Economics 101
Spring 2016
Answers to Homework #4
Due April 5, 2016

Directions: The homework will be collected in a box **before** the lecture. Please place your name, TA name and section number on top of the homework (legibly). Make sure you write your name as it appears on your ID so that you can receive the correct grade. Late homework will not be accepted so make plans ahead of time. **Please show your work.** Good luck!
Please realize that you are essentially creating "your brand" when you submit this homework. Do you want your homework to convey that you are competent, careful, professional? Or, do you want to convey the image that you are careless, sloppy, and less than professional. For the rest of your life you will be creating your brand: please think about what you are saying about yourself when you do any work for someone else!

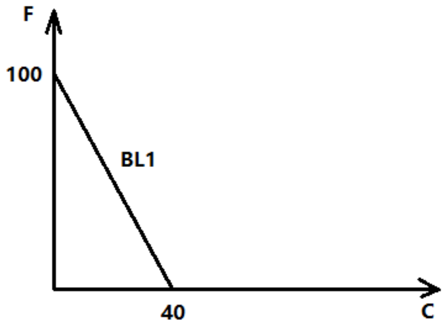
1. Jar Jar Binks is a resident of the Gungan kingdom. He earns $1000 per year working as a senator, and he spends this income on coconuts (C) and fish (F). The price ofcoconuts is $25 and the price of fish is $10.

Suppose that Jar Jar values coconuts and fish equally. His utility function is given by the following equation where U is the amount of utility:

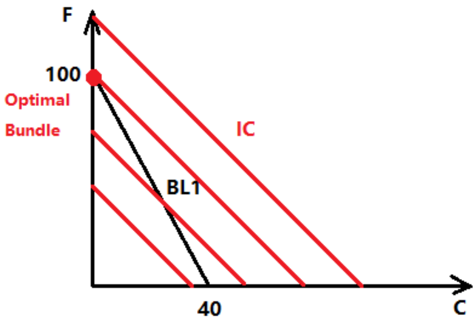
U = 2C + 2F

a) Given the information, what is the equation for Jar Jar's budget line? Assume that Jar Jar spends all of his income on coconuts and/or fish. Graph this budget line with coconuts on the x-axis and fish on the y-axis. Label this budget line BL1.

The equation for his budget line can be written as follows:
Income = PfishQfish + PcoconutsQcoconuts
1000 = 10F + 25C
Solving this equation for fish will put the equation in slope-intercept form:
10F = 1000 – 25C
F = 100 – 2.5C
If Jar Jar spends all his income on coconuts, he could afford $1000/$25 = 40 coconuts and zero fish. If he spends all his income on fish, he could afford $1000/$10 = 100 fish and zero coconuts. Connect the two points (40, 0) and (0, 100) with a straight line, and you have BL1. This budget line has slope -2.5.

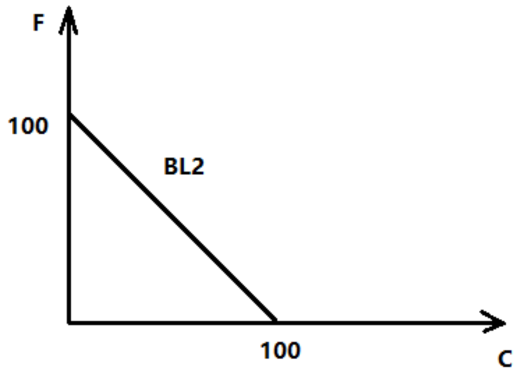


b) Given the above information, draw at least four of Jar Jar's indifference curves in a graph that also contains BL1. Given the budget BL1, how many coconuts and fish will Jar Jar Binks choose to consume? What is his utility level given this consumption bundle?



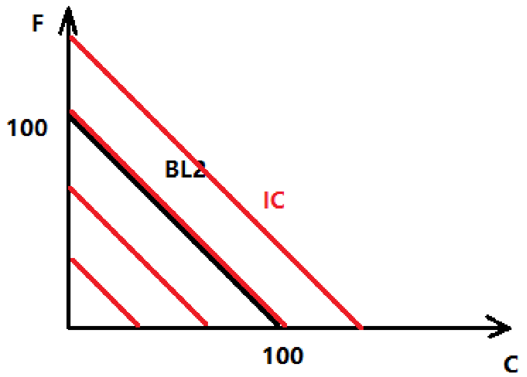
Jar Jar has a utility function of U = 2F + 2C, which means that he values coconuts and fish equally (i.e. they are perfect substitutes). To give up one coconut, Jar Jar must obtain one more fish to maintain the same utility level. Therefore, his indifference curves are parallel, straight lines with slope -1. The optimal bundle is the point on BL1 that lies on the highest utility curve, i.e. point (0, 100) on the graph. At this point, Jar Jar eats no coconuts and 100 fish, and his utility is 2*0 + 2*100 = 200 utils of satisfaction.

c) Now, suppose that Jar Jar's income doubles. At the same time, the price of coconuts drops to $20, while the price for fish rises to $20. Find the new budget line for Jar Jar and graph it in a new graph where coconuts are measured on the x-axis and fish are measured on the y-axis. Label this budget line BL2. Write the equation for BL2.



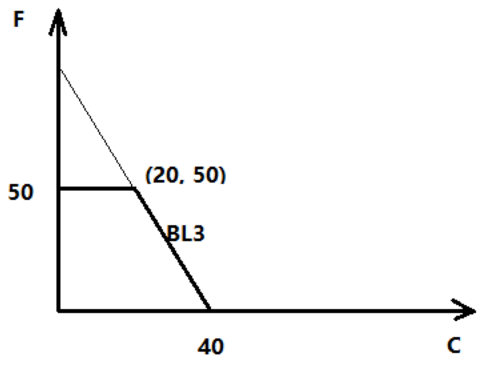
Now Jar Jar has an income of \$2000. If he spends all this money on coconuts, he gets 100 coconuts and no fish. If he spends all this money on fish, he gets 100 fish and no coconuts. Therefore, (0, 100) and (100, 0) are the intercepts of BL2. This budget line has slope -1, because the prices of coconuts and fish are the same. BL2 can be written as: F = 100 – C.

d) Given budget BL2, how many coconuts and fish does Jar Jar eat? What is his utility level? Illustrate his optimal consumption bundle(s) with a graph. (Hint: think about this one very carefully!)



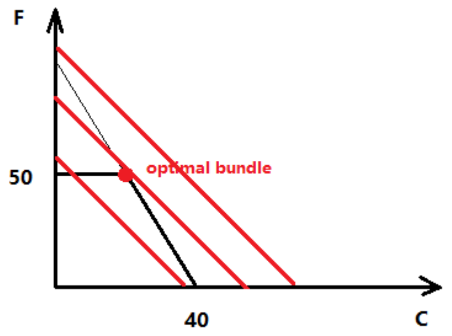
The indifference curves are the same as before, i.e. parallel, straight lines with slope -1. As you can see, because the budget line has the same slope as the indifference curves, every point on BL2 is on the same indifference curve. Therefore, every point on BL2 is an optimal point, meaning that every bundle uses all of Jar Jar's income and is also an optimal consumption bundle. For example, (50, 50) is one optimal bundle, and the utility level is 2*50+2*50 = 200. (100, 0) is another optimal bundle, with the same utility level 2*100+2*0 = 200.

e) Go back to the original situation (income of \$1000, price of \$25 and \$10 for coconuts and fish, respectively). Now, the Gungan king dictates that nobody can eat more than 50 fish per year, to protect the local fish species. Graph the new budget line for Jar Jar. Label it BL3.



Return to the budget line in part a). However, this time the budget line is truncated at F = 50, because Jar Jar is not allowed to eat more than 50 fish. So at F = 50 we have a horizontal line segment as part of the budget curve, where decrease in coconut consumption does not automatically lead to more fish consumption. Notice we have a kink point at (20, 50). Here, Jar Jar eats 50 fish, and spends the rest of his income, 1000 – 10*50 = $500, on coconuts. He can therefore eat 50 fish and 20 coconuts, which is the point where the kink occurs.

f) Under the budget BL3, how many coconuts and fish would Jar Jar Binks eat? What is his utility level?



As you can see in the graph, the point on BL3 that lies on the highest indifference curve is the kink point (20, 50). So under BL3, Jar Jar eats 20 coconuts and 50 fish. His utility level is 2*20 + 2*50 = 140.

g) Compare you answers in b) and f). Is Jar Jar Binks better off from the new restriction on fish consumption, or is he worse off? Briefly explain why.

Jar Jar's utility drops from 200 to 140, so he is obviously worse off under the new restriction. This is quite intuitive. Under the given income and price conditions, the best option for Jar Jar is (0, 100), i.e. eat zero coconuts and 100 fish. However, this is not allowed under the new restriction, so Jar Jar can only choose a less preferred consumption bundle and therefore must be worse off.

2. A poor graduate student at UW Madison survives on cheap pizzas (x) and Badger Market coffee (y). His utility from consuming these two goods is given by:

U = x^2y^2

His marginal utility from eating one slice of pizza is 2xy^2, and his marginal utility from drinking one cup of coffee is $2x^2$y.

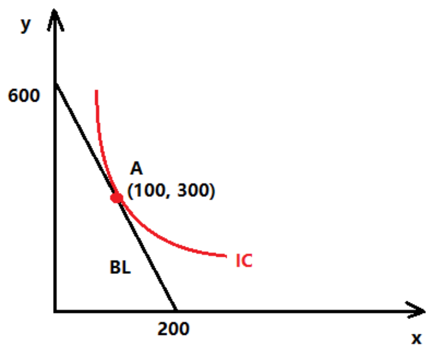
a) The current price for one slice of pizza is $3, and the price for one cup of coffee is $1. The graduate student can spend $600 on these two goods. Given this information, provide an equation relating the graduate student's consumption of x and y if he is maximizing his utility. This equation is called an "optimality condition".

The optimality condition is given by MUx/MUy = Px/Py, or MUx/Px = MUy/Py.

MUx = 2xy^2, and MUy = $2x^2$y, so MUx/MUy = y/x. Px/Py = 3/1 = 3. Therefore y/x = 3, or y = 3x.

b) When the graduate student is behaving optimally, many slices of pizza and cups of coffee does he choose to purchase? What is his utility level? In a graph draw the graduate student's budget line and an indifference curve showing his utility maximizing choice. Put pizza (x) on the horizontal axis. Label the optimal bundle of pizza and coffee as point A.

Since the student spends all of $600, we have 3x+y = 600. We can use our optimality condition that y = 3x and this budget line to find that y + y = 600 or y = 300. This implies that x = 100. The optimal consumption bundle is therefore (100, 300). The utility level is 100^2 * 300^2 = 900,000,000 utils of satisfaction.

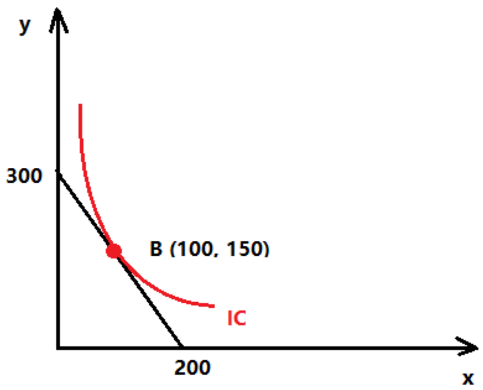


c) Badger Market decides to raise the price of one cup of coffee to $2. How many pizza and coffee does the graduate student now purchase, if he is maximizing his utility? How much is he worse off? Draw the new budget line and the indifference curves. Label the new optimal bundle as point B.

The optimality condition is still MUx/MUy = Px/Py, but now Px/Py = 3/2. So y/x = 3/2, and we have 2y = 3x.

Since the student spends \$600, we have 3x + 2y = 600, so 2y = 3x = 300, and x = 100, y = 150.

The utility level is 100^2 * 150^2 = 225,000,000. The student is worse off by 675,000,000 utils.

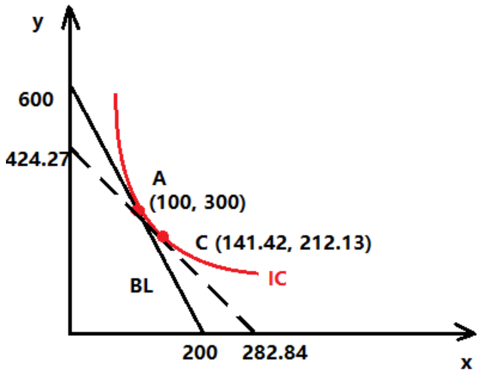


d) We now want to compute the income and substitution effects of the price change in part (c). To do this we must compute the point "C" that lies on an "imaginary" budget line. There are two features of point C: (i) it gives the same utility as the bundle chosen under the original prices, so it lies on the same indifference curve as point A; (ii) the imaginary budget line has the same slope as the second budget line, so the slope of the indifference curve at point C is equal to the slope of budget line 2. Importantly that means that **the imaginary budget line does not pass through point A!** Indeed, we do not know the income of the imaginary budget line, though we do know its slope. This information about slope gives you an optimality condition and the second equation you need is that the utility from the bundle at C is the same as the utility from the bundle at A. Use these two equations to find point C and illustrate on a diagram. Once you find the coordinates of bundle C, calculate the income of this imaginary budget line. Hint: the numbers here will get messy-use a calculator and carry all numbers out to two places past the decimal.

The imaginary budget line has the same slope as the second budget line, so its slope must be equal to -3/2, which means Px/Py = 3/2. Assume that Px = 3 and Py = 2 for simplicity.

Point C is the tangent point between this imaginary budget line and the indifference curve in part b), so it also satisfies the optimality condition MUx/MUy = Px/Py = 3/2, which implies y/x = 3/2, and 2y = 3x.

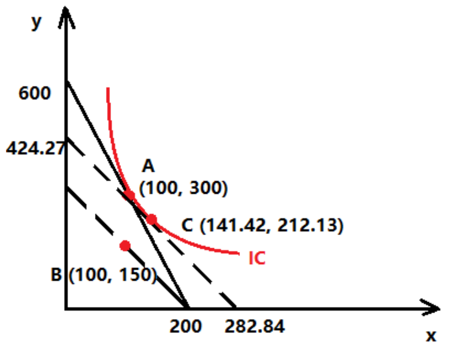
Let the imaginary budget line be of the form 3x + 2y = I, with I being the income. Then from 2y = 3x, we have 2y + 2y = I. So y = I/4, x = I/6. We also know that C is on the indifference curve of part b), so it should give a utility level of 900,000,000 utils, which means (I/6)^2 * (I/4)*2 = 900,000,000, or (I^4)/576 = 900,000,000. Solving this equation, we get income I = 848.53. x = I/6 = 141.42. y = I/4 = 212.13. So C has the coordinates (141.42, 212.13).



e) Use point C to calculate the income and substitution effects of the increase in coffee price (in terms of the change in the quantity of coffee consumed). Hint: in this problem you will calculate the income and substitution effects along the y-axis and, if you can do this, you should be ready to do the simpler examples where the income and substitution effects are calculated along the x-axis.

The substitution effect is the change in quantity of coffee consumed between points A and C. From A to C, y decreases by 300 – 212.13 = 87.87.

The income effect is the change in quantity of coffee consumed between points C and B. From C to B, y decreases by 212.13 - 150 = 62.13.



f) Due to the loud protests from impoverished graduate students, the university decides to raise their stipend (salary). How much should the stipend raise be, such that our graduate student could return to his original utility level before the price increase? (In other words, by how much would his stipend have to change to be able to afford the bundle at point C?)

Part e) has already answered this question. Point C is on a hypothetical budget line given by 3x + 2y = 848.53, and it gives the same amount of utility as point A. So the student needs an income of 848.53 to compensate for the price increase and to return to the original utility level of 900,000,000. Therefore, the stipend needs to increase by 848.53 – 600 = $248.53.

3. The Silph Company employs workers to make scopes. The following table shows the short-run relationship for the company between workers employed and scopes produced. You are told that the wage for each worker is $10 and that the raw material to make a single scope costs $20.

Quantity of Variable Input: Labor	Quantity of Output: Scopes	FC	VC	TC	AFC	AVC	ATC	MC	MPL
0	0				---	---	---	---	---
1	1								
2	2						80		
3	3								
5	4								
8	5								
13	6								
21	7								
34	8								

a) Fill out the table above using the given information.

When q = 2, the company employs two workers and pays 2*10 = $20 as wage. There is also a raw material cost of 2*20 = $40. So the variable cost (VC) = 20+40 = $60. The ATC = $80, so TC = $160, and FC = TC – VC = 160 – 60 = $100.

Since the company is operating in the short run, FC should be constant for every output level. The filled table is like below:

Quantity of variable input: Labor	Quantity of output: scopes	FC	VC	TC	AFC	AVC	ATC	MC	MPL
0	0	100	0	100	---	---	---	---	---
1	1	100	30	130	100	30	130	30	1
2	2	100	60	160	50	30	80	30	1
3	3	100	90	190	33.33	30	63.33	30	1
5	4	100	130	230	25	32.50	57.50	40	0.5
8	5	100	180	280	20	36	56	50	0.33
13	6	100	250	350	16.67	41.67	58.33	70	0.2
21	7	100	350	450	14.29	50	64.29	100	0.125
34	8	100	500	600	12.50	62.50	75	150	0.077

b) Does Silph Co. experience diminishing returns to labor? Use the last column of the table to answer this question.

MPL is calculated as the increase in output per one additional worker. Since MPL decreases as the number of workers increases, the company does experience diminishing returns to labor.

c) Assume that Silph Co. is in a perfectly competitive market for scopes, and the current market price for one scope is \$60. How many scopes should Silph Co. produce? How many workers does it hire? Does the company make a profit?

In a perfectly competitive market, firms are price takers, and they produce at the level such that P = MC. Here, P = \$60. If the company produces at q = 6, MC would exceed P, and the company would lose money by producing the last of the six scopes. So the company produces at q=5, where MC = \$50, just below P. The associated total cost is 280, while the revenue is 5*60 = \$300, so the company makes a profit of 300-280 = \$20.

d) In the long run, what is the minimum price for one scope such that the company could stay open?

In the long run, Silph Co. still produces at P = MC since it is in a perfectly competitive market. However, the firm would only stay open if it does not suffer a loss, i.e. its revenue is at least as large as its total cost. In other words, P must be at least as large as the ATC.

So we have $MC = P \geq ATC$ as the condition for the firm to stay open. This is only true for q larger or equal to 6, and for the firm to produce at least 6 scopes, the price of one scope must exceed \$70. Therefore, \$70 is the minimum price for the company to stay open.

4. Consider the taco stands on State Street. Suppose that the taco market on State Street is a perfect competitive market, where all taco stands are exactly the same (i.e. producing the exact same tacos and having the exact same cost features). The market demand for tacos is given by the demand curve where P is the price per taco and Q is the market quantity of tacos:

$$P = 100 – 2Q$$

Each taco stand faces a marginal cost curve given by the following equation where q is the quantity of tacos produced by the firm:

$$MC = 4q$$

and a total cost curve given by:

$$TC = 2q^2+8$$

a). For each taco stand, what is the fixed cost (FC)? What is the average fixed cost (AFC) curve and the average total cost (ATC) curve?

The total cost curve is made up of a term that increases in q (i.e. $2q^2$) and a term invariant in q (i.e. 8). Therefore, $2q^2$ is the variable cost, and 8 is the fixed cost.

For the average costs, simply divide by the quantity produced by the firm, q:

AFC = FC/q = 8/q
AVC = 2q
ATC = TC/q = $(2q^2+8)/q$ = 2q + 8/q = AVC + AFC

b) What is the break-even price for each taco stand?

At the break-even price, each taco stand makes exactly zero profit, i.e. the revenue just covers the total cost, Pq = TC.

Dividing by q, we have P = ATC. Since we are in a perfectly competitive market, each taco stand is a price taker, and we have P = MC for each taco stand. Therefore we must have MC = ATC at the break-even price.

MC = 4q. ATC = 2q + 8/q. Solve 4q = 2q +8/q, we get q = 2 tacos.

The break-even price is P = MC = 4q = 4*2 = $8 per taco.

c) What is the shut-down price? (For simplicity, assume that q does not need to be an integer)

At the shut-down price, the revenue of each taco stand just covers the fixed cost, i.e. Pq = VC. Dividing by q, we have P = AVC. Again, since we are in a perfectly competitive market, P = MC for each taco stand. Therefore we must have MC = AVC at the shut-down price.
MC = 4q. AVC = 2q. Solve 4q = 2q, we get q = 0. The shut-down price is P = MC = 4q = 4*(0) = $0 per taco.

d) Assume that the current price for one taco on State Street is $10. How many tacos does each taco stand produce? Suppose that there is no cost in setting up/closing down a taco stand. Will we witness entry of new taco stands into the market, or exit of existing stands from the market?

In a competitive market, P=MC, so $10=MC=4q, and q=2.5 tacos. Each taco stand makes 2.5 tacos and generates total revenue of 2.5*10 = $25. The total cost associated with q = 2.5 is 2*(2.5^2) + 8 = $20.50, so each taco stand makes a profit of 25.00 -

20.50 = \$4.50.

Since each taco stand makes a profit, and there is no entry cost into the taco market, we expect to see entry of new taco stands into the market to share the profit.

e) Imagine that you are one of the taco stand owners. The current price for one taco has dropped from \$10 to \$7. Describe what you would do in the short run and in the long run.

\$7 is below the break-even price and above the shut-down price. So in the short run, you should cut back on production and accept the loss. Specifically, you should cut production from $q = 10/4 = 2.5$ tacos to $q = 7/4 = 1.75$ tacos.

In the long run, because \$7 is below the break-even price, you should close down the taco stand and stop making the loss.

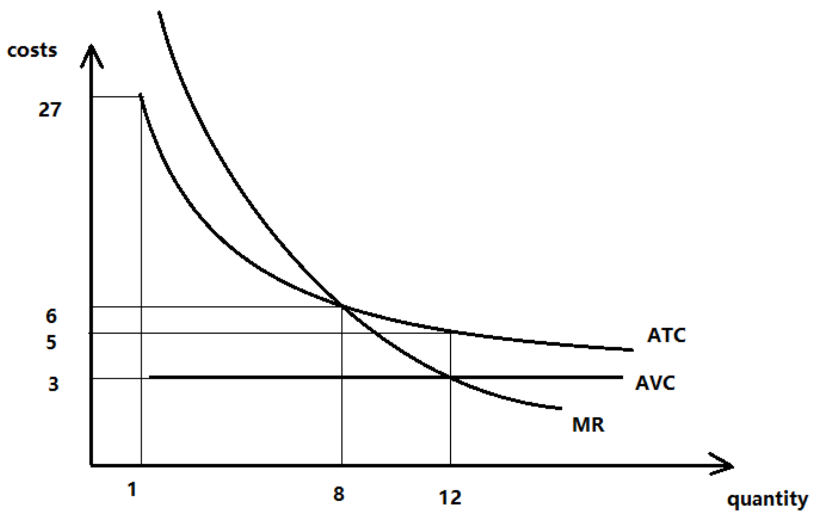
f). In the long run, what would the price for one taco be on State Street? How many taco stands will stay in the market?

In the long run, the free of entry of new taco stands would cause profit to drop to zero. In other words, every taco stand will make exactly zero profit and have P = ATC. Since P = MC, in the long run each taco stand produces at MC = ATC.

That is, we have $4q = 2q + 8q$ or $q = 2$ tacos. The associated price level is $P = MC = 4q = \$8$ per taco.

So in the long run, the price for one taco is \$8. Plug this into the demand curve, we get that the market demand for tacos is 46 tacos. Since each taco stand produces q=2 tacos, the number of taco stands must be 46/2 = 23 taco stands.

5. Observe the following cost graph of a manufacturing firm:



a) Is the firm in a perfectly competitive market? Explain your reasoning.

No. The firm cannot be located in a perfectly competitive market, because the marginal revenue curve is downward sloping in the graph. In a perfectly competitive market, P=MR, so MR must be a flat horizontal line.

b) Find the average fixed cost (AFC) curve.

From the graph, we know that when q = 1, AVC = 3 and ATC = 27.
VC = q*AVC = 1*3 = $3
TC = q*ATC = 1*18 = $27
So the fixed cost is TC - VC = 27 - 3 = $24. The AFC is FC/q = 24/q.

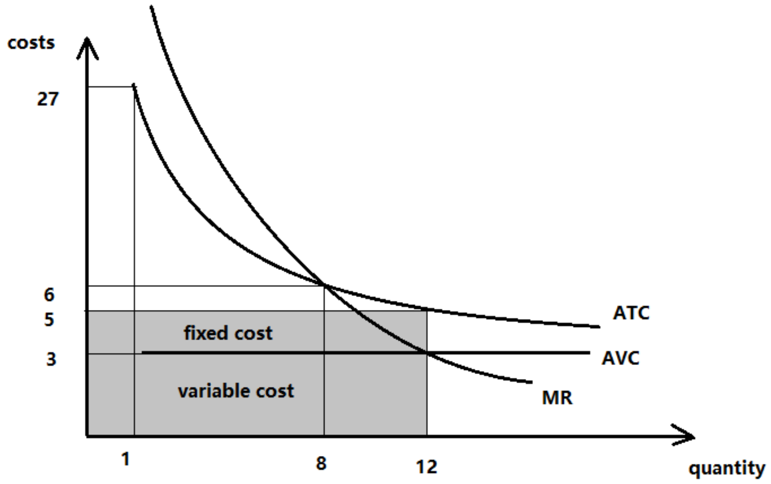
c) Find the marginal cost (MC) curve.

Observe that the AVC curve is a flat horizontal line. This implies that marginal cost is also constant, and that marginal cost is equal to average variable cost. So here, MC = AVC = 3.

d) If the firm chooses to maximize its profit, at what quantity should it produce? Label the areas in the graph representing revenue, total fixed cost and total variable cost when the firm chooses the optimal quantity. Assume the firm can only charge one price for the good.

The firm chooses the optimal quantity where MC = MR. In the graph, we find that MC = MR when q = 12, so the optimal quantity is 12 units of the good.

At q = 12 units, the average total cost is $5, so the total cost is equal to 5*12 = $60. The total cost is represented by the shaded rectangle area in the following graph. The shaded area above the AVC curve is the total fixed cost, and the shared area below the AVC curve is the total variable cost.



The total revenue is equal to P*q, or ($3 per unit)(12 units) or $36. Total revenue is equal to variable cost for this firm. The firm has revenue that is less than its total cost.