Economics 101
Spring 2016
Answers to Homework #1
Due Tuesday, February 9, 2016

Directions:

- The homework will be collected in a box **before** the large lecture.
- Please place <u>your name</u>, <u>TA name</u> and <u>section number</u> on top of the homework (legibly). Make sure you write your name as it appears on your ID so that you can receive the correct grade.
- Late homework will not be accepted so make plans ahead of time. **Please show your work.** Good luck!

Please realize that you are essentially creating "your brand" when you submit this homework. Do you want your homework to convey that you are competent, careful, professional? Or, do you want to convey the image that you are careless, sloppy, and less than professional. For the rest of your life you will be creating your brand: please think about what you are saying about yourself when you do any work for someone else!

1. **Math Review Question:**

a. You are told that there are two linear relationships between Y and X where Y is the variable measured on the vertical axis and X is the variable measured on the horizontal axis. The first linear relationship is given by the equation $Y = 50 – 2X$. You are told that the second linear relationship goes through the origin and that for every 1 unit increase in the X variable, the Y variable increases by 8 units. What is the equation for the second line and what is the solution (X, Y) for your two equations?

Answer:
For the second equation you know that the definition of slope is rise/run. The rise, or the change in Y, is 8 units; the run, or the change in X, is 1 unit. Thus, the slope of this equation is 8. Since the equation goes through the origin, the y-intercept must be equal to 0. So, the equation for the second line is $Y = 8X$.
Using these two equations, you can find the values of X and Y where the two lines intersect.
$8X = 50-2X$
$X=5$
This will occur at (5, 40).

b. You are told that there are two linear relationships between Y and X where Y is the variable measured on the vertical axis and X is the variable measured on the horizontal axis. The first linear relationship contains the points (125, 75) and (50, 150). The second linear relationship contains the points (50, 100) and (150, 150). Find the equations for the two lines and then calculate the solution (X, Y) for these two equations.

Answer:

The slope of line 1 is equal to $(75 – 150)/(125 – 50) = -75/75 = -1$. Then, to find the y-intercept use the general y-intercept form ($y = mx + b$) and substitute in the slope measure you calculated and one of the points you were given. Thus, the equation can be written as $Y = 200 – X$.

The slope of line 2 is equal to $(100 – 150)/(50 – 150) = -50/-100 = ½$. Then, to find the y-intercept use the general y-intercept form ($y = mx + b$) and substitute in the slope measure you calculated and one of the points you were given. Thus, the equation can be written as $Y = (1/2)X + 75$.

To find the solution for these two equations, set them equal to one another. Thus,

$200 – X = (1/2)X + 75$

$125 = (3/2)X$

$X = 125(2/3) = 83.3$

$Y = 200 – X = 200 – 83.3 = 116.7$ or

$Y = (1/2)X + 75 = (1/2)(83.3) + 75 = 116.7$

The solution to the two equations is (83.3, 116.7).

c. You are told that there are two linear relationships between Y and X where Y is the variable measured on the vertical axis and X is the variable measured on the horizontal axis. The first linear relationship is described as follows: the Y variable is equal to 5 more than twice the X variable. The second linear relationship is described as follows: the X variable is equal to 5 less than twice the Y variable. Find the equations for the two lines and then calculate the solution (X, Y) for these two equations.

Answer:

The first line can be written as $Y = 5 + 2X$ while the second line can be written as $X = 2Y – 5$. You can rewrite the second equation, solving for Y, in order to have this equation in slope-intercept form. Thus, $Y = (1/2) X + (5/2)$. Take a moment to verify that each equation actually fits the description you were given in the problem.

Use these two equations to find the solution:

$5 + 2X = (1/2)X + (5/2)$

$10 + 4X = X + 5$

$3X = -5$

$X = -5/3$

$Y = 5 + 2X = 5 + 2(-5/3) = 5/3$ or

$Y = (1/2)X + (5/2) = (1/2)(-5/3) + 5/2 = (-5/6 + 15/6) = 10/6 = 5/3$

$(X, Y) = (-5/3, 5/3)$.

If you are having difficulties with the fractions you might want to take some time to review the rules about adding, subtracting, multiplying and dividing fractions. I will assume that you know these rules and that you can use them accurately.

2. Math Review Question:

a. (Physics Question☺) [Note: Professor Kelly found physics impossible! So, be assured that you do not need to know ANY physics to answer this question-just apply the standard slope-intercept form equation to this new setting. Be brave, you non-physicists!] An experimental physicist is attempting to determine the relationship between the mass and kinetic energy of a particle in a laboratory setting. After two trials, she has observed the following data, written as an ordered pair (mass, kinetic energy): (2, 4) and (4, 10). As her lab assistant, what is the slope-intercept form of the straight line that expresses kinetic energy as a function of mass? Based on this estimate, what kinetic energy would we expect for a particle that has a mass of 8 units? Theoretically, a particle with zero mass should have zero kinetic energy. Is our experimental model consistent with this?

Answer:

We know that slope $m = (\Delta y)/(\Delta x)$ (the change in y over the change in x) and that slope intercept form is $y = mx + b$. Given data points (2,4) and (4,10), use the equation for slope to compute m.

$m = (10-4)/(4-2) = 6/2 = 3$

We now know m, but not b. To find y-intercept b, plug one of the data points into the slope-intercept equation and solve for b (it does not matter which data point you use; we use (2,4)) using the m you just found.

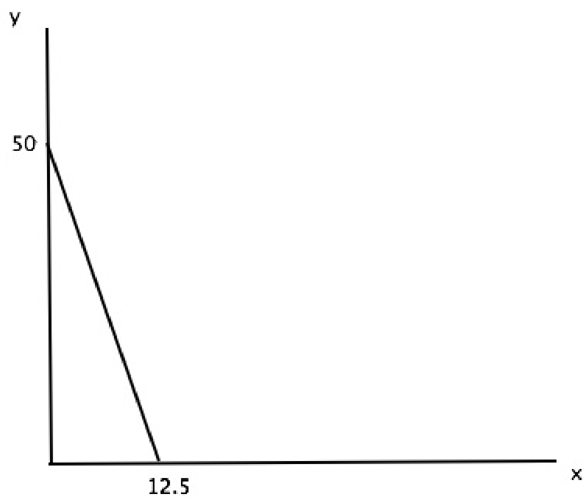
$4 = (3)(2) + b = 6 + b$

$b = -2$

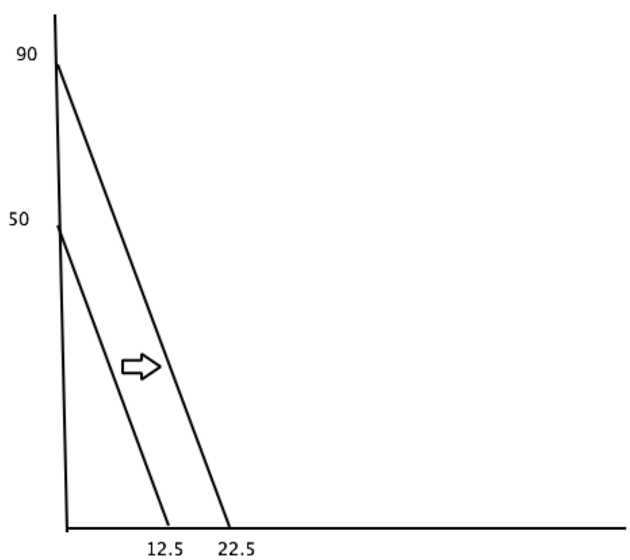
Therefore, the equation $y = 3x - 2$ expresses kinetic energy, y, as a function of mass, x, in slope-intercept form. Next, we want an estimate for y given that $x = 8$. Plugging 8 into the equation above, we find that $y = (3)(8) – 2 = 22$ is our estimate for the particle's kinetic energy. Theory tells us that a particle with zero mass should have zero kinetic energy; in other words, the point (0,0) should be on our estimated line. This is equivalent to saying that our line has a y-intercept of zero, which it does not ($b = -2$), so our experimental model is not consistent with the theory.

b. Suppose you are given a line described by the equation $y = 50 - 4x$ and you are told that the x value has increased by 10 units at every y value. What is the equation for the new line? Show your work.

It is helpful to have a sketch to guide your work here. The initial line is given in the graph below.

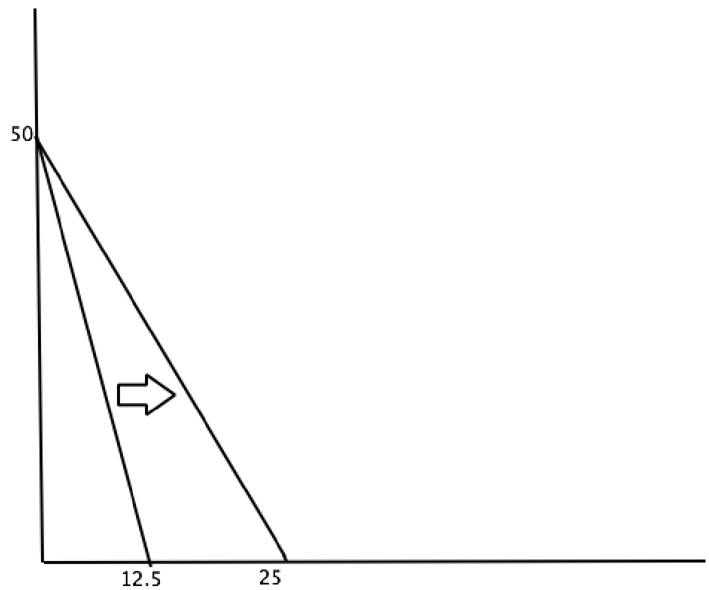


The line has shifted in a parallel fashion to the right so the slope is unchanged but the new line contains the point (22.5, 0). The new equation is y = -4x + b and you can find the value of b by substituting in the point that lies on the new line. Thus, 0 = -4((22.5) + b or b = 90. The new equation is y = 90 - 4x. The graph below represents the new line.



c. Suppose you are given a line described by the equation y = 50 - 4x and you are told that the x value has doubled at every y value. What is the equation for the new line? Show your work.

The new line has the same y-intercept as the initial line but the x-intercept is now 25 instead of 12.5. Therefore the new line has slope = -2 and the equation for the new line is y = 50 – 2x. The diagram below illustrates this new line.



d. You are given two equations.

Equation 1: y = 10 + 2x

Equation 2: y = 26 – 2x

But, you are also told that equation 1 has changed and now the y value is 10 units bigger at every x value than it was initially.

i. Write the equation that represents the new Equation1'.

ii. Given the new Equation 1' and Equation 2, find the (x,y) solution that represents the intersection of these two lines.

i. We know that (0,10) was on the original line represented by Equation 1; the new Equation 1' would contain the point (0, 20) since the y value at every x value has increased by 10 units. The slope of Equation 1' is the same as the slope of Equation 1. Thus, y = b' + 2x where b' is the y-intercept of the new Equation 1'. Use the point (0, 20) to find the value of b'. Thus, 20 = b' + 2(0) or b' = 20. The equation for Equation 1' is y = 2x + 20.

ii. To find where Equation 1' and Equation 2 intersect set the two equations equal to one another:

2x + 20 = 26 – 2x

4x = 6

x = 3/2

y = 2x + 20 = 2(3/2) + 20 = 23

Or, y = 26 – 2x = 26 – 2(3/2) = 23

The solution for these two equation is (x, y) = (3/2, 23).

3. **First Percentage Change Problem:**This question reviews some pretty simple math in an attempt to make your aware that you know how to change an index number. This question allows you to practice this technique in a familiar setting so that later in the course when we do this in an economics context it might seem more familiar. Tom's chemistry class has had three exams. The exams in the class are all given the same weight in the grading scale, but each exam has a different number of total possible points. On the first exam Tom made a 60 out of 75 points, on the second exam Tom made a 51 out of a possible 60 points, and on the third exam Tom made a 40 out of 50 points.

    a. What is Tom's grade on the first exam if the first exam score was converted to a 100 point scale?
    b. What is Tom's grade on the second exam if the second exam score was converted to a 100 point scale?
    c. What is Tom's grade on the third exam if the third exam score was converted to a 100 point scale?
    d. On a 100 point scale with each exam given the same weight in the calculation, what is Tom's average grade?
    e. If Tom wants to raise his average grade and the fourth exam has 60 points, how many points must Tom get on this exam?

Answer:

a. To convert Tom's score to a 100 point scale you can set up a ratio: 60/75 = x/100 and then solve for x. In this case x is equal to 80.
b. To convert Tom's score to a 100 point scale you can set up a ratio: 51/60 = y/100 and then solve for y. In this case y is equal to 85.
c. To convert Tom's score to a 100 point scale you can set up a ratio: 40/50 = z/100 and then solve for z. In this case z is equal to 80.
d. Now that we know Tom's exam scores on a 100 point scale we can add up those exam scores and divide by the number of exams (3) in order to find his average: (80 + 85 + 80)/3 = 81.67.
e. To find the score that Tom needs to raise his average we can take advantage of the ratio technique once again. This time our ratio will be w/60 = 81.67/100. When we solve for w we are finding the exam score on a 60 point scale that will result in Tom's average staying constant. Solving for w we get w equal to 49. For Tom to raise his average he must score higher than 49 on the fourth exam.

4. **Second Percentage Change Problem:**Bernie stays confused about percentages and he is struggling to figure out what he needs to do on his final exam in Chemistry in order to get the B he needs. Here is the information he has: he scored a 40 out of a possible 50 points on his first midterm in the class; he scored a 15 out of 25 points on the second midterm (it was tough!) and on the third midterm he got an 85 out of a 100 points. He knows that his final will have 50

points. And, he also knows that each midterm has equivalent weight to all the other midterms and that this weight is 20% of his final grade; he also knows that the final exam will be weighted as 40% of his final grade; and to get a B in the class he knows that his total weighted average must be at least an 84 on a 100 point scale. So, what score will Bernie need to make on that final exam if he is going to get a B in the class? Show your work! Here you will find it helpful to work in decimals instead of fractions: try to do this without a calculator though!

Answer: To answer this question we need to do a lot of indexing of the midterm grades. To begin with let's calculate Bernie's score on each midterm based on a 100 point scale. Midterm 1: 40/50 points is the same as 80/100 points. To see this recognize that to transform a 50 point test into a 100 point test requires us to multiply by a factor of 2. But, if we are going to multiply the denominator by 2, we must also multiply the numerator by 2 in order to keep the same value: hence, (40/50)(2/2) = 80/100. Effectively we are just rescaling the exam to 100 points. Midterm 2: 15/25 points is the same as 60/100 points. Here the rescaling factor is 4 since 25 * 4 = 100. Midterm 3: 85/100 points needs no rescaling since it is already on a 100 point scale. So, Bernie's three midterms scored on a 100 point scale are respectively: 80, 60, and 85. Now, we need to compute the final weighted grade: Final weighted grade = (score on first midterm on 100 point scale)(weight of first midterm) + (score on second midterm on 100 point scale)(weight of second midterm) + (score on third 6 midterm on 100 point scale)(weight of third midterm) + (score on final exam on 100 point scale)(weight of final exam) Thus, to get a B in the class Bernie has the following equation: 84 = 80(.2) + 60(.2) + 85(.2) + (score on final exam on 100 point scale)(.4) 84 = 16 + 12 + 17 + (score on final exam on 100 point scale)(.4) 39 = (score on final exam on 100 point scale)(.4) 97.5 = score on final exam on 100 point scale But, Bernie's final exam has only 50 points in all-so, we need to rescale this 97.5 out of 100 points to the number out of 50 points he needs to get. Thus, (score on final exam on 50 point scale)/50 = 97.5/100 or score on final exam on 50 point scale = 97.5/2 = 48.75. Whoa, Bernie is really going to have to perform on this final exam if he hopes to get the B in the class!

5. **Opportunity Cost:**

Pareto can travel from Madison to Minneapolis in one hour by taking an airplane. The same trip takes 5 hours by bus. Airfare is $90 and the bus fare is $40. If he is not travelling, Pareto can work to earn $25/hour.

Answer the following questions:

a. What is the opportunity cost if Pareto travels by bus?
   $40(bus fare) + 5(25) = $165
b. What is the opportunity cost if Pareto travels by plane?

$90(airfare) + 1(25) = $115

c. Which of these two travel options is cheaper for Pareto if Pareto considers the opportunity costs involved in this travel?

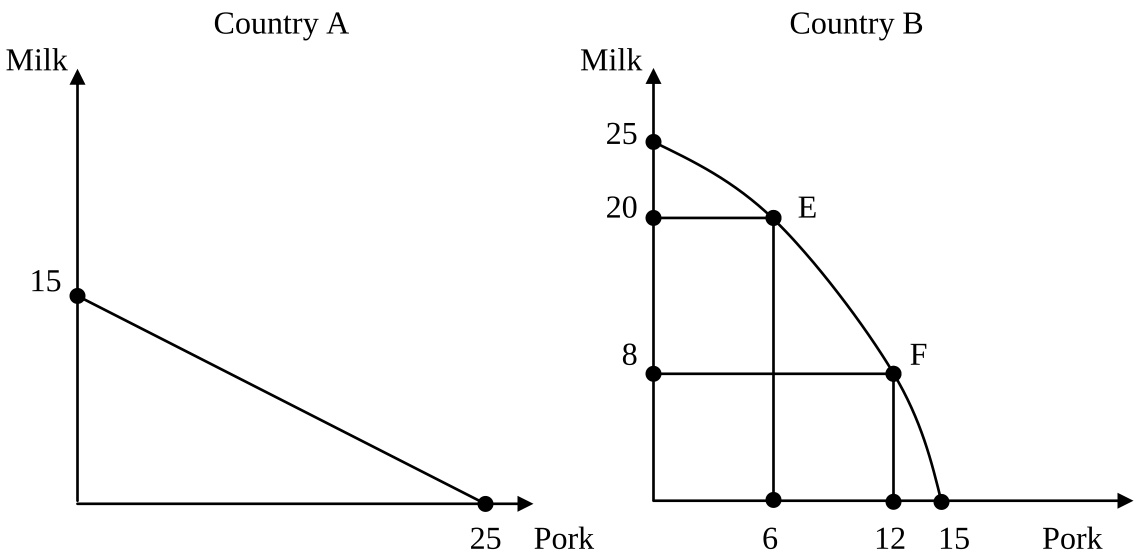
In terms of opportunity cost, travelling by air is cheaper for Tina.

d. Suppose Walras is considering the same trip but Walras only earns $7/hour when he is not travelling. Which of these two travel options is cheaper for Walras given this information? Explain the intuition behind the difference in answers you get for Pareto and Walras.

For Walras, the opportunity cost of travelling by bus is $40 + 5(7) = $75 and the opportunity cost of travelling by air is $90 + $7 = $97. Thus travelling by bus is cheaper.

Even though the bus and plane tickets cost the same for Pareto and Walras, Pareto's time has a much higher valued than Walras. Although both of them are great economists, Pareto would rather spend less time on the road.

6. PPF and Opportunity Cost: The following two graphs represent production possibility frontiers for countries A and B. Both of these countries produce milk (measured in gallons) and pork (measured in pounds).



a. Explain what a production possibility frontier represents.

PPF is a graph representing production tradeoffs of an economy given fixed resources. It is a graphical representation of the maximum mix of outputs that an economy can achieve using its existing resources to full extent and in the most efficient way.

b. What is country A's opportunity cost of producing one gallon of milk in terms of pork? What is country A's opportunity cost of producing one pound of pork in terms of milk?

I will use the slope interpretation of opportunity cost for this question. For intuitive explanation of opportunity cost, please see Question 7. The slope of the line represents opportunity cost of producing one pound of pork in terms of milk. The slope is rise/run. Then 3/5 gallons of milk is the opportunity cost of producing one pound of pork. Then, the opportunity cost of producing one gallon of milk is the reciprocal of 3/5, that is 5/3 pounds of pork.

c. Is country B's opportunity cost of producing one gallon of milk higher at point E than at point F? Explain.

Yes. As country B moves to the left on its production possibility frontier, the opportunity cost of producing one gallon of milk increases. (Think in terms of slope of the curve at point E and F!)

7. **PPF and Comparative Advantage:** The dwarves of Erebor devote 10 hours each day to producing either beer or wine. They can produce a barrel of beer in 2 hours, but need 5 hours to make a bottle of wine. The nearby (more laid-back) elves of Mirkwood devote only 6 hours each day to working. However, they need only 2 hours to produce either a barrel of beer or a bottle of wine.

a. Who has the comparative advantage in producing wine? Who has the comparative advantage in producing beer?

Answer:

The dwarves work 10 hours a day. At maximum, they could produce 10/5=2 bottles of wine and no beer, or 10/2=5 barrels of beer and no wine. Therefore, to produce 1 additional bottle of wine, the dwarves need to give up 5/2 = 2.5 barrels of beer. In other words, the opportunity of producing a bottle of wine is 2.5 barrels of beer for the dwarves.

Similarly, to produce 1 additional barrel of beer, the dwarves need to give up 2/5 = 0.4 bottle of wine. In other words, the opportunity of producing a barrel of beer is 0.4 bottle of wine for the dwarves.

The elves work 6 hours a day. At maximum, they could produce 6/2=3 bottles of wine and no beer, or 6/2=3 barrels of beer and no wine. The opportunity cost of producing one bottle of wine is one barrel of beer, and the opportunity cost of producing one barrel of beer is one bottle of wine, for the elves.

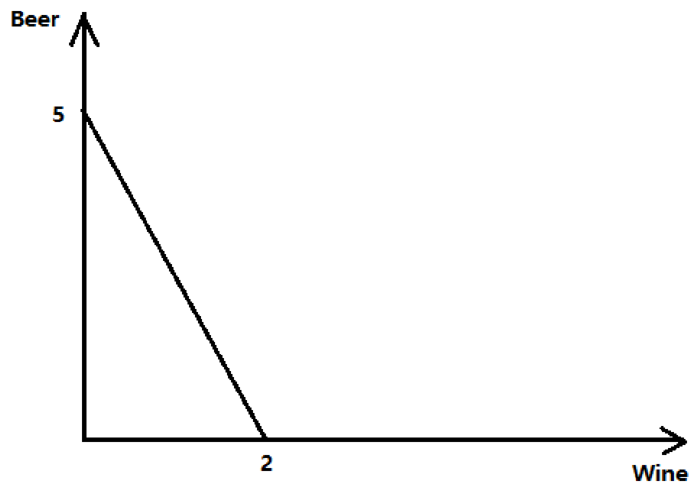
We can organize the results in the following table:

	Dwarves of Erebor	Elves of Mirkwood
Opportunity cost of wine	2.5 barrels of beer	1 barrel of beer
Opportunity cost of beer	0.4 bottle of wine	1 bottle of wine

Since 1<2.5, the opportunity cost of producing wine is lower for the elves than for the dwarves. Therefore the elves have a comparative advantage in producing wine. On the other hand, 0.4<1, so the dwarves have a comparative advantage in producing beer.

b. Draw the PPF for dwarves with wine on the x-axis. On a separate graph, do the same for the elves. What do the slopes signify?

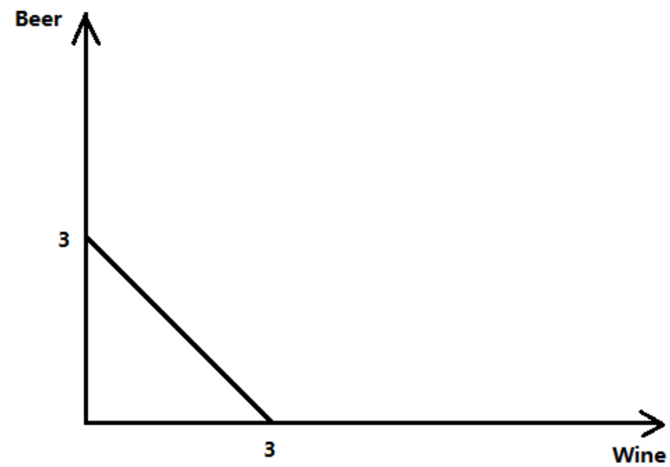
The PPF for the dwarvesof Erebor:



We know that the dwarves could produce 5 barrels of beer and no wine, or 2 bottles of wine and no beer. So points (0, 5) and (2, 0) should naturally be on the PPF. Connect the two points, and you have the PPF for the dwarves.

The slope of the PPF here represents the amount of beer that the dwarves have to give up in order to produce one bottle of wine. In other words, the slope is the dwarves' opportunity cost of producing wine in terms of beer, times -1, which is 2.5*(-1) = -2.5. The function form of the PPF is B = -2.5W+5.

The PPF for the elves of Mirkwood:



The PPF is a straight line segment that connects points (0, 3) and (3, 0). The slope is the elves' opportunity cost of producing wine in terms of beer, times -1, which is 1*(-1) = -1. The function form of the PPF is B = -W+3.

c. The dwarves and elves trade. Draw the joint PPF for both dwarves and elves. Label the kink point on this graph. What do the slopes of this joint PPF signify?

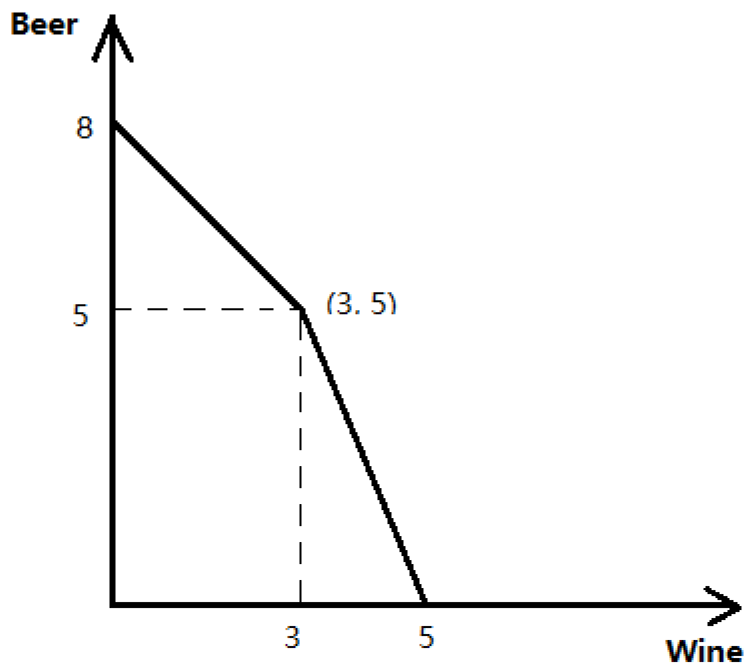
To solve for the joint PPF, we can start by finding some key points on the joint PPF curve.

The horizontal intercept: This represents the total amount of wine produced, if both the dwarves and elves produce only wine and no beer. Hence the horizontal intercept is 2+3 = 5.

The vertical intercept: This represents the total amount of beer produced, if both the dwarves and elves produce only beer and no wine. This value is 5+3=8.

The kink point: At this point, the dwarves and the elves each specialize in producing the product that they have a comparative advantage in. In other words, the dwarves produce only beer, while the elves produce only wine.
The amount of beer produced by dwarves: 5 barrels of beer.
The amount of wine produced by elves: 3 bottles of wine.
Therefore, the kink point is (3, 5).

Connect the three points, and you have the joint PPF curve:



To the left of the kink point, the dwarves only produce beer, while the elves produce both beer and wine. The slope is the amount of beer that the elves have to give up in order to produce one additional bottle of wine, i.e. the elves' opportunity cost of producing wine times -1, which is -1. The function form is B = -W+8.

To the right of the kink point, the elves only produce wine, while the dwarves produce both wine and beer. The slope is the amount of beer that the dwarves have to give up in order to produce on additional bottle of wine, i.e. the dwarves' opportunity cost of producing wine times -1, which is -2.5. The function form is B = -2.5W+12.5.

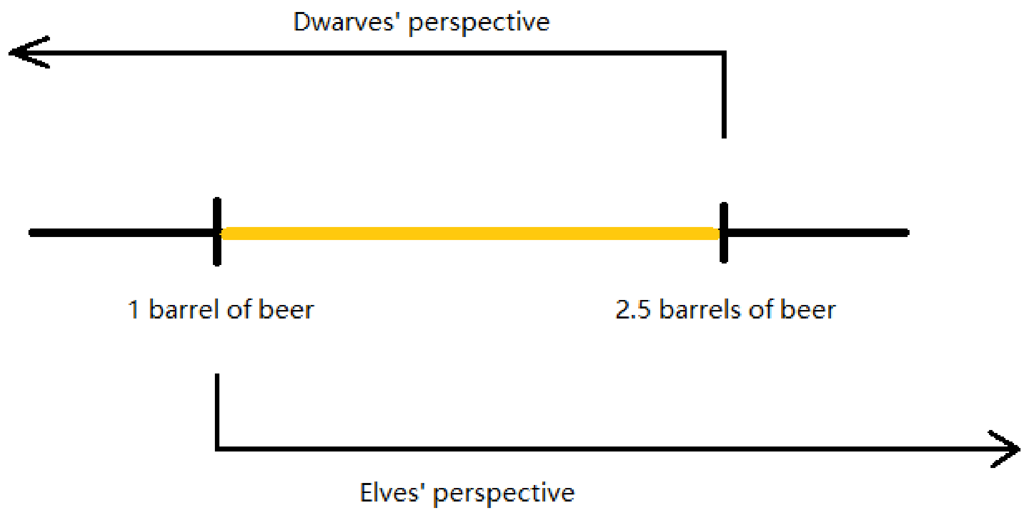
d. What is the range of trading price for one bottle of wine in terms of barrels of beer?

The elves have a comparative advantage in producing wine. Therefore, they would specialize in producing wine and sell it to the dwarves. The elves are willing to sell wine only if the market price of wine (in terms of beer) is above the opportunity cost of producing wine (in terms of beer). Therefore, the trading price must exceed 1.

The dwarves specialize in producing beer and import wine from the elves. The dwarves are willing to buy wine only when the market price for wine (in terms of beer) is below the dwarves' opportunity cost of producing wine (in terms of beer). Therefore, the trading price must be less than 2.5.

The trading range of price for one bottle of wine is therefore between 1 barrel of beer and 2.5 barrels of beer.

You can visualize the result using the following graph:



e. With trade, would it be possible for each nation to consume 1 barrel of beer and 2 bottles of wine? Is the production at the efficient level?

If each nation consumes 1 barrel of beer and 2 bottles of wine, in total they must produce 2 barrels of beer and 4 bottles of wine. Find point (4, 2) on the joint PPF graph. We know that point (4, 2.5) is on the lower segment of the joint PPF curve. Therefore, (4, 2) is on the inside of the PPF curve. The two nations are not producing efficiently.

Now, (spoiler alert) the dark lord Sauron has fallen, and the men of Gondor decide to join the international trade network of the Middle Earth. Men work 8 hours a day. They need 2 hours to produce a barrel of beer and 4 hours to produce a bottle of wine.

f. Find the joint PPF for the dwarves, the elves, and men. Label all the kink points. Draw a graph of this joint PPF and then provide the equations for each segment of the joint PPF.

The men of Gondor work 8 hours a day. At maximum, they could produce 8/4 = 2 bottles of wine and no beer, or 8/2 = 4 barrels of beer and no wine. The opportunity cost of producing one bottle of wine is 4/2 = 2 barrels of beer, while the opportunity cost of producing one barrel of beer is 2/4 = 0.5 bottles of wine. Add men to the opportunity cost table:

	Dwarves	Elves	Men
Opportunity cost of wine	2.5 barrels	1 barrel	2 barrels
Opportunity cost of beer	0.4 bottle	1 bottle	0.5 bottle

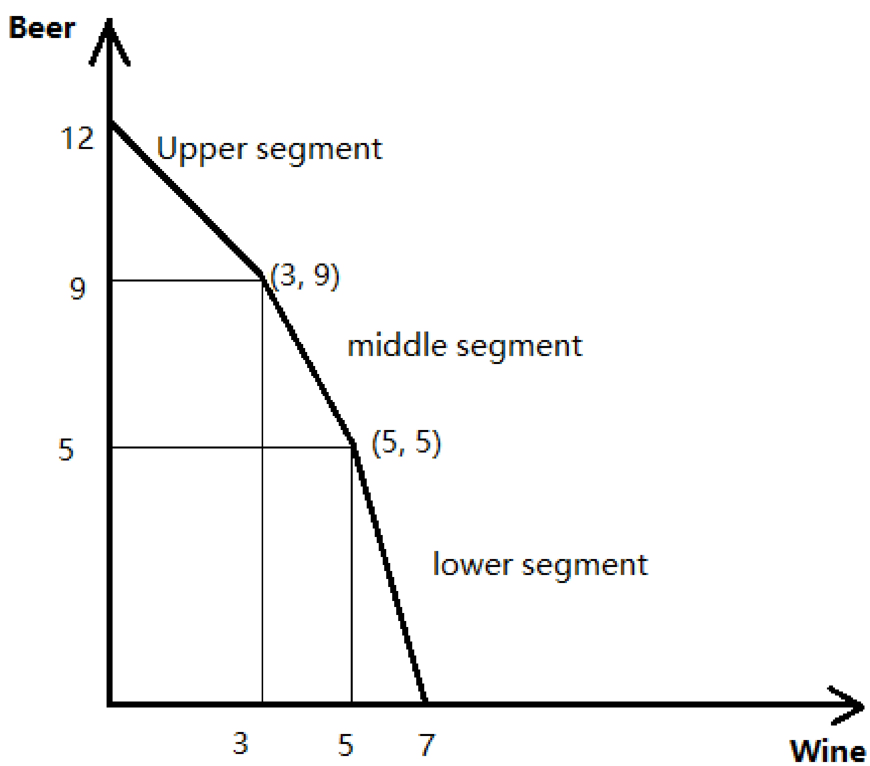
Therefore, comparing to the dwarves, the men have a comparative advantage in producing wine as 2<2.5. Comparing to the elves, the men have a comparative advantage in producing beer as 0.5<1. In other words, the men somehow occupy a middle position between dwarves and elves.

As market demand for beer grows and demand for wine weakens, the dwarves would be the first to switch to producing only beer. After them, the men would also switch out of wine into beer. Finally, if market demand for beer becomes strong enough, even the elves would start producing beer, until it reaches the point where all three nations produce only beer and no wine.

To find the new joint PPF curve, we need the following points on the curve:

1. Horizontal intercept: All three nations produce only wine. Together, they produce 2+3+2 = 7 bottles of wine and no beer. We get point (7, 0).
2. Vertical intercept: All three nations produce only beer. Together, they produce 5+3+4 = 12 barrels of beer and no wine. We get point (0, 12).
3. Kink points:
    a. The point where dwarves produce only beer, while men and elves produce only wine. Beer produced = 5. Wine produced = 2+3 = 5. We get kink point (5, 5).
    b. The point where dwarves and men produce only beer, while elves produce only wine. Beer produced = 5+4 = 9. Wine produced = 3. We get kink point (3, 9).

Connect the four points, we get the new joint PPF curve:



The slope of the upper segment is the elves' opportunity cost of producing wine in terms of beer times -1, which is (-1)*1 = -1. The function form is B = -W+12.

The slope of the middle segment is the men's opportunity cost of producing wine in terms of beer times -1, which is (-1)*2 = -2. The function form is B = -2W+15.

The slope of the lower segment is the dwarves' opportunity cost of producing wine in terms of beer times -1, which is (-1)*2.5 = -2.5. The function form is B = -2.5W+17.5.

g. Find the range of trading price forone barrel of beer in terms of bottles of wine.
The elves are willing to sell wine only if the market price of wine (in terms of beer) is above the opportunity cost of producing wine (in terms of beer). In other words, they sell if the price of wine per bottle is above 1 barrel of beer.

The dwarves are willing to buy wine only when the market price for wine (in terms of beer) is below the dwarves' opportunity cost of producing wine (in terms of beer). In other words, they buy if the price of wine per bottle is below 2.5 barrel of beer.

For men, the opportunity cost of producing one bottle of wine is 2 barrels of beer. If market price of wine exceeds this amount, men are willing to sell wine in exchange for beer. If market price of wine is below this amount, men are willing to buy wine and pay with beer.

The trading range is hence (1 barrel of beer, 2.5 barrels of beer). Within (1 barrel of beer, 2 barrels of beer), elves sell wine, men and dwarves buy wine. Within (2 barrels of beer, 2.5 barrels of beer), elves and men sell wine, while dwarves buy wine.

Use the following graph to visualize the results.