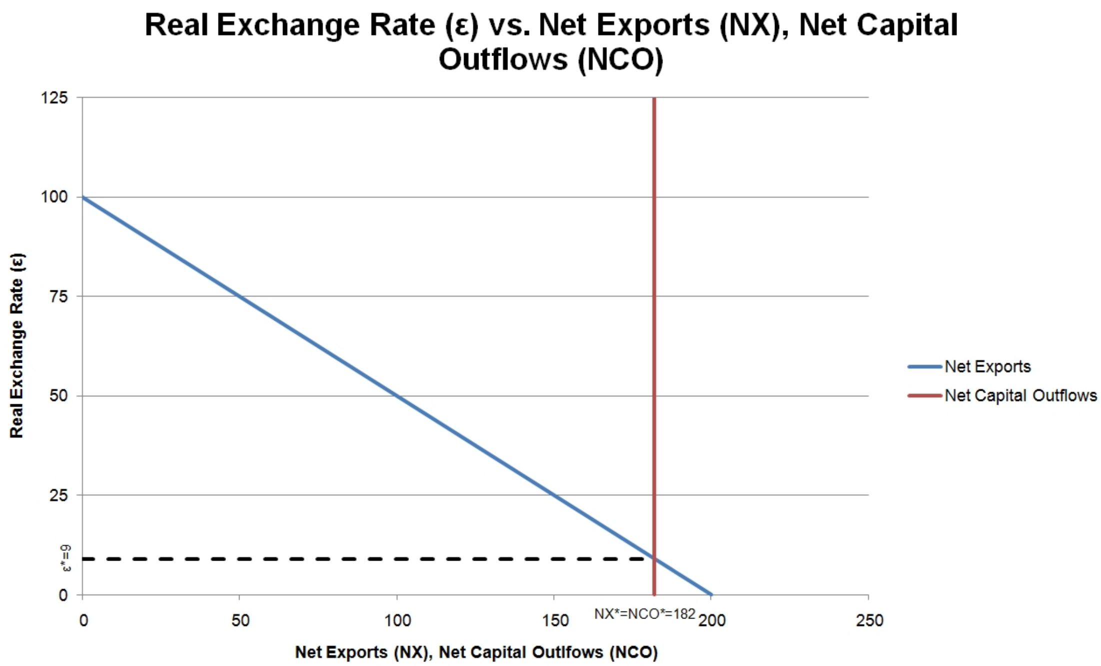
$$G = Y - (C + I + NX) = 2000 - (1195 + 225 + 182) = 398$$

Therefore, we conclude that the level of government expenditure in the US is $G = 398$ units.

2.4 Draw a graph of the real foreign exchange market, plotting the real exchange rate ϵ versus net exports $NX(\epsilon)$ and net capital outflows NCO, which should not depend on ϵ. Label you axes. Identify the equilibrium levels ϵ^* and $NX^* = NX(\epsilon^*)$ clearly on the graph.

$$NX = 200 - 2\epsilon \;\Rightarrow\; \epsilon = 100 - 0.5NX$$

$$NCO = S - I = (Y - C - G) - I = (2000 - 1195 - 398) - 225 = 182$$



Note that the intersection point is to the right of the NX = 0 line (y-axis), so we are in the first quadrant.

3 Steady-State in the Labor Market

Assume the usual labor market identity, which says that individuals in the labor force are either employed or unemployed.

$$LF = U + E \tag{15}$$

Let ϕ (Greek letter phi, for "finding work") denote the job-finding rate, the proportion of unemployed individuals who find work every month. In other words, ϕU individuals move from U to E on a monthly basis. Let λ (Greek letter lambda, for "leaving work") denote the separation rate, the proportion of employed individuals who lose their jobs every month. Similarly, λE individuals move from E to U every month. Define *steady-state in the labor market* as a situation where the levels of unemployment and employment are constant. We can write the *steady-state condition* as follows.

$$\phi U = \lambda E \tag{16}$$

In words, this equation says that the flows in and out of the groups U and E are equal; the number of individuals entering U equals the number of individuals leaving U, for example. ϕU jobs are created and λE jobs are destroyed each month. Steady-state implies that job creation equals job destruction.

3.1 Assume that the steady-state condition $\phi U = \lambda E$ holds. Solve for the steady-state unemployment rate $UR = \frac{U}{LF}$ in terms of parameters ϕ and λ. This is also called the natural rate of unemployment.

$$\phi U = \lambda E \;\Rightarrow\; E = \frac{\phi}{\lambda} U$$

$$LF = U + E = U + \frac{\phi}{\lambda} U = (1 + \frac{\phi}{\lambda}) U$$

$$1 = (1 + \frac{\phi}{\lambda}) \frac{U}{LF}$$

$$UR = \frac{U}{LF} = \frac{1}{1 + \frac{\phi}{\lambda}} = \frac{\lambda}{\lambda + \phi}$$

3.2 Compute partial derivatives $\frac{\partial UR}{\partial \phi}$ and $\frac{\partial UR}{\partial \lambda}$. These will be used in the next part for policy evaluation.

It is helpful to rewrite the unemployment rate as a product before taking the partial derivative (you could also just use the quotient rule directly).

$$UR = (\lambda)(\phi + \lambda)^{-1}$$

Recall the product rule for differentiation; denote $\frac{\partial f(x)}{\partial x} \equiv f'(x)$.

$$\frac{\partial}{\partial x}[f(x)g(x)] = f'(x)g(x) + f(x)g'(x)$$

We can now differentiate UR with respect to ϕ ($x = \phi$ in the formula, treating λ as a constant) and λ ($x = \lambda$, ϕ constant).

3.2.1 *Calculating* $\frac{\partial UR}{\partial \phi}$

$$\frac{\partial UR}{\partial \phi} = (0)(\phi+\lambda)^{-1} + (\lambda)(-1)(\phi+\lambda)^{-2}$$

$$\frac{\partial UR}{\partial \phi} = -\frac{\lambda}{(\phi+\lambda)^2}$$

3.2.2 *Calculating* $\frac{\partial UR}{\partial \lambda}$

$$\frac{\partial UR}{\partial \lambda} = (1)(\phi+\lambda)^{-1} + (\lambda)(-1)(\phi+\lambda)^{-2}$$

$$\frac{\partial UR}{\partial \lambda} = (\phi+\lambda)^{-1} - (\lambda)(\phi+\lambda)^{-2}$$

$$\frac{\partial UR}{\partial \lambda} = (\phi+\lambda)^{-2}[(\phi+\lambda) - \lambda]$$

$$\frac{\partial UR}{\partial \lambda} = \frac{\phi}{(\phi+\lambda)^2}$$

3.3 You have been hired as a labor economist by the US Bureau of Labor Statistics (BLS). The organization needs your help to reduce the US unemployment rate, which is currently at 9.8%. The BLS has estimated $\phi = 0.6$ and $\lambda = 0.05$ from historical data. Using your results from the previous part, which policy would be most beneficial in terms of lowering the steady-state unemployment rate: a very small increase in ϕ or a very small decrease in λ?

$$\frac{\partial UR}{\partial \phi} = -\frac{(0.05)}{[(0.6)+(0.05)]^2} = -0.12$$

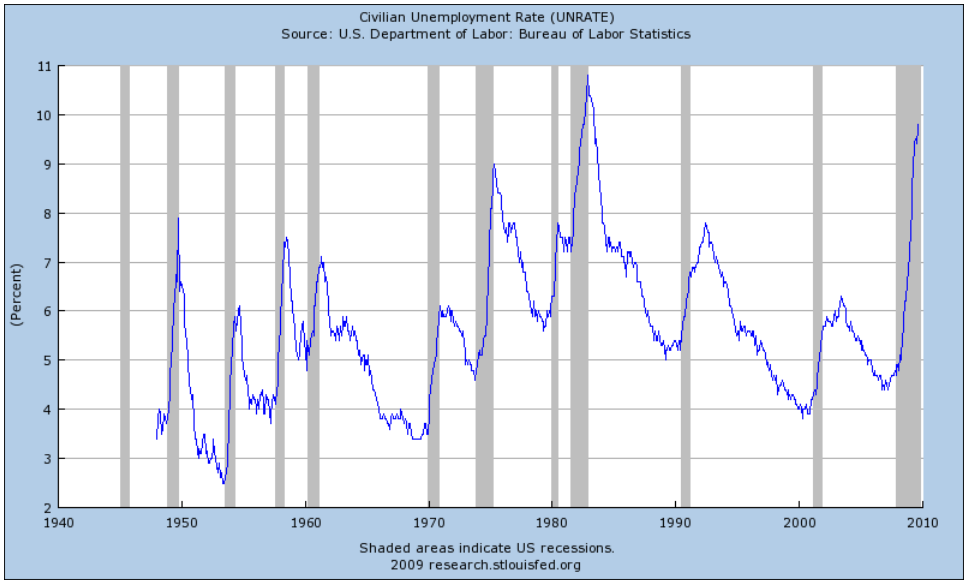
$$\frac{\partial UR}{\partial \lambda} = \frac{(0.6)}{[(0.6)+(0.05)]^2} = 1.42$$

Since $|\frac{\partial UR}{\partial \phi}| < |\frac{\partial UR}{\partial \lambda}|$*, the policy that decreases* λ *by a very small amount will have the largest impact on the unemployment rate. You should recommend the* λ *policy to the BLS as the most beneficial in terms of driving down US unemployment.*

3.4 Provided that $\phi = 0.6$ and $\lambda = 0.05$, calculate UR in steady-state.

$$UR = \frac{\lambda}{\lambda+\phi} = \frac{0.05}{0.05+0.6} = 0.0769 = 7.69\%$$

3.5 Given what you know about historical US unemployment rates, are the BLS estimates of ϕ and λ reasonable? You can use the graph below to inform your response.



This is not too far from the US natural rate of unemployment, which is near 5.5%. Their estimate is high by two percentage points. If you think that the financial crisis means a higher natural rate of unemployment post-crisis (i.e. "jobless recovery"), the BLS estimate could be accurate. However, nothing in the data can support this directly.

4 Computational Labor Market Model (do #3 first)

The BLS has sent you off to work with their latest model of the labor market (see Excel file, Q4a tab). The model can track the movement of U and E over time, if you know ϕ and λ. Let the subscript t stand for time, which is indexed in months. Here are the initial conditions.

$$\phi_0 = 0.6 \tag{17}$$

$$\lambda_0 = 0.05 \tag{18}$$

$$U_0 = 100 \tag{19}$$

$$E_0 = 920 \tag{20}$$

You don't have to worry about this, but here is how U_t and E_t change over time in the model. These equations might give you some intuition about what is going on.

$$U_t = (1 - \phi)U_{t-1} + \lambda E_{t-1} \tag{21}$$

$$E_t = (1 - \lambda)E_{t-1} + \phi U_{t-1} \tag{22}$$

The columns have labels U (unemployed), E (employed), LF (labor force), UR (unemployment rate), and ER (employment rate). You can change parameters ϕ (rate of job finding) and λ(rate of job loss) and the model will adjust accordingly. For this problem, $0 \leq t \leq 30$.

4.1 Before you do anything in Excel, how long do you think it will take (t) for the model to reach steady-state in terms of the unemployment rate UR? $t = 2$? $t = 15$? $t = \infty$? Use your intuition (no wrong answer here). Explain.

Any answer here is okay, as long as the explanation is well-reasoned and coherent. It turns out that the correct answer is closest to t = 15.

4.2 Run the model by copying the formula in the $t = 1$ row all the way down to the $t = 30$ row. How long did the model actually take to converge to the steady-state unemployment rate?

We have convergence to the steady-state unemployment rate at roughly t = 9, at least to two decimal places. Higher precision convergence will take longer.

4.3 Graph UR versus time. On your graph, include a horizontal line for the steady-state unemployment rate that you calculated in Q3. Is the outcome of the model consistent with your previous calculation of the steady-state unemployment rate? Given the initial conditions, are you converging to the steady-state value from above or from below?

See Excel file for graph. The steady-state UR result should be consistent with your previous calculation of 7.69%. With $U_0 = 100$ and $E_0 = 920$, the model converges to the steady-state unemployment rate from above relative to UR=7.69%.

4.4 Make columns for ΔU and ΔE in the Excel file. Graph ΔU and ΔE versus time (try to do this on one graph). Given this graph, make an argument about how the model reaches the steady-state unemployment rate.

See Excel file for graph. As t increases, both ΔU and ΔE go to zero. They are both small in magnitude by the time t = 9. $\Delta U = -\Delta E$, so the changes in U offset the changes in E (this is because the labor force LF is fixed at 1020). E is increasing towards its steady-state value and U is decreasing towards its steady-state value.

4.5 Now look at the Q4b tab. You'll see that the values of ϕ and λ are varying over time; this is more realistic. Run the model by copying the formula in the $t = 1$ row all the way down to the $t = 30$ row. Graph UR versus time. Interpret your graph. Do you observe convergence to a steady-state?

See Excel file for graph. *As t increases, ΔU is increasing and ΔE is decreasing; both ΔU and ΔE do not go to zero. The unemployment rate initially declines, reaches a minimum at $t = 3$, and then grows without bound. UR does not converge to a steady-state even at $t = 30$.* A steady-state is only possible when parameters ϕ and λ are constant for an extended period of time (20-30 months). In real-world data, we observe that ϕ and λ are constantly in flux (changing over time). Therefore, we'd expect that a steady-state is observationally impossible; we can only achieve it in a model.