Prelim Question – Summer 2008 – Dan Quint

Consider the classic Cournot model of quantity competition, with asymmetric marginal costs. Two firms $i \in \{1, 2\}$ simultaneously choose production levels $q_i \in \Re^+$; each firm i produces those units at a constant marginal cost of c_i per unit, and sells them at the market price, which is determined by the inverse demand function

$$P = \max \{0, \ \ 100 - q_1 - q_2\}$$

1. Calculate firm i's best-response given a production level q_j of his opponent. (Be sure to account for the cases where it is optimal to set $q_i = 0$.)

2. Let $c_1 = 25$ and $c_2 = 55$. Find the Nash equilibrium in which both players produce, and calculate both firms' profits.

3. If firm 1 produced 45 units, firm 2's best-response would be not to produce at all. Calculate firm 1's profits in this event. Is it higher or lower than your answer to part 2? Is $(q_1, q_2) = (45, 0)$ an equilibrium? Why or why not?

Now let G be any two-player simultaneous-move game, with strategy spaces $A_i = \Re^+$ and payoff functions $u_i : A_i \times A_j \to \Re$ which are continuous and differentiable. Let G_1 be a variation on the game G where player 1 moves first, then player 2 observes 1's action and moves second. (When G is the Cournot game, G_1 is known as the Stackelberg game.)

4. Let $BR_i : A_j \rightrightarrows A_i$ be player i's best-response correspondence for the game G, that is, $BR_i(a_j) = \arg\max_{a_i \in A_i} u_i(a_i, a_j)$. Show that if BR_2 is single-valued ($BR_2(a_1)$ is a singleton for every a_1), then player 1's **payoff** in any subgame-perfect equilibrium of G_1 is at least as high as his payoff in any pure-strategy Nash equilibrium of G.

5. Now suppose BR_1 and BR_2 are both single-valued. Show that if BR_2 is weakly decreasing and u_1 is weakly decreasing in a_2, then player 1's **strategy** in any subgame-perfect equilibrium of G_1 is at least as high as his strategy in any pure-strategy Nash equilibrium of G.

6. Let G be the Cournot game described above, with marginal costs c_1 and c_2, and suppose G has a unique equilibrium in which both firms produce strictly positive quantities. Show that the statements in parts 4 and 5 hold strictly: firm 1 produces strictly more in the Stackelberg game than in the simultaneous-move Cournot game, and earns strictly higher profits.