POOLING WITH ESSENTIAL AND NONESSENTIAL PATENTS

Daniel Quint[1]

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Abstract. Several recent technological standards were accompanied by *patent pools* – arrangements to license relevant intellectual property as a package. A key distinction made by regulators – between patents *essential* to a standard and patents with substitutes – has not been addressed in the theoretical literature. I show that pools of essential patents are always welfare-increasing, while pools which include nonessential patents can be welfare-reducing – even pools limited to complementary patents and stable under compulsory individual licensing. If pools gain commitment power and price as Stackelberg leaders, this reduces, and can reverse, the gains from welfare-increasing pools.

[1]Department of Economics, University of Wisconsin. Correspond to: dquint@ssc.wisc.edu. 7428 Social Sciences, 1180 Observatory Drive, Madison WI 53706. This project was supported by the Bradley Graduate and Post Graduate Fellowship Program, through a grant to the Stanford Institute for Economic Policy Research; and by the John M. Olin Program in Law and Economics at Stanford Law School. The author thanks Tim Bresnahan, Steven Durlauf, Liran Einav, Amit Gandhi, Jon Levin, Paul Milgrom, Ilya Segal, Ricardo Serrano-Padial, Carl Shapiro, and seminar participants at Caltech, Harvard Business School, Maryland, MIT Sloan, NYU Stern, Ohio State, Stanford, UCLA, UCSD, Wisconsin, and Yale SOM for helpful comments.

1 Introduction

When firms with market power sell complementary goods, their combined price will typically be higher than if both were sold by a single monopolist. This effect, first understood by Cournot (1838) and later termed double marginalization, can be particularly severe in the context of intellectual property. In high-tech fields where innovation is rapid and cumulative, a large number of patents may touch on the same new technology; double marginalization can make the technology expensive to commercialize, harming downstream producers and consumers as well as the innovators the patent system was designed to reward.

One tool to address this problem is a *patent pool* – an agreement by multiple patentholders to share a group of patents among themselves or to license them as a package to third parties. Patent pools in the U.S. go back to the 1850s, the first one involving sewing machine patents. Lerner and Tirole (2007) claim that "in the early days of the twentieth century . . . many (if not most) important manufacturing industries had a patent pooling arrangement." In 1917, with airplanes needed for World War I, Franklin Roosevelt (then Assistant Secretary of the Navy) pushed U.S. aircraft manufacturers into a patent pool because ongoing litigation between the Wright Company and Curtiss Company had choked off aircraft production. In the last decade, several patent pools have formed in conjunction with technological standards, beginning with the MPEG-2 video and DVD standards in the late 1990s;[2] Lerner and Tirole cite one estimate that "sales in 2001 of devices based in whole or in part on pooled

[2]In addition to the pioneering MPEG-2 pool and two pools related to DVD-ROM and DVD-Video, recent standard-based pools have included Firewire, Bluetooth, RFID, AVC, DVB-T, and MPEG-4.

patents were at least $100 billion."

When patents in a pool are complements, the pool can lower their combined price and increase licensing revenues – as well as reduce transaction costs (by reducing the number of individual licensing agreements required to make use of the technology) and the risk of holdup by the final patentholders. However, patent pools have also been used to eliminate competition between rival technologies, facilitate collusion, and even administer cartels. In the early 1900s, a pool administered by National Harrow specified what products its licensees could produce and fixed the prices in the downstream market.[3] The Hartford-Empire pool dominated glassware manufacturing in the 1940s and used licensing terms to set production quotas and discourage entry into the market. In the early 1990s, Summit and VISX, which by 1996 were the only two companies with FDA-approved technologies for laser eye surgery, formed a patent pool and set a standard licensing fee of $250 for each use of either firm's technology; according to the FTC, "Instead of competing with each other, the firms placed their competing patents in the patent pool in order to share the proceeds each and every time a Summit or VISX laser was used."

Antitrust treatment of patent pools has evolved significantly over time. In 1902, the Supreme Court upheld the National Harrow pool, ruling that patent law trumped the Sherman Act:

> The general rule is absolute freedom in the use or sale of rights under the patent laws of the United States. The very object of these laws is monopoly . . . . The fact that the conditions in the contracts keep up the monopoly or fix prices does not render them

[3]The pool accounted for 90% of the U.S. market for float spring tooth harrows, agricultural devices used to cultivate surface soil before planting.

illegal. *E. Bement & Sons v. National Harrow*, 186 U.S. 70 (1902)

Within ten years, the court had backtracked and begun to examine overly restrictive licensing terms. The Hartford-Empire pool was ruled to be an antitrust violation because it restricted downstream production; the Supreme Court ruled that the pool itself was legal and Hartford-Empire was free to charge whatever royalty rate it chose, but could not restrict its licensees further. (By the time the appeal was ruled on, Hartford had already changed its licensing terms so that "every licensee shall be at liberty to use the machinery for the manufacture of any kind or quantity of glassware comprehended within the decree" (*Hartford-Empire Co. et al. v. United States*, 323 U.S. 386 (1945)). Several other patent pools were found to be antitrust violations in the 1940s and early 50s, and very few new pools formed from the mid-1950s until the 1990s. In 1998, the Summit and VISX pool was dissolved following a settlement with the FTC. The 1995 FTC/DOJ Guidelines for the Licensing of Intellectual Property explicitly recognized the procompetitive possibilities of patent pools, and a number of pools related to technological standards appeared thereafter, beginning with MPEG-2 in 1997.

When evaluating the recent standard-based pools, regulators have drawn a key distinction between patents which are *essential* to comply with the standard and patents for which suitable substitutes exist. The recent pools have all been limited to essential patents, and provide for independent experts to determine which patents should be included on this basis; this is seen as a "competitive safeguard" to ensure that the pool does not have anticompetitive effects.[4]

[4]In business letters issued by the Department of Justice in 1998 and 1999 in response to the proposed DVD pools, Joel Klein writes, "One way to ensure that the proposed pool will integrate only complementary patent rights is to limit the pool to patents that are essential

As I discuss in the next section, the existing literature on patent pools fails to distinguish between essential and nonessential patents, treating only the polar cases where every patent is essential (users need licenses to all of the patents) or no patent is essential (any set of patents of a particular size is sufficient). The aim of this paper is to understand the effects of a patent pool in an environment in which essential and nonessential patents co-exist; given the emphasis placed by regulators on distinguishing essential from nonessential, this seems to be an empirically important case. I find the following:

- Patent pools containing **only essential patents** lead to lower prices for every product using the technology; greater consumer surplus in the downstream market; and higher licensing revenue for all patentholders outside the pool. Such pools are always welfare-enhancing. As in earlier models, however, such pools may be inefficiently small when individual patentholders can opt out without disrupting pool formation.

- Patent pools containing patents required only for one of several competing products, or **nonessential patents which are perfect complements**, will reduce the price of one product, but can increase or decrease the prices of others. Such pools increase total consumer surplus, but may harm some individual consumers; they increase revenues of outsiders whose patents are complementary to the pool, but decrease revenues to others. The overall welfare effect can be either positive or negative. This is in contrast to previous papers, which found that complementarity of the patents within the pool was sufficient for a pool to be welfare-enhancing. (The difference is that my model allows for the

to compliance with the Standard Specifications. Essential patents by definition have no substitutes; one needs licenses to each of them in order to comply with the standard."

patents in a pool to be complements to each other, yet have substitutes outside the pool; in earlier models, this was impossible.)

- As in earlier models, pools containing patents which are **substitutes** will tend to reduce consumer surplus and overall welfare.

- In contrast to previous models, robustness to **compulsory individual licensing** (the forced availability of pooled patents individually as well as through the pool) is *not* a sufficient screen for efficiency: pools of complementary nonessential patents are robust to individual licensing, and may be welfare-negative.

- To the extent that a pool is less flexible than individual patentholders and therefore acts as a **Stackelberg-style leader**, the welfare gains are reduced, and may even be reversed.

This suggests that, from a policy point of view, pools of essential patents should generally be allowed, and encouraged to be as inclusive as possible; pools which include complementary nonessential patents should be considered more cautiously, though not necessarily ruled out; and terms of a pooling arrangement which make the pool less flexible with respect to pricing (terms which "bind the pool's hands") should be eyed suspiciously, since (*provided the same pool would have formed without these terms*) they lead to higher prices and lower welfare.

The rest of this paper proceeds as follows. Section 2 discusses existing literature on patent pools. Section 3 introduces a model of price competition in a differentiated-products setting and characterizes its equilibrium. Section 4 presents the results following from this model. Section 5 concludes with a discussion of the limitations of the model and avenues for future work. All claims in the text (including those stated informally) are proved in the appendix.

2 Related Literature

A few recent papers have presented theoretical models of patent pools. Gilbert (2004) (along with a detailed history of antitrust litigation) and Shapiro (2001) give simple models of competition with perfect substitutes and perfect complements, emphasizing the double-marginalization problem in the latter case. Lerner and Tirole (2004) model a world with n identical patents which need not be perfect substitutes nor perfect complements; they show that a pool containing all the patents is more likely welfare-enhancing when patents are more complementary; and that forcing pool participants to also offer their patents individually (compulsory individual licensing) destabilizes "bad" pools without affecting "good" ones. Brenner (2009) extends the Lerner and Tirole framework to consider smaller ("incomplete") pools containing only some of the patents; he explicitly models the fact that some patentholders may do better by remaining outside of the pool, and examines which pools will form under different formation procedures. Brenner compares the welfare achieved under a particular formation protocol to the outcome without a pool, and shows that compulsory individual licensing is a good screen for efficiency under this formation rule. Aoki and Nagaoka (2005) use a coalition-formation model (based on Maskin's (2003) framework of coalition games with externalities) to show that even when the patents are all essential and the grand coalition is therefore desireable, it will not form when the number of patents is large. Kim (2004) shows that in the presence of a patent pool, vertical integration – the presence of patentholding firms in the downstream market – always lowers the price of the final product. Dequiedt and Versaevel (2008) show that allowing pool formation increases firms' R&D investments prior to the pool being formed.

In all of these models, patents are assumed to be interchangeable; that is, users derive value based on the number of patents they license, not which ones. This means that either all or none of the patents are essential. Under this assumption, the authors generally find that as long as the patents are complements, pools are socially desirable. (One exception is Choi (2003), who observes that pooling of weak patents (patents which are unlikely to be upheld in court) may reduce the incentive for producers to challenge them, leading to a loss in welfare even though the patents may be complements.) My model differs from these in allowing a given patent to have some complements and some substitutes.

In addition to this theoretical work, there have been several recent empirical examinations of patents pools. Lerner, Strojwas and Tirole (2005) examine the licensing rules of various patent pools, finding that pools of complementary patents are more likely to allow independent licensing and require grantbacks, consistent with their theory. Layne-Farrer and Lerner (2011) examine the rules by which revenues are allocated and the effect this has on participation. Lampe and Moser (2010, 2011) find that patent pools appear to suppress further patenting and innovation related to the pool technology.

The discrete-choice logit model I use for consumer demand is similar to the one considered in Anderson, de Palma and Thisse (1992) and Nevo (2000), among many others. These papers situate the various products in a multi-dimensional space of product characteristics, assuming that consumers have differentiated (generally linear) preferences over these characteristics; I focus on the simpler case where each product is its own dimension, that is, consumer preferences are over the products themselves. (Also see Berry and Pakes (1993) for a discussion of the use of these and other techniques in merger analysis.)

Also related is the problem of tying and bundling of consumer goods – see Kobayashi (2005) for a recent survey of the bundling literature, and Chen and Nalebuff (2006) for a recent contribution. The key difference between my model and all of these is that here, products are overlapping sets of different components, which may be produced (and priced) by different firms.

3 Model

In this section, I introduce my model. Patentholders license their patents to downstream manufacturers, who sell products based on the patented technologies to consumers. The products are substitutes, but consumers have differentiated tastes for them, so competition between patentholders is imperfect; I focus on this upstream competition by assuming the downstream sector is perfectly competitive. (Under perfect competition, the results are the same as if patentholders licensed their patents directly to end users, as in the previous literature.) The model is analogous to a model of two-level production: patentholders correspond to upstream (wholesale) suppliers of product components, which are assembled into products and sold by downstream retailers. (The results below are for zero marginal costs, but extend without difficulty to cases where the wholesale suppliers face constant marginal costs.)

3.1 Patents, Products, Patentholders

I begin with a finite set $\mathcal{K} = \{1, 2, \ldots, K\}$ of distinct **products**, each covered by a set T_k of required **patents**. The set of patents required for different products may overlap, although below, I place restrictions on this overlap.

The patents are owned by a set of **patentholders** $\mathcal{J} = \{1, 2, \ldots, J\}$ who face no costs and whose only revenue is from licensing. A single patentholder may own multiple patents. Let S_j denote the set of patents owned by patentholder j; then $\{S_j\}_{j \in \mathcal{J}}$ forms a partition of the set of all patents $\bigcup_{k \in \mathcal{K}} T_k$.

Patentholders may price individual patents separately, or may offer a menu of prices for different subsets of their portfolio. Since a product can only be manufactured if all its intellectual property is licensed, all that will matter is the lowest price at which each patentholder j makes available all its patents covering a given product k; I will refer to this price as p_k^j. Without loss of generality, then, assume each patentholder j names prices $\left(p_1^j, p_2^j, \ldots, p_K^j\right)$, where $p_k^j = 0$ if $S_j \cap T_k = \emptyset$ (patentholder j has no patents blocking product k). The patentholders name prices simultaneously, seeking to maximize their total revenue

$$\pi_j(\cdot) = \sum_{k \in \mathcal{K}} p_k^j Q_k \tag{1}$$

where Q_k is the demand for product k.

3.2 Manufacturers

Manufacturers (or retailers) license patents from patentholders and sell the products $k \in \mathcal{K}$ to downstream consumers. I assume that manufacturers are perfectly competitive, with no fixed costs and constant marginal costs for each product which are the same across manufacturers (but may vary across the K products). Under perfect competition, manufacturers will charge consumers the patentholders' licensing fees plus their own marginal costs; for ease of exposition, I will assume that consumer utility terms (described below) are net of these marginal costs, so that the retail price of each product k is

$$P_k \equiv \sum_{j \in \mathcal{J}} p_k^j \tag{2}$$

3.3 Consumers

Demand for the products comes from a measure 1 of consumers $l \in L$, who have idiosyncratic preferences for the various products. If consumer l buys product k at price P_k, her utility is

$$v_k^l - P_k \tag{3}$$

Each consumer can buy at most one product; if consumer l declines to buy any of the products, her utility is v_0^l.

While many of the comparative statics we will show hold for more general demand systems (see Quint (2011) for a discussion), we will make a strong assumption here about the demand system in order to give sharper results.

Assumption 1. **[Logit Demand]** *For $k \in \mathcal{K} \cup \{0\}$, $v_k^l = v_k + \epsilon_k^l$, where $v_0 = 0$, $(v_1, \ldots, v_K)$ are constants, and ϵ_k^l are i.i.d. draws (across k and l) from the standard double-exponential distribution.*[5]

Under Assumption 1, a fraction

$$Q_k = \frac{e^{v_k - P_k}}{1 + \sum_{k' \in \mathcal{K}} e^{v_{k'} - P_{k'}}} \tag{4}$$

of consumers demand product k, and price elasticities have simple closed-form expressions.

3.4 Equilibrium

Assumption 1 gives us enough structure to talk about the equilibrium of the patentholders' pricing game. Retailers and consumers are nonstrategic, so

[5] $F(x) = \exp(-\exp(-(x+\gamma)))$, where γ is Euler's constant. See McFadden (1974) or Anderson, de Palma and Thisse (1992) for more on logit demand.

the licensing fees set by patentholders constitute the equilibrium, although consumer surplus will be included in welfare calculations. Under logit demand, patentholders will choose to set a single price for their entire patent portfolio, rather than use the different patents to price-discriminate. Thus, without loss, we can consider the pricing game as a game with single-dimensional strategy spaces: the players are the patentholders $j \in \mathcal{J}$; player j's action space is $\Re_+$; and player j's payoff is $p^j \sum_{k:S_j \cap T_k \neq \emptyset} Q_k$.

Lemma 1. *Under Assumption 1, a unique equilibrium exists, and is characterized by the first-order conditions.*

See Appendix A.1 for proof.

3.5 Essential and Nonessential Patents

In order to demonstrate clear comparative statics, I will make a further assumption on the types of patents in the universe and the way they are partitioned among patentholders. This assumption leads to a natural definition for an essential patent, and a sharp distinction between essential and nonessential patents.

Assumption 2. *Define* $T^E \equiv T_1 \cap T_2 \cup \ldots \cap T_K$.

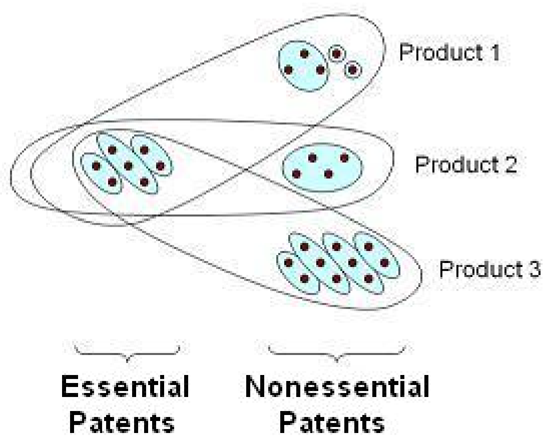
1. *For any two products* $k' \neq k$, $(T_k - T^E) \cap (T_{k'} - T^E) = \emptyset$.

2. *For any firm* $j \in \mathcal{J}$, *either* $S_j \subset T^E$ *or* $S_j \subset T_k - T^E$ *for some* k.

Assumption 2 is fairly strong, but buys us a lot. It will lead to a supermodular structure on the pricing game, giving a number of sharp comparative statics. It also leads to a natural distinction between essential and nonessential patents. The patents in T^E are required for every product, and therefore have no substitutes; I will therefore refer to these as **essential**. Define $T_k^N \equiv T_k - T^E$; the patents in T_k^N, while required for product k, have substitutes (the patents in $T_{k'}^N$, $k' \neq k$), and are therefore **nonessential**.

Figure 1 below shows a set of products, patents, and patentholders satisfying Assumption 2. The brown dots represent patents; the light-blue ovals patentholders; and the white ovals products.

Figure 1: Products, Patentholders, and Patents Satisfying Assumption 2



One can think of the different products as competing applications or implementations of a common core technology – different cell phones that work on a particular network, for example. Essential patents are patents on the core technology (or the technological standard the products are built to comply with); nonessential patents might be tied to other aspects of product design (a touch-screen interface for a cell phone).[6]

[6]One recent example that fits Assumption 2 is the intellectual property related to third-generation, or 3G, mobile telephony, which was designed to use five different radio interfaces, each backward-compatible with one second-generation network. Some patents are tied to one of these interfaces, while others relate to the 3G network as a whole.

Under Assumption 2, all the patents owned by a single patentholder are perfect complements; so demand for each product (and therefore payoffs) depends only on the price demanded for all the patents together, not the individual price to license each one. Thus, we will let p^j denote the price demanded by patentholder j for his entire portfolio; when we refer to equilibrium being unique, we will mean, unique up to changes in individual patent prices that preserve p^j.

Define the following additional notation:

- $\mathcal{J}^E \equiv \{j \in \mathcal{J} \ : \ S_j \subset T^E\}$, $n_E \equiv \left|\mathcal{J}^E\right|$, and $P^E \equiv \sum_{j \in \mathcal{J}^E} p^j$ the set, number, and combined price of all the patentholders with essential patents

- $\mathcal{J}_k^N \equiv \{j \in \mathcal{J} \ : \ S_j \subset T_k^N\}$, $n_k \equiv \left|\mathcal{J}_k^N\right|$, and $P_k^N \equiv \sum_{j \in \mathcal{J}_k^N} p^j$ the set, number, and combined price of all the patentholders with nonessential patents covering a particular product $k \in \mathcal{K}$

The price of product k can be written as $P_k = P^E + P_k^N$.

Lemma 2. *Under Assumptions 1 and 2, equilibrium values of aggregate prices are the unique solution to the system of equations*

$$\begin{aligned} P_1^N(1-Q_1) &= n_1 \\ P_2^N(1-Q_2) &= n_2 \\ &\vdots \\ P_K^N(1-Q_K) &= n_K \\ P^E(1-Q_1-Q_2-\ldots-Q_K) &= n_E \end{aligned} \tag{5}$$

and individual patentholder prices are $p^j = \frac{1}{n_k}P_k^N$ *for* $j \in \mathcal{J}_k^N$ *and* $p^j = \frac{1}{n_E}P^E$ *for* $j \in \mathcal{J}^E$.

See Appendix A.1 for proof. While $(n_E, n_1, \ldots, n_K)$ are integers, the solutions to Equation 5 vary continuously with these parameters; by differentiating these equations with respect to each of these parameters, we can solve explicitly for the marginal effect of n_E or n_k on equilibrium prices, allowing us to understand the effect of pools on equilibrium outcomes in more detail.

4 Results

For simplicity, I assume that when a firm joins a pool, all of its patents are included, and that pools offer their entire portfolio at a single price. I will use $\mathbf{v}$ to denote the vector $(v_1, v_2, \ldots, v_K)$ of average surplus generated by each product, and $\mathbf{n}$ to denote the vector $(n_E, n_1, \ldots, n_K)$. A pool of n essential patentholders behaves as if it were a single essential patentholder, so its formation has the same effect on equilibrium values of aggregate prices as a reduction in n_E by $n - 1$. Similarly, a pool of n patentholders in $\mathcal{J}_k^N$ has the same effect on aggregate prices as a reduction in n_k by $n - 1$. Equation 5 allows us to understand the effect of $\mathbf{n}$ on equilibrium outcomes, leading to the following results.

(Note that Theorems 1, 2, 5, and 6 below – the price and welfare effects of pools of complements, and the effects of compulsory individual licensing and Stackelberg pools – rely mainly on the signs of the marginal effects of n_E and n_k on equilibrium outcomes. They therefore extend almost completely to more general demand systems satisfying certain regularity conditions – see Quint (2011) for a discussion.)

4.1 Welfare Effects of Pools of Complements

Previous work has suggested that pools are generally welfare-positive when the patents being pooled are sufficiently strong complements. In my setting, this is not always the case even when patents within the pool are *perfect* complements: pools of essential patents are always welfare-positive, but pools of complementary nonessential patents can be welfare-negative. The results are mostly proved through brute-force calculation specific to the logit demand system, but intuition for what drives the results can be gained from Quint (2011).

Theorem 1. *A pool containing only essential patents will:*

1. *Lower the price P_k of every product and increase the surplus to each consumer*
2. *Increase the demand Q_k for every product*
3. *Increase the profits of every patentholder (essential and nonessential) outside the pool*
4. *Increase total welfare*

If the pool is profitable for its members, it therefore represents a Pareto-improvement.

Later on, I address the question of when such a pool is likely to be profitable to its members. While a pool of essential patents has only positive externalities and is always welfare-increasing, a pool of nonessential complements has some positive and some negative externalities, and may or may not increase total welfare:

Theorem 2. *Choose a product k, and consider a pool of patentholders in $\mathcal{J}_k^N$. For each $k' \neq k$, such a pool will:*

1. *Decrease P_k^N and $P_{k'}^N$ and increase P^E*
2. *Increase Q_k and decrease $Q_{k'}$*
3. *Decrease P_k and have an ambiguous effect on $P_{k'}$*
4. *Increase total consumer surplus, although some individual consumers may be made worse off*
5. *Increase profits of essential patentholders and outsiders in $\mathcal{J}_k^N$ and decrease profits of patentholders in $\mathcal{J}_{k'}^N$*
6. *Have an ambiguous effect on total welfare*

To clarify the final point, if we treat n_k as a continuous variable (affecting prices continuously through equation 5), then the welfare effect of a pool consisting of m of the n patentholders in $\mathcal{J}_k^N$ can then be written as

$$\int_{n-m+1}^{n} \left[\sum_{k' \in \mathcal{K} - \{k\}} w_{k'} \left(P_k - P_{k'} \right) + w_0 P_k \right] dn_k \tag{6}$$

where $w_{k'}$ and w_0 are positive weights which vary with n_k. Thus, when product k is expensive relative to its alternatives, a pool of patents blocking only product k, while not Pareto-improving, will be welfare-positive overall.[7] When k is relatively cheap, such a pool can be either welfare-positive or welfare-negative, since either term in the integrand can dominate:

[7] Since the dominant effect of such a pool is a reduction in the price P_k, this can be seen as an analog to the "concertina theorem" in international trade (originally posited by Meade (1955)): that in the absence of complementarities, when different goods face different tariffs, reducing the highest tariff is unambiguously welfare-positive. See Bertrand and Vanek (1971) for a discussion.

Example 1. *Let $K = 3$, $\mathbf{v} = (10, 10, 5)$, $n_1 = n_2 = n_3 = 3$, and $n_E = 1$. The following table gives equilibrium prices, market shares, patentholder profits, consumer surplus, and total welfare before and after the formation of a pool of the three patentholders in $\mathcal{J}_3^N$. (Π_E refers to the combined profits of all essential patentholders, Π_k to the combined profits of all patentholders in $\mathcal{J}_k^N$.)*

n	P^E	P_1^N	P_2^N	P_3^N	Q_1	Q_2	Q_3	Π_E	Π_1	Π_2	Π_3	CS	W
(1,3,3,3)	4.578	4.861	4.861	3.048	.383	.383	.016	3.578	1.861	1.861	**0.048**	1.521	**8.869**
(1,3,3,1)	4.818	4.647	4.647	1.091	.354	.354	.084	3.818	1.647	1.647	**0.091**	1.572	**8.775**

The pool significantly reduces P_3, increases total consumer surplus, and nearly doubles the profit of its members; but it increases P_1 and P_2, and is overall welfare-negative. On the other hand, if $n_E = 3$, the same pool would be welfare-positive:

n	P^E	P_1^N	P_2^N	P_3^N	Q_1	Q_2	Q_3	Π_E	Π_1	Π_2	Π_3	CS	W
(3,3,3,3)	6.472	4.085	4.085	3.016	.266	.266	.005	3.472	1.085	1.085	0.016	0.769	**6.426**
(3,3,3,1)	6.562	4.024	4.024	1.035	.254	.254	.034	3.562	1.024	1.024	0.035	0.783	**6.428**

The addition of patentholders in $\mathcal{J}_k^N$ into an existing pool of essential patents will have the same effects as in Theorem 2. The formation of a pool containing patentholders in both $\mathcal{J}^E$ and $\mathcal{J}_k^N$ will be a hybrid of Theorems 1 and 2, and will result in a decrease in P_k, an increase in Q_k, an increase in total consumer surplus, and an increase in profits to outsiders in $\mathcal{J}^E$ and $\mathcal{J}_k^N$; the other effects will be ambiguous.

4.2 Profitability of Pools of Complements

All the results in this section are stated formally, with proofs, in Appendix A.4.

Not surprisingly, when n_E is large, the double-marginalization problem among essential patentholders is severe, and a sufficiently large pool of essential patentholders will always be profitable. More surprisingly, when n_E is sufficiently large, even small pools are profitable. However, as in many merger situations,[8] much of the benefit of a pool goes not to the insiders but the outsiders, that is, the essential patentholders who remain outside the pool. This creates a freerider problem which may prevent pools from reaching their optimal size.

Aoki and Nagaoka (2005) find that when all patents are essential, the grand coalition (which is both profit-maximizing and efficient) will only occur when the number of patentholders is small; for large numbers of patentholders, "the emergence of [an] outsider is inevitable, so that. . . voluntary negotiation cannot secure the socially efficient outcome." Brenner (2009) proposes an "exclusionary" formation rule, under which a patent pool proposed by one player must be unanimously accepted by its members or fail to form; this allows the grand coalition to be achieved when it is efficient but could not be reached by sequential negotiations. In both these papers, the grand coalition maximizes total patentholder profits but still may fail to form. In my setting, the problem is more severe, as a pool of all essential patents, while more efficient than a smaller pool, does not necessarily maximize the joint profits of the essential patentholders. This is due to strategic effects: as the pool grows and P^E falls, nonessential patentholders raise their prices in response.

[8]See, for example, Deneckere and Davidson (1985) for a discussion.

Example 2. *Let $K = 3$, $\mathbf{v} = (10, 10, 5)$, $n_1 = n_2 = n_3 = 3$, and $n_E = 6$. The following table gives equilibrium outcomes given no pool, a pool of five essential patentholders, and a pool of all six:*

n	P^E	P_1^N	P_2^N	P_3^N	Q_1	Q_2	Q_3	Π_E	Π_1	Π_2	Π_3	CS	W
(6,3,3,3)	8.224	3.466	3.466	3.004	.135	.135	.001	**2.224**	0.466	0.466	0.004	0.315	3.477
(2,3,3,3)	5.685	4.410	4.410	3.026	.320	.320	.009	**3.685**	1.410	1.410	0.026	1.045	7.577
(1,3,3,3)	4.578	4.861	4.861	3.048	.383	.383	.016	**3.578**	1.861	1.861	0.048	1.521	8.869

A pool of all six essential patentholders increases their combined profits by 66%. However, once five of them have formed a pool, the addition of the sixth **reduces** *their combined profits. (A pool of five would increase the combined profits of all six essential patentholders, but not the combined profits of the five participants, which would fall from $\frac{5}{6}(2.224) = 1.853$ to $\frac{1}{2}(3.685) = 1.843$.)*

Thus, as in previous models, pools of essential patents are welfare-positive, but may fail to achieve their efficient size (or form at all) if individual patentholders can opt out without destroying the pool.

Like with essential patents, when n_k is sufficiently large, pools of nonessential patents blocking product k are profitable. Interestingly, when there are only two competing products, welfare-negative pools of nonessential complements (as in Example 1) are always unprofitable:

Theorem 3. *When $K = 2$, a pool of nonessential patents blocking the same product is welfare-positive if it is profitable for its members.*

The converse does not hold, and, as Example 1 showed, the result does not hold for $K = 3$.

When a pool already contains all of the essential patents, and is able to add all the nonessential patents blocking a single product, this will always increase the combined profits of the pool and the patentholders being added. However, when the pool does not contain every essential patent, or when only a single patent is added, the combined profit of the pool and the patentholder being added can rise or fall.

The effect of one pool on the profitability of another is unpredictable:

Example 3. *Let $K = 3$, $\mathbf{v} = (10, 10, 5)$, and $(n_E, n_1, n_2, n_3) = (3, 5, 5, 3)$. A pool of all three essential patentholders is profitable, but following a pool of the five patentholders in $\mathcal{J}_1^N$, it is not.*

On the other hand, when $(n_E, n_1, n_2, n_3) = (3, 5, 3, 5)$, the result is reversed: a pool of all three essential patentholders is unprofitable, but following a pool of the five patentholders in $\mathcal{J}_1^N$, it is profitable. (Equilibrium prices and payoffs for this example are in the appendix.)

4.3 Pools Containing Substitutes

Earlier papers have shown that when pooled patents are not available to be licensed separately, pools of substitute patents lead to higher prices and lower welfare. I give two examples along the same lines in the current setting.

Theorem 4.

1. *A pool consisting of two patentholders with nonessential patents blocking different products always decreases total consumer surplus.*
2. *A pool consisting of one nonessential patent blocking each product always decreases total consumer surplus and total welfare.*

Pools of substitute patents will, however, tend to be profitable.

A pool containing several nonessential patents blocking each product might well be welfare-enhancing overall. However, one could decompose the formation of such a pool into two steps: the formation of K smaller pools (each containing nonessential patents blocking a single product), and the merging of these pools into a single larger one. Theorem 4 states that while the net effect may be welfare-positive, the second step – combining competing pools into a single one – would be inefficient, leaving total welfare lower than if separate pools were maintained. In fact, this is effectively what was done in the 3G case. The general structure was originally intended to be a single "patent platform" with a single body determining standard (a la carte) licensing fees for all relevant patents; antitrust concerns led instead to the establishment of five separate Licensing Administrators to oversee selection of patents and licensing fees separately for each of the five radio interfaces making up the 3G standard.

4.4 Compulsory Individual Licensing

Lerner and Tirole (2004) propose compulsory individual licensing – mandating that participants in a patent pool also offer their patents individually – as a solution to welfare-negative pools. Define a pool to be *stable* to individual licensing if equilibrium prices are the same with and without individual licensing, and *weakly unstable* to individual licensing if in at least some equilibria, after the pool names its profit-maximizing price, individual licensing causes prices to revert to their pre-pool levels. Lerner and Tirole show that in their setting, welfare-positive pools are stable to compulsory individual licensing, while welfare-negative pools are weakly unstable. Thus, they claim that compulsory individual licensing functions as a screen for efficient pools.

In Brenner's (2009) setting (an extension of Lerner and Tirole's model where pools smaller than the grand coalition are considered), all welfare-positive pools are stable to individual licensing, but some welfare-negative pools may be as well. However, pools which form in equilibrium under an "exclusive" formation rule – where participants must agree unanimously to a proposed pool or it does not form, rather than patentholders being able to opt in or out individually – are stable to individual licensing if and only if they are welfare-enhancing.

In my setting, compulsory individual licensing does not always distinguish welfare-positive from welfare-negative pools. In particular:

Theorem 5. *Any pool containing only essential patents is stable to individual licensing.*

Any pool containing only complementary nonessential patents is stable to individual licensing.

Both these types of pools are unaffected by compulsory individual licensing; but Theorem 2 established that pools of complementary nonessential patents can be welfare-negative. Thus, compulsory individual licensing is not a sufficient screen against socially inefficient pools.

Like in Lerner and Tirole, a pool containing substitute patents in my setting would be unstable to individual licensing. Consider a pool containing a single nonessential patent blocking each product. Each patentholder in the pool has an incentive to slightly undercut the pool's price, to capture the entire revenue from manufacturers building his product rather than sharing it with the rest of the pool; prices would collapse to those set in the absence of the pool. Thus, compulsory individual licensing in my setting is disruptive to some welfare-negative pools, but is not a perfect screen for efficiency.

4.5 Pools as Stackelberg Leaders

Since a patent pool involves joint action by multiple firms, a pool might be less able to easily change the licensing fees it demands than an individual patentholder would be. This would give a pool commitment power, allowing it to act like a Stackelberg leader, expecting the outside patentholders to best-respond. Such commitment power would make any pool weakly profitable (since at worst it could commit to its pre-pool prices), and most pools strictly profitable; but it would also typically result in the pool demanding a higher price than the static model suggests, reducing and sometimes reversing the welfare results.

Theorem 6. *Define a* **Stackelberg pool** *as a patent pool which commits to prices before individual patentholders (and cannot adjust them afterwards). Fix all primitives but n_E, and fix the number n_O of essential patentholders who remain outside the pool. Then*

1. *A Stackelberg pool of all but n_O of the essential patentholders is always more profitable than a regular pool of the same patentholders, but the latter is always more efficient*

2. *There exists an integer $n^* \geq n_O + 2$ such that*

    - *If $n_E \geq n^*$, a Stackelberg pool of all but n_O of the essential patentholders is a Pareto-improvement over no pool*

    - *If $n_E < n^*$, a Stackelberg pool of all but n_O of the essential patentholders is less efficient than no pool, raises all prices P_k, and reduces the payoffs to everyone (consumers and patentholders) outside the pool*

The following example illustrates Theorem 6 when $n_O = 0$ and $n^* = 3$, so that a Stackelberg pool of all the essential patents represents a welfare gain when $n_E \geq 3$ but a welfare loss when $n_E = 2$:

Example 4. *Let $K = 3$, $\mathbf{v} = (10, 10, 5)$, $n_1 = n_2 = 6$, and $n_3 = 9$. The following table gives equilibrium outcomes with no pool for various values of n_E, as well as equilibrium outcomes given a Stackelberg pool and a regular pool containing all the essential patents:*

	P^E	P_1^N	P_2^N	P_3^N	Q_1	Q_2	Q_3	Π_E	Π_1	Π_2	Π_3	CS	W
No Pool, n_E=4	5.165	6.762	6.762	9.001	.113	.113	.000	1.165	0.762	0.762	0.001	0.256	**2.946**
No Pool, n_E=3	4.367	7.113	7.113	9.001	.156	.156	.000	1.367	1.113	1.113	0.001	0.375	**3.969**
No Pool, n_E=2	3.438	7.585	7.585	9.003	.209	.209	.000	1.438	1.585	1.585	0.003	0.542	**5.154**
Stackelberg Pool	3.569	7.515	7.515	9.003	.202	.202	.000	1.440	1.515	1.515	0.003	0.517	**4.990**
Regular Pool	2.223	8.271	8.271	9.008	.275	.275	.001	1.223	2.271	2.271	0.008	0.799	**6.572**

The analogous result holds for pools of nonessential complements as well. Choosing k and fixing everything but n_k, there exists an n^* such that when $n_k \geq n^*$, the effects of a Stackelberg pool containing all but n_O patentholders in $\mathcal{J}_k^N$ are described by Theorem 2; when $n_k < n^*$, the effects are the opposite of those in Theorem 2; in either case, the pool is profitable.

5 Conclusion

I have introduced a differentiated-products model for price competition in settings where each product is a combination of components priced by separate suppliers. I've applied the model to understand the static effects of patent pools on pricing. The same framework could also be used to model competition among certain “aggregate” goods such as personal computers, which are made up of components built by different wholesalers, and to understand the effect of mergers between such wholesale suppliers. (For simplicity, I’ve presented a model in which components (patents) come with no marginal cost, but the results extend easily to the case where each component is built at a constant marginal cost.)

As it pertains to patent pools, the model does have certain limitations. I ignore uncertainty about patent scope and enforceability, treating each patent as unassailable and having bright-line boundaries. Firms that own nonessential patents blocking different products do not fit within my framework. If the downstream manufacturing sector were not assumed to be perfectly competitive, we would need to consider the separate incentives of vertically-integrated firms and patentholders who do not participate in downstream production.

In this paper, I do not attempt to explicitly model pool formation or predict which pool or pools will form in a given setting. My model defines a mapping from coalition structures (a particular pool or set of pools) to payoffs (patentholder profit and consumer surplus); any cooperative or noncooperative model of coalition formation could therefore be applied, taking these payoffs as given, to attempt to answer this question.

Like most of the existing literature on patent pools, this paper treats both the technology and the set of patents as exogenous; that is, it abstracts away from the question of innovation (and with it the endogeneity of which patents are essential). Again, this paper can be thought of as specifying the final-stage payoffs, this time of a dynamic, multi-firm patent or R&D race; a formal model of this race, taking the “endgame” profits as given, would allow examination of the effects of pools on innovation.

APPENDIX – Proofs

A.1 Characterizing Equilibrium

Patentholders With Multiple Patents Don't Price-Discriminate

One of the properties of logit demand is that

$$\frac{\partial Q_k}{\partial P_{k'}} = \begin{cases} Q_k Q_{k'} & if \quad k' \neq k \\ -Q_k(1-Q_k) & if \quad k' = k \end{cases}$$

Consider a patentholder j with patents covering some subset $\mathcal{K}_j \subset \mathcal{K}$ of the products. He names prices $\left(p_k^j\right)_{k \in \mathcal{K}_j}$ to solve

$$\max \sum_{k \in \mathcal{K}_j} p_k^j Q_k$$

Differentiating with respect to p_k^j gives first-order condition

$$\begin{aligned} 0 &= Q_k - p_k^j Q_k(1-Q_k) + \textstyle\sum_{k' \in \mathcal{K}_j - \{k\}} p_k^j Q_k Q_{k'} \\ 0 &= Q_k - p_k^j Q_k + p_k^j Q_k \textstyle\sum_{k' \in \mathcal{K}_j} Q_{k'} \\ 1 &= p_k^j - p_k^j \textstyle\sum_{k' \in \mathcal{K}_j} Q_{k'} \\ \frac{1}{p_k^j} &= 1 - \textstyle\sum_{k' \in \mathcal{K}_j} Q_{k'} \end{aligned}$$

Since the right-hand side does not depend on k, optimality demands p_k^j be the same for every $k \in \mathcal{K}_j$; henceforth, without loss of generality, we will focus on one-dimensional strategies $p^j \in \Re_+$.

Lemma 1

Consider two patentholders j and j', with patents covering sets of products $\mathcal{K}_j$ and $\mathcal{K}_{j'}$, respectively. Let $q_j = \sum_{k\in\mathcal{K}_j} Q_k$.

$$\begin{aligned}
\tfrac{\partial q_j}{\partial p^{j'}} &= \textstyle\sum_{k\in\mathcal{K}_j}\sum_{k'\in\mathcal{K}_{j'}} \tfrac{\partial Q_k}{\partial P_{k'}} \\
&= \textstyle\sum_{k\in\mathcal{K}_j}\sum_{k'\in\mathcal{K}_{j'}} Q_k Q_{k'} - \sum_{k\in\mathcal{K}_j\cap\mathcal{K}_{j'}} Q_k \\
&= q_j q_{j'} - q_{j,j'}
\end{aligned}$$

where $q_{j,j'} = \sum_{k\in\mathcal{K}_j\cap\mathcal{K}_{j'}} Q_k$. Then

$$\frac{\partial}{\partial p^j}\log\pi_j = \frac{\partial}{\partial p^j}\log p^j + \frac{\partial}{\partial p^j}\log q_j = \frac{1}{p^j} + \frac{q_j^2 - q_j}{q_j} = \frac{1}{p^j} + q_j - 1$$

and so

$$\frac{\partial^2}{\partial p^j \partial p^{j'}}\log\pi_j = q_j q_{j'} - q_{j,j'} = \frac{\partial^2}{\partial p^j \partial p^{j'}}\log\pi_{j'}$$

Per Monderer and Shapley (1996), this is necessary and sufficient for the existence of a continuous and differentiable potential function $P : \Re_+^{|\mathcal{J}|} \to \Re$ with

$$\frac{\partial P}{\partial p^j} = \frac{\partial \log\pi_j}{\partial p^j}$$

Since q_j is decreasing in p^j, $\log\pi_j$ is strictly concave, and $\log\pi_j$ is strictly increasing close to $p^j = 0$. Since

$$q_j = \frac{\sum_{k\in\mathcal{K}_j}\exp(v_k - P_k)}{1+\sum_{k\in\mathcal{K}}\exp(v_k - P_k)}$$

and for $k \in \mathcal{K}_j$, $P_k \geq p^j$, we can bound $q_j < e^{-p^j}\sum_{k\in\mathcal{K}_j} e^{v_k}$, so the solution to $\frac{1}{p^j} + e^{-p^j}\sum_{k\in\mathcal{K}_j} e^{v_k} = 1$ gives an upper bound on undominated strategies for player j. Since best-responses are interior, they are equivalent to $\frac{\partial \log\pi_j}{\partial p^j} = 0$, so equilibria are equivalent to solutions to $\nabla P = 0$. We can calculate that

$$D^2 P = -\begin{bmatrix} (p^1)^{-2} & & & \\ & (p^2)^{-2} & & \\ & & \ddots & \\ & & & (p^N)^{-2} \end{bmatrix} - \begin{bmatrix} q_1(1-q_1) & q_{1,2} - q_1 q_2 & \dots & q_{1,N} - q_1 q_N \\ q_{1,2} - q_1 q_2 & q_2(1-q_2) & \dots & q_{2,N} - q_2 q_N \\ \vdots & \vdots & & \vdots \\ q_{1,N} - q_1 q_N & q_{2,N} - q_2 q_N & \dots & q_N(1-q_N) \end{bmatrix}$$

where $N = |\mathcal{J}|$. The matrix on the right has the same form as a variance-covariance matrix, and is therefore positive semidefinite, so $D^2 P$ is negative definite; P is therefore strictly concave, leading to $\nabla P = 0$ holding only at the unique global maximum. Having calculated an upper bound on best-responses, we can make the strategy space compact without loss of generality, ensuring existence of a maximum and therefore the unique equilibrium.

Lemma 2

From before, best-responses are solutions to

$$\frac{1}{p^j} = -\frac{\partial}{\partial p^j} \log q_j = 1 - q_j$$

The right-hand side is $1 - Q_k$ for all $j \in \mathcal{J}_k^N$, so all patentholders $j \in \mathcal{J}_k^N$ ask the same price, so $P_k^N = n_k p^j$. Similarly, the right-hand side is $1 - \sum_{k \in \mathcal{K}} Q_k$ for all $j \in \mathcal{J}^E$, so all essential patentholders ask the same price and $P^E = n_E p^j$. Since first-order conditions are necessary and sufficient for best-responses, equilibrium therefore corresponds to solutions to the $K+1$ simultaneous equations

$$n_k = P_k^N (1 - Q_k) \quad and \quad n_E = P^E \left(1 - \sum_{k \in \mathcal{K}} Q_k \right)$$

coupled with

$$p^j = \begin{cases} \frac{1}{n_k} P_k^N & if \quad j \in \mathcal{J}_k^N \\ \frac{1}{n_E} P^E & if \quad j \in \mathcal{J}^E \end{cases}$$

A.2 Theorem 1

Marginal Effect of n_E on Prices

Payoffs to patentholders and consumers can be written as functions of aggregate prices $\left(P^E, P_1^N, \ldots, P_K^N\right)$. A pool of essential patents affects aggregate prices like a decrease in n_E. From the supermodular-games approach in Quint (2011), we know that P^E is increasing in n_E and P_k^N decreasing in n_E. To calculate the exact effects, we can treat n_E as a continuous variable, affecting equilibrium prices through the simultaneous equations $P_k^N(1-Q_k) = n_k$ and $P^E Q_0 = n_E$, and differentiate these with respect to n_E:

$$\begin{aligned}
P_k^N(1-Q_k) &= n_k \\
(1-Q_k)\tfrac{\partial P_k^N}{\partial n_E} - P_k^N\left(\textstyle\sum_{k'\in\mathcal{K}} \tfrac{\partial Q_k}{\partial P_{k'}^N}\tfrac{\partial P_{k'}^N}{\partial n_E} + \tfrac{\partial Q_k}{\partial P^E}\tfrac{\partial P^E}{\partial n_E}\right) &= 0 \\
(1-Q_k)\tfrac{\partial P_k^N}{\partial n_E} - P_k^N\left(-Q_k(1-Q_k)\tfrac{\partial P_k^N}{\partial n_E} + \textstyle\sum_{k'\neq k} Q_k Q_{k'}\tfrac{\partial P_{k'}^N}{\partial n_E} - Q_k Q_0\tfrac{\partial P^E}{\partial n_E}\right) &= 0 \\
(1-Q_k+P_k^N Q_k)\tfrac{\partial P_k^N}{\partial n_E} - P_k^N Q_k\left(\textstyle\sum_{k'\in\mathcal{K}} Q_{k'}\tfrac{\partial P_{k'}^N}{\partial n_E} - Q_0\tfrac{\partial P^E}{\partial n_E}\right) &= 0 \\
\textit{and} & \\
P^E Q_0 &= n_E \\
Q_0\tfrac{\partial P^E}{\partial n_E} + P^E\left(\tfrac{\partial Q_0}{\partial P^E}\tfrac{\partial P^E}{\partial n_E} + \textstyle\sum_{k'\in\mathcal{K}} \tfrac{\partial Q_0}{\partial P_{k'}^N}\tfrac{\partial P_{k'}^N}{\partial n_E}\right) &= 1 \\
Q_0\tfrac{\partial P^E}{\partial n_E} + P^E\left(Q_0(1-Q_0)\tfrac{\partial P^E}{\partial n_E} + \textstyle\sum_{k'\in\mathcal{K}} Q_0 Q_{k'}\tfrac{\partial P_{k'}^N}{\partial n_E}\right) &= 1 \\
\left(Q_0 + P^E Q_0\right)\tfrac{\partial P^E}{\partial n_E} - P^E Q_0\left(Q_0\tfrac{\partial P^E}{\partial n_E} - \textstyle\sum_{k'\in\mathcal{K}} Q_{k'}\tfrac{\partial P_{k'}^N}{\partial n_E}\right) &= 1
\end{aligned}$$

Letting $\Delta = Q_0\frac{\partial P^E}{\partial n_E} - \sum_{k'\in\mathcal{K}} Q_{k'}\frac{\partial P_{k'}^N}{\partial n_E}$, which we know from supermodularity is positive, gives

$$\frac{\partial P_k^N}{\partial n_E} = -\frac{P_k^N Q_k}{1-Q_k+P_k^N Q_k}\Delta \quad \textit{and} \quad \frac{\partial P^E}{\partial n_E} = \frac{1}{Q_0+P^E Q_0} + \frac{P^E}{1+P^E}\Delta$$

Plugging these terms back into the definition of Δ and rearranging yields

$$\frac{1}{1+P^E} = \Delta\left(\frac{Q_0}{1+P^E} + \sum_{k'\in\mathcal{K}} Q_{k'}\frac{1-Q_{k'}}{1-Q_{k'}+P_{k'}^N Q_{k'}}\right)$$

and therefore

$$\frac{\partial P_k^N}{\partial n_E} \;=\; -\frac{P_k^N Q_k}{1-Q_k+P_k^N Q_k}\frac{\delta}{1+P^E} \quad and \quad \frac{\partial P^E}{\partial n_E} \;=\; \frac{1}{Q_0+P^E Q_0}+\frac{P^E}{1+P^E}\frac{\delta}{1+P^E}$$

where $\frac{1}{\delta}=\frac{Q_0}{1+P^E}+\sum_{k'\in\mathcal{K}} Q_{k'}\frac{1-Q_{k'}}{1-Q_{k'}+P_{k'}^N Q_{k'}}$. We can rearrange $\frac{\partial P^E}{\partial n_E}$ to

$$\frac{\partial P^E}{\partial n_E} \;=\; \left(1+\frac{1}{Q_0}\sum_{k'\in\mathcal{K}} Q_{k'}\frac{1-Q_{k'}}{1-Q_{k'}+P_{k'}^N Q_{k'}}\right)\frac{\delta}{1+P^E}$$

to show that $\frac{\partial\left(P_k^N+P^E\right)}{\partial n_E}=\frac{\delta}{1+P^E}\left(-\frac{P_k^N Q_k}{1-Q_k+P_k^N Q_k}+1+\frac{1}{Q_0}\sum_{k'\in\mathcal{K}}\frac{Q_{k'}(1-Q_{k'})}{1-Q_{k'}+P_{k'}^N Q_{k'}}\right)>0$. Finally, since $P_k^N(1-Q_k)=n_k$ does not change with n_E, $\frac{\partial\left(\log P_k^N+\log(1-Q_k)\right)}{\partial n_E}=\frac{1}{P_k^N}\frac{\partial P_k^N}{\partial n_E}-\frac{1}{1-Q_k}\frac{\partial Q_k}{\partial n_E}=0$, which implies $\frac{\partial Q_k}{\partial n_E}=\frac{1-Q_k}{P_k^N}\frac{\partial P_k^N}{\partial n_E}=-\frac{(1-Q_k)Q_k}{1-Q_k+P_k^N Q_k}\frac{\delta}{1+P^E}$.

Welfare

First, note that consumer surplus can be written as

$$CS=\mathbb{E}_{\epsilon_0^l,\epsilon_1^l,\epsilon_2^l,\ldots,\epsilon_K^l}\max\left\{\epsilon_0^l,\max_{k'\in\mathcal{K}}\left\{v_{k'}-P_{k'}+\epsilon_{k'}^l\right\}\right\}=\log\left(1+\sum_{k'\in\mathcal{K}} e^{v_{k'}-P_{k'}}\right)$$

and differentiating, $\frac{\partial CS}{\partial P_k}=-Q_k$. Total patentholder profits are $P^E\left(\sum_{k'\in\mathcal{K}} Q_{k'}\right)+\sum_{k'\in\mathcal{K}} P_{k'}^N Q_{k'}=\sum_{k'\in\mathcal{K}} P_{k'}Q_{k'}$, so the effect of one price on total welfare is

$$\begin{aligned}\frac{\partial Welfare}{\partial P_k} &= \frac{\partial CS}{\partial P_k}+\frac{\partial}{\partial P_k}\sum_{k'\in\mathcal{K}} P_{k'}Q_{k'}\\ &= -Q_k-P_kQ_k(1-Q_k)+Q_k+\sum_{k'\neq k} P_{k'}Q_{k'}Q_k\\ &= -P_kQ_k+\sum_{k'\in\mathcal{K}} P_{k'}Q_{k'}Q_k\\ &= Q_k\left(\overline{P}-P_k\right)\end{aligned}$$

where $\overline{P}\equiv\sum_{k'\in\mathcal{K}} Q_{k'}P_{k'}$. Summing over k,

$$\frac{\partial Welfare}{\partial P^E} \;=\; \sum_{k\in\mathcal{K}}\frac{\partial Welfare}{\partial P_k} \;=\; \sum_{k\in\mathcal{K}} Q_k\left(\overline{P}-P_k\right) \;=\; \overline{P}\sum_{k\in\mathcal{K}} Q_k-\overline{P} \;=\; -Q_0\overline{P}$$

Using these,

$$\begin{aligned}
\frac{\partial Welfare}{\partial n_E} &= \frac{\partial Welfare}{\partial P^E}\frac{\partial P^E}{\partial n_E} + \sum_{k'\in\mathcal{K}} \frac{\partial Welfare}{\partial P^N_{k'}}\frac{\partial P^N_{k'}}{\partial n_E} \\
&= -Q_0\overline{P}\frac{\partial P^E}{\partial n_E} + \sum_{k'\in\mathcal{K}} Q_{k'}\left(\overline{P} - P_{k'}\right)\frac{\partial P^N_{k'}}{\partial n_E} \\
&= -Q_0\overline{P}\left(1 + \frac{1}{Q_0}\sum_{k'\in\mathcal{K}} Q_{k'}\frac{1-Q_{k'}}{1-Q_{k'}+P^N_{k'}Q_{k'}}\right)\frac{\delta}{1+P^E} \\
&\quad - \sum_{k'\in\mathcal{K}} Q_{k'}\left(\overline{P} - P_{k'}\right)\frac{P^N_{k'}Q_{k'}}{1-Q_{k'}+P^N_{k'}Q_{k'}}\frac{\delta}{1+P^E}
\end{aligned}$$

which simplifies to

$$\frac{\partial Welfare}{\partial n_E} = \frac{\delta}{1+P^E}\sum_{k'\in\mathcal{K}} Q_{k'}P_{k'}\left(\frac{P^N_{k'}Q_{k'}}{1-Q_{k'}+P^N_{k'}Q_{k'}} - 1\right) < 0$$

Pools of essential patents have the same impact on equilibrium prices as a decrease in n_E; we can integrate the marginal effect to calculate the effect of a pool.

We showed above that each price $P_k = P^N_k + P^E$ is increasing in n_E, so each price P_k is lower following the formation of a pool; so total consumer surplus (as well as each individual consumer's payoff) is increased by a pool.

We showed that for each k, P^N_k and Q_k are both decreasing in n_E, and therefore higher after a pool. For $j \in \mathcal{J}^N_k$, $\pi_j = \frac{1}{n_k}P^N_k Q_k$, so π_j is higher after a pool for every $j \in \mathcal{J} - \mathcal{J}^E$. As for essential patentholders who remain outside of a pool, they earn profits of $\frac{1}{n_E}P^E(1-Q_0)$ both before and after the pool forms. Since $P^E Q_0 = n_E$, we can rewrite this as $\frac{1}{Q_0}(1-Q_0) = \frac{1}{Q_0} - 1$. Since each Q_k is decreasing in n_E, Q_0 is increasing in n_E, so $\frac{1}{n_E}P^E(1-Q_0) = \frac{1}{Q_0} - 1$ is decreasing in n_E and therefore higher after a pool.

Finally, we showed that total welfare is decreasing in n_E, and therefore higher after a pool.

A.3 Theorem 2

Marginal Effect of n_1 on Equilibrium Prices

Again, we begin by calculating the marginal effect, this time of n_1, on equilibrium prices through the simultaneous equations characterizing them. Differentiating, we find

$$\begin{aligned}
P_1^N(1-Q_1) &= n_1 \\
(1-Q_1)\frac{\partial P_1^N}{\partial n_1} - P_1^N\left(\sum_{k'\in\mathcal{K}} \frac{\partial Q_1}{\partial P_{k'}^N}\frac{\partial P_{k'}^N}{\partial n_1} + \frac{\partial Q_1}{\partial P^E}\frac{\partial P^E}{\partial n_1}\right) &= 1 \\
(1-Q_1)\frac{\partial P_1^N}{\partial n_1} - P_1^N\left(-Q_1(1-Q_1)\frac{\partial P_1^N}{\partial n_1} + \sum_{k'\neq 1} Q_1 Q_{k'}\frac{\partial P_{k'}^N}{\partial n_1} - Q_1 Q_0\frac{\partial P^E}{\partial n_1}\right) &= 1 \\
(1-Q_1+P_1^N Q_1)\frac{\partial P_1^N}{\partial n_1} - P_1^N Q_1\left(\sum_{k'\in\mathcal{K}} Q_{k'}\frac{\partial P_{k'}^N}{\partial n_1} - Q_0\frac{\partial P^E}{\partial n_1}\right) &= 1
\end{aligned}$$

$$\begin{aligned}
P_k^N(1-Q_k) &= n_k \\
(1-Q_k)\frac{\partial P_k^N}{\partial n_1} - P_k^N\left(\sum_{k'\in\mathcal{K}} \frac{\partial Q_k}{\partial P_{k'}^N}\frac{\partial P_{k'}^N}{\partial n_1} + \frac{\partial Q_k}{\partial P^E}\frac{\partial P^E}{\partial n_1}\right) &= 0 \\
(1-Q_k)\frac{\partial P_k^N}{\partial n_1} - P_k^N\left(-Q_k(1-Q_k)\frac{\partial P_k^N}{\partial n_1} + \sum_{k'\neq k} Q_k Q_{k'}\frac{\partial P_{k'}^N}{\partial n_1} - Q_k Q_0\frac{\partial P^E}{\partial n_1}\right) &= 0 \\
(1-Q_k+P_k^N Q_k)\frac{\partial P_k^N}{\partial n_1} - P_k^N Q_k\left(\sum_{k'\in\mathcal{K}} Q_{k'}\frac{\partial P_{k'}^N}{\partial n_1} - Q_0\frac{\partial P^E}{\partial n_1}\right) &= 0
\end{aligned}$$

$$\begin{aligned}
P^E Q_0 &= n_E \\
Q_0\frac{\partial P^E}{\partial n_1} + P^E\left(\sum_{k'\in\mathcal{K}} \frac{\partial Q_0}{\partial P_{k'}^N}\frac{\partial P_{k'}^N}{\partial n_1} + \frac{\partial Q_0}{\partial P^E}\frac{\partial P^E}{\partial n_1}\right) &= 0 \\
Q_0\frac{\partial P^E}{\partial n_1} + P^E\left(\sum_{k'\in\mathcal{K}} Q_0 Q_{k'}\frac{\partial P_{k'}^N}{\partial n_1} + Q_0(1-Q_0)\frac{\partial P^E}{\partial n_1}\right) &= 0 \\
(Q_0+P^E Q_0)\frac{\partial P^E}{\partial n_1} + P^E Q_0\left(\sum_{k'\in\mathcal{K}} Q_{k'}\frac{\partial P_{k'}^N}{\partial n_1} - Q_0\frac{\partial P^E}{\partial n_1}\right) &= 0
\end{aligned}$$

Let $\Lambda = \sum_{k'\in\mathcal{K}} Q_{k'}\frac{\partial P_{k'}^N}{\partial n_1} - Q_0\frac{\partial P^E}{\partial n_1}$, which (as before) we know from supermodularity is positive; then

$$\begin{aligned}
\frac{\partial P_1^N}{\partial n_1} &= \frac{1}{1-Q_1+P_1^N Q_1} + \frac{P_1^N Q_1}{1-Q_1+P_1^N Q_1}\Lambda \\
\frac{\partial P_k^N}{\partial n_1} &= \frac{P_k^N Q_k}{1-Q_k+P_k^N Q_k}\Lambda \\
\frac{\partial P^E}{\partial n_1} &= -\frac{P^E}{1+P^E}\Lambda
\end{aligned}$$

and, plugging these into the definition of Λ and solving for Λ, $\Lambda = \frac{Q_1}{1-Q_1+P_1^N Q_1}\delta$, with (as before) $\frac{1}{\delta} = \sum_{k'\in\mathcal{K}} \frac{Q_{k'}(1-Q_{k'})}{1-Q_{k'}+P_{k'}^N Q_{k'}} + \frac{Q_0}{1+P^E}$. We can also rewrite $\frac{\partial P_1^N}{\partial n_1}$ as

$$\frac{\partial P_1^N}{\partial n_1} = \left(1 + \frac{1}{Q_1}\left(\sum_{k'\neq 1} \frac{Q_{k'}(1-Q_{k'})}{1-Q_{k'}+P_{k'}^N Q_{k'}} + \frac{Q_0}{1+P^E}\right)\right)\Lambda$$

to show $\frac{\partial P_1^N}{\partial n_1} > \Lambda$, and therefore that $P_1 = P_1^N + P^E$ is increasing in n_1. For $k \neq 1$,

$$\frac{\partial(\log P_k^N + \log(1-Q_k))}{\partial n_1} = \frac{\partial \log n_k}{\partial n_1} = 0 \;\rightarrow\; \frac{\partial Q_k}{\partial n_1} = \frac{1-Q_k}{P_k^N}\frac{\partial P_k^N}{\partial n_1} = \frac{Q_k(1-Q_k)}{1-Q_k+P_k^N Q_k}\Lambda$$

As for Q_1, $\frac{\partial Q_1}{\partial n_1} = \frac{\partial Q_1}{\partial P_1^N}\frac{\partial P_1^N}{\partial n_1} + \sum_{k'\neq 1}\frac{\partial Q_1}{\partial P_{k'}^N}\frac{\partial P_{k'}^N}{\partial n_1} + \frac{\partial Q_1}{\partial P^E}\frac{\partial P^E}{\partial n_1}$, which simplifies to

$$\frac{\partial Q_1}{\partial n_1} = -\left(\sum_{k'\neq 1} \frac{Q_{k'}(1-Q_{k'})}{1-Q_{k'}+P_{k'}^N Q_{k'}} + \frac{Q_0}{1+P^E}\right)\Lambda < 0$$

Consumer Surplus

As noted above, $\frac{\partial CS}{\partial P_k} = -Q_k$, so $\frac{\partial CS}{\partial n_1} = -Q_1\frac{\partial P_1^N}{\partial n_1} - \sum_{k'\neq 1} Q_{k'}\frac{\partial P_{k'}^N}{\partial n_1} - \sum_{k'\in\mathcal{K}} Q_{k'}\frac{\partial P^E}{\partial n_1}$; plugging in the expressions above and simplifying gives $\frac{\partial CS}{\partial n_1} = -\frac{1}{1+P^E}\Lambda < 0$.

Welfare

As noted above, $\frac{\partial Welfare}{\partial P_k} = Q_k(\overline{P} - P_k)$ and $\frac{\partial Welfare}{\partial P^E} = -Q_0\overline{P}$, so

$$\frac{\partial Welfare}{\partial n_1} = Q_1(\overline{P} - P_1)\frac{\partial P_1^N}{\partial n_1} + \sum_{k'\neq 1} Q_{k'}(\overline{P} - P_{k'})\frac{\partial P_{k'}^N}{\partial n_1} - Q_0\overline{P}\frac{\partial P^E}{\partial n_1}$$

Plugging in the expressions above and simplifying gives

$$\frac{\partial Welfare}{\partial n_1} = \left(\sum_{k'\neq 1} P_{k'}\frac{Q_{k'}(1-Q_{k'})}{1-Q_{k'}+P_{k'}^N Q_{k'}} - \sum_{k'\neq 1} P_1 \frac{Q_{k'}(1-Q_{k'})}{1-Q_{k'}+P_{k'}^N Q_{k'}} - P_1\frac{Q_0}{1+P^E}\right)\Lambda$$

Theorem

A pool of patentholders in $\mathcal{J}_1^N$ has the same effect on equilibrium prices as a decrease in n_1. We showed above that P_k^N is increasing in n_1 for every k

(including 1); P^E is decreasing in n_1; Q_1 is decreasing in n_1; Q_k $(k \neq 1)$ is increasing in n_1; and P_1 is increasing in n_1. Examples in the text show that P_k can increase or decrease in n_1. We showed above that total consumer surplus is decreasing in n_1; but a pool which increases P_k $(k \neq 1)$ will harm some individual consumers (at a minimum, those who still purchase product k after the pool).

For $k \neq 1$, P_k^N and Q_k are both increasing in n_1, so $\frac{1}{n_k} P_k^N Q_k$ is increasing in n_1, so profits of patentholders in $\mathcal{J}_k^N$ are lower following the pool. Since $P^E Q_0 = n_E$ and P^E is decreasing in n_1, Q_0 must be increasing in n_1, so $\frac{1}{n_E} P^E (1 - Q_0)$ is decreasing in n_1, so profits of patentholders in $\mathcal{J}^E$ are higher after the pool. As for outsiders within $\mathcal{J}_1^N$, they earn $\frac{1}{n_1} P_1^N Q_1$; since $P_1^N(1 - Q_1) = n_1$, we can rewrite this as $\frac{Q_1}{1-Q_1}$; since Q_1 is higher after the pool, these outsiders earn more. Examples in the text show both welfare-positive and welfare-negative pools.

A.4 Profitability of Pools of Complements

Claim 1. *Fix $m \geq 0$. If n_E is sufficiently large, a pool containing all but m of the essential patents increases the profits of its participants.*

Let π be the equilibrium profits of each essential patentholder in the game with the same primitives as G except that there are only $m+1$ essential patents. Note that $\pi > 0$. Should a pool form containing all but m of the n_E essential patents, π would be the pool's total revenue; divided up equally, this would give each participant a payoff of $\frac{\pi}{n_E - m}$. In the absence of a pool, each essential patentholder's profits are If no pool forms, each essential patentholder's profits are $\frac{P^E}{n_E}\sum_{k\in\mathcal{K}} Q_k$. We know that

$$\sum_{k\in\mathcal{K}} Q_k = \frac{\sum_{k\in\mathcal{K}} e^{v_k - P_k^N - P^E}}{1+\sum_{k\in\mathcal{K}} e^{v_k - P_k^N - P^E}} < \sum_{k\in\mathcal{K}} e^{v_k - P_k^N - P^E} < e^{-P^E}\sum_{k\in\mathcal{K}} e^{v_k}$$

and so

$$\frac{P^E}{n_E}\sum_{k\in\mathcal{K}} Q_k < \frac{1}{n_E}P^E e^{-P^E} V$$

where $V \equiv \sum_{k\in\mathcal{K}} e^{v_k}$. To prove the result, we need to show that for n_E sufficiently large, $\frac{\pi}{n_E - m} > \frac{1}{n_E}P^E e^{-P^E} V$. Since $n_E > n_E - m$, a sufficient condition is $\frac{\pi}{V} > P^E e^{-P^E}$. $\frac{\pi}{V}$ is a constant, and xe^{-x} decreases to 0 as x increases; since $P^E > n_E$ increases without bound as $n_E \to \infty$, the result follows.

Claim 2. *Fix $m \geq 1$. If n_E is sufficiently large, the formation of a pool of $m+1$ essential patents increases the profits of its participants.*

(Note that this is the same as saying, the addition of m essential patents to an already-essential pool increases the joint profits of the new entrants and the pool.)

I will treat n_E as the name of the variable, $\tilde{n}_E$ as its starting value, and $\tilde{n}_E - m$ as its ending value (after pool formation). Let $\Pi_E(\cdot)$ be the equilibrium value of $P^E \sum_{k'\in\mathcal{K}} Q_{k'}$, as a function of n_E. Combined profits of the patentholders forming the pool are $\frac{m+1}{\tilde{n}_E}\Pi^E(\tilde{n}_E)$ before pool formation, and $\frac{1}{\tilde{n}_E - m}\Pi^E(\tilde{n}_E - m)$ after. We can therefore write the gain in profits as

$$d\pi \quad = \quad (m+1)^r \frac{\Pi^E(\tilde{n}_E-(1-r)m)}{\tilde{n}_E-(1-r)m}\Big|_{r=1}^{r=0} \quad = \quad -\int_0^1 \frac{d}{dr}\left((m+1)^r \frac{\Pi^E(\tilde{n}_E-(1-r)m)}{\tilde{n}_E-(1-r)m}\right) dr$$

Thus, it will suffice to show that for n_E sufficiently large, $(m+1)^r \frac{\Pi^E(\tilde{n}_E-(1-r)m)}{\tilde{n}_E-(1-r)m}$ is decreasing in r for all $r \in (0,1)$. Letting $n_E = \tilde{n}_E - (1-r)m$, then, it will suffice to show that

$$0 \quad > \quad \frac{\partial}{\partial r}\left((m+1)^r \frac{\Pi^E(\tilde{n}_E-(1-r)m)}{\tilde{n}_E-(1-r)m}\right) \quad = \quad \frac{d}{dr}\left(e^{r\log(m+1)} \frac{\Pi^E(\tilde{n}_E-m+rm)}{\tilde{n}_E-m+rm}\right)$$

Taking the derivative, plugging in the expressions above, and simplifying, we can show that the derivative is proportional to

$$\frac{\log(m+1)}{m}Q_0^2\left[\frac{\sum_{k'\in\mathcal{K}}Q_{k'}\frac{P_{k'}^N Q_{k'}}{1-Q_{k'}+P_{k'}^N Q_{k'}}}{\sum_{k'\in\mathcal{K}}Q_{k'}\frac{1-Q_{k'}}{1-Q_{k'}+P_{k'}^N Q_{k'}}}\right]+\frac{\log(m+1)}{m}Q_0P^E(1-Q_0)-\left(1-\frac{\log(m+1)}{m}Q_0\right)$$

which we can rewrite as

$$\frac{\log(m+1)}{m}Q_0^2\left[\frac{\textit{wtd avg } P_{k'}^N Q_{k'}}{\textit{wtd avg } 1-Q_{k'}}\right]+\frac{\log(m+1)}{m}Q_0P^E(1-Q_0)-\left(1-\frac{\log(m+1)}{m}Q_0\right)$$

where the weighted averages are taken with respect to weights $\frac{Q_{k'}}{1-Q_{k'}+P_{k'}^N Q_{k'}}$, normalized to sum to 1.

Now, $Q_k = \frac{\exp(v_k-P_k)}{1+\sum_{k'\in\mathcal{K}}\exp(v_{k'}-P_{k'})} < e^{v_k}e^{-P^E}$ and $P^E > n_E$, so as n_E grows without bound, $P_k^N Q_k$ and $P^E(1-Q_0)$ go to 0 and $1-Q_k$ does not, so the first two terms vanish. For $m \geq 1$, $\frac{\log(m+1)}{m} < 0.7$, so the term in parentheses is at least 0.3. Pick $\tilde{n}_E$ sufficiently large so that the whole expression is negative for $n_E \geq \tilde{n}_E - m$ and the integrand is negative on $(0,1)$, making the pool profitable.

(Conversely, since the lead term is positive, the integrand will be positive whenever $\frac{\log(m+1)}{m}(Q_0 + P^E(1-Q_0)Q_0) > 1$. For $m = 1$, this will hold for all $r \in (0,1)$ if prior to pool formation, $(\tilde{n}_E - 2)(1-Q_0) > \frac{1}{\log 2} - 1 \approx 0.442$.)

Claim 3. *Fix $m \geq 0$. If n_k is sufficiently large, a pool containing all but m of the patentholders in $\mathcal{J}_k^N$ increases the profits of its participants.*

The claim is the same as the one proven in the text for essential patents: fixing m, if n_k is sufficiently large, a pool of all but m of the patentholders in $\mathcal{J}_k^N$ increases the profits of its participants.

This time, let π denote equilibrium profits to each player in $\mathcal{J}_k^N$ after the pool formed (that is, with $n_k = m+1$). Since there are $n_k - m$ pool members, each earns $\frac{1}{n_k - m}\pi$ given the pool. Without the pool, each earns $\frac{1}{n_k}P_k^N Q_k$. Pick n_k sufficiently large that $\pi e^{-v_k} > n_k e^{-n_k}$. Since $P_k^N > n_k$ and xe^x is decreasing above 1, $\pi e^{-v_k} > P_k^N e^{-P_k^N}$, so $\pi > P_k^N e^{v_k - P_k^N} > P_k^N Q_k$, so

$$\frac{1}{n_k - m}\pi > \frac{1}{n_k}\pi > \frac{1}{n_k}P_k^N Q_k$$

and the pool is profitable.

Claim 4. *Fix $m \geq 1$. If n_k is sufficiently large, the formation of a pool of $m+1$ patentholders in $\mathcal{J}_k^N$ increases the profits of its participants.*

Mimicking the proof above of Claim 2, note that when $m+1$ nonessential patents blocking technology 1 form a pool, the change in their joint profits is

$$d\pi \quad = \quad (m+1)^r \frac{\Pi^1(\tilde{n}_1 - (1-r)m)}{\tilde{n}_1 - (1-r)m}\Bigg|_{r=1}^{r=0} \quad = \quad -\int_0^1 \frac{d}{dr}\left((m+1)^r \frac{\Pi^1(\tilde{n}_1 - (1-r)m)}{\tilde{n}_1 - (1-r)m}\right) dr$$

where Π^1 is the equilibrium value of $P_1^N Q_1$ as a function of n_1. Differentiating and letting $n_1 = \tilde{n}_1 - (1-r)m$, the integrand is

$$\log(m+1) e^{r \log(m+1)} \frac{P_1^N Q_1}{n_1} + e^{r \log(m+1)} m \frac{\partial}{\partial n_1}\left(\frac{P_1^N Q_1}{n_1}\right)$$

which, after a lot of simplification, is proportional to

$$\frac{\log(m+1)}{m}(1 - Q_1) - \frac{1}{1 - Q_1 + P_1^N Q_1}\left(1 - \frac{\frac{Q_1(1-Q_1)}{1-Q_1+P_1^N Q_1}}{\sum_{k\in\mathcal{K}} \frac{Q_k(1-Q_k)}{1-Q_k+P_k^N Q_k} + \frac{Q_0}{1+P^E}}\right)$$

As n_1 increases to infinity, P_1^N grows unboundedly and Q_1 and $P_1^N Q_1$ shrink to 0. The term $\frac{Q_1(1-Q_1)}{1-Q_1+P_1^N Q_1} < Q_1$ vanishes; since Q_0 increases in n_1 and P^E decreases, $\frac{Q_0}{1+P^E}$ does not vanish; so the messy fraction goes to 0 and the term within parentheses goes to 1. $\frac{1}{1-Q_1+P_1^N Q_1}$ goes to 1 as well, so the entire second expression goes to 1, while the first expression is bounded above by $\frac{\log(m+1)}{m} < 0.7$. So when n_1 is sufficiently large, the integrand is negative and the pool is profitable.

Claim 5. *Fixing k, the addition of all patentholders in $\mathcal{J}_k^N$ to a pool already containing all essential patents is always profitable.*

Consider the effects of the addition of these patents to the pool as occurring in four stages:

1. The profits of these nonessential patents begin to accrue to the pool, without any changes in any prices

2. The pool reduces the prices of these patents to 0 and adjusts the price of the pool, as a static best-response to the prices of the nonessential patents blocking other technologies

3. The nonessential patentholders blocking other technologies change their prices to the new equilibrium levels

4. The pool changes its prices to its new equilibrium level

Stage 1 does not change the joint profits of the pool and the patents being added, it just allows us to focus on the profits going to essential patentholders in the last three stages.

We showed above that an essential patentholder cannot gain by charging different prices to producers accessing different technologies. Thus, stage 2 increases essential patentholder (pool) profits, increasing joint profits.

Since the new equilibrium corresponds to a decrease in n_k, the prices of nonessential competing patents $P_{k'}^N$ decrease in equilibrium; this increases essential patentholder profits. So stage 3 increases the joint profits we're looking at.

Finally, stage 4 increases joint profits, since the pool will only change prices to increase profits.

Thus, each stage increases the joint profits, so the net effect is to increase joint profits.

Claim 6. *Adding some patentholders in $\mathcal{J}_k^N$ to a pool containing all or some of the essential patentholders is sometimes unprofitable.*

If a single essential patent (or a pool) joins with a single essential patent blocking technology 1, we can write the change in their joint profits as

$$d\pi = \int_0^1 \frac{d}{dr}\left(\frac{1}{n_E}\Pi^E(\tilde{n}_1 - r) + (1-r)\frac{\Pi^1(\tilde{n}_1 - r)}{\tilde{n}_1 - r}\right) dr$$

where $\tilde{n}_1$ is the starting value of n_1 and Π^E and Π^1 are the equilibrium values of $P^E(1-Q_0)$ and $P_1^N Q_1$, respectively, as a function of n_1. We find conditions for this to be negative. Taking the derivative and letting $n_1 = \tilde{n}_1 - r$, we can show the integrand is proportional to

$$\begin{aligned}&\tfrac{1}{Q_0}\tfrac{1}{1+P^E}\tfrac{1-Q_1}{1-Q_1+P_1^N Q_1} - \left(\textstyle\sum_{k\in\mathcal{K}} \tfrac{Q_k(1-Q_k)}{1-Q_k+P_k^N Q_k} + \tfrac{Q_0}{1+P^E}\right)\\ &+(1-r)\tfrac{1}{1-Q_1}\tfrac{1}{1-Q_1+P_1^N Q_1}\left(\textstyle\sum_{k\in\mathcal{K}-\{1\}} \tfrac{Q_k(1-Q_k)}{1-Q_k+P_k^N Q_k} + \tfrac{Q_0}{1+P^E}\right)\end{aligned}$$

If $Q_1 \geq \frac{1}{Q_0}\frac{1}{1+P^E} = \frac{1}{Q_0+n_E}$ and $1 \geq (1-r)\frac{1}{1-Q_1}\frac{1}{1-Q_1+P_1^N Q_1}$, then the $k=1$ piece of the second term dominates the first term, and the remainder of the second term dominates the third term, making the whole expression negative. The latter requires that

$$1 \;\geq\; \tfrac{1-r}{(1-Q_1)^2+P_1^N Q_1(1-Q_1)} \;=\; \tfrac{1-r}{1-2Q_1+Q_1^2+n_1 Q_1} \;=\; \tfrac{1-r}{1-2Q_1+Q_1^2+(\tilde{n}_1-r)Q_1}$$

If $\tilde{n}_1 \geq 2$, the condition $1 \geq \frac{1-r}{1+(\tilde{n}_1-2)Q_1+Q_1^2-rQ_1}$ holds for all $r \in [0,1]$. Thus, $\tilde{n}_1 \geq 2$, along with the condition that $Q_1(Q_0+n_E) \geq 1$ at each $n_1 = \tilde{n}_1 - r$, guarantees all negative integrands. Since Q_1 is decreasing in n_1, if $Q_1 n_E \geq 1$ at $r = 0$ (before the pool forms/grows), then $Q_1 n_E \geq 1$ at all r, which will suffice.

So when $n_E > \frac{1}{Q_k}$ (there are at least $\frac{1}{Q_k} - 1$ outsiders to a pool of essential patents) and $n_k \geq 2$, the addition of a single patentholder in $\mathcal{J}_k^N$ to the pool decreases combined profits.

For a concrete example, let $K = 3$, $\mathbf{v} = (10, 10, 5)$, $n_E = 1$ (a pool already exists containing all essential patentholders), and $n_1 = n_2 = n_3 = 3$. Adding a single one of the patentholders in $\mathcal{J}_2^N$ to the existing pool decreases their combined profit:

n	P^E	P_1^N	P_2^N	P_3^N	Q_1	Q_2	Q_3	Π_E	Π_1	Π_2	Π_3	CS	W
(1,3,3,3)	4.578	4.861	4.861	3.048	.383	.383	.016	**3.578**	1.861	**1.861**	0.048	1.521	8.869
(1,3,2,3)	5.171	4.354	3.904	3.024	.311	.488	.008	**4.171**	1.354	1.904	0.024	1.643	9.096

Combined profits of the pool and a single patentholder in $\mathcal{J}$ are $3.578 + \frac{1}{3}(1.861) = 4.198$ to begin, and 4.171 after the patentholder is added to the pool.

Example 3. First, let $(n_E, n_1, n_2, n_3) = (3, 5, 5, 3)$, and consider the profitability of a pool containing the three essential patentholders:

n	P^E	P_1^N	P_2^N	P_3^N	Q_1	Q_2	Q_3	Π_E	Π_1	Π_2	Π_3	CS	W
(3,5,5,3)	5.012	6.151	6.151	3.084	.187	.187	.027	**2.012**	1.151	1.151	0.084	0.513	4.911
(1,5,5,3)	3.047	7.068	7.068	3.285	.293	.293	.087	**2.047**	2.068	2.068	0.285	1.114	7.582

Next, suppose that a pool of the five patentholders in $\mathcal{J}_1^N$ has already formed, and consider the profitability of the same pool:

n	P^E	P_1^N	P_2^N	P_3^N	Q_1	Q_2	Q_3	Π_E	Π_1	Π_2	Π_3	CS	W
(3,1,5,3)	7.397	2.279	5.162	3.005	.561	.031	.002	**4.397**	1.279	0.162	0.005	0.902	6.745
(1,1,5,3)	5.235	3.452	5.503	3.022	.710	.091	.007	**4.235**	2.452	0.503	0.022	1.655	8.867

Now do the same, but starting with $(n_E, n_1, n_2, n_3) = (3, 5, 3, 5)$. Before the pool of patentholders in $\mathcal{J}_1^N$:

n	P^E	P_1^N	P_2^N	P_3^N	Q_1	Q_2	Q_3	Π_E	Π_1	Π_2	Π_3	CS	W
(3,5,3,5)	5.748	5.700	4.641	5.008	.123	.354	.002	**2.748**	0.700	1.641	0.008	0.650	5.747
(1,5,3,5)	3.632	6.530	5.806	5.035	.234	.483	.007	**2.632**	1.530	2.806	0.035	1.290	8.293

And after the pool of all five patentholders in $\mathcal{J}_1^N$:

n	P^E	P_1^N	P_2^N	P_3^N	Q_1	Q_2	Q_3	Π_E	Π_1	Π_2	Π_3	CS	W
(3,1,3,5)	7.772	1.980	3.405	5.001	.495	.119	.000	**4.772**	0.980	0.405	0.001	0.952	7.109
(1,1,3,5)	6.001	2.675	3.782	5.002	.626	.207	.000	**5.001**	1.675	0.782	0.002	1.792	9.253

A.5 Theorem 3 (welfare and profitability when $K = 2$)

Using the above, we can calculate (for general K)

$$\begin{aligned}\frac{\partial\left(Welfare - P_1^N Q_1\right)}{\partial n_1} \quad = \quad & \Bigg(\sum_{k'\neq 1} P_{k'} \frac{Q_{k'}(1-Q_{k'})}{1-Q_{k'}+P_{k'}^N Q_{k'}} - \sum_{k'\neq 1} P_1 \frac{Q_{k'}(1-Q_{k'})}{1-Q_{k'}+P_{k'}^N Q_{k'}} - P_1 \frac{Q_0}{1+P^E} \\ & - P_1^N \left(- \sum_{k'\neq 1} \frac{Q_{k'}(1-Q_{k'})}{1-Q_{k'}+P_{k'}^N Q_{k'}} - \frac{Q_0}{1+P^E} \right) \\ & - Q_1 \left(1 + \frac{1}{Q_1} \left(\sum_{k'\neq 1} \frac{Q_{k'}(1-Q_{k'})}{1-Q_{k'}+P_{k'}^N Q_{k'}} + \frac{Q_0}{1+P^E} \right) \right) \Bigg) \Lambda\end{aligned}$$

which eventually simplifies to $\frac{\partial\left(Welfare - P_1^N Q_1\right)}{\partial n_1} = \left(\sum_{k'\neq 1} \frac{P_{k'}^N Q_k}{1-Q_{k'}+P_{k'}^N Q_{k'}} - 1 \right) \Lambda$. When $K = 2$, the summation has only one term, which is less than 1, so the expression is negative; so a decrease in n_1 (a pool of patentholders in $\mathcal{J}_1^N$) has a net positive externality on everyone outside of $\mathcal{J}_1^N$. Since we already showed that it has a positive externality on outsiders within $\mathcal{J}_1^N$, if it increases the profits of its members, the overall welfare impact must be positive.

A.6 Theorem 4 (Pools Containing Substitutes)

For the first claim, suppose the patents block technologies 1 and 2. The pool sets price p^* to maximize $p(Q_1 + Q_2)$, leading to first-order condition

$p^*(1-Q_1-Q_2)=1$, which is equivalent to

$$\begin{array}{rcccl} p^*(1-Q_1) & = & \frac{1-Q_1}{1-Q_1-Q_2} & = & 1+\frac{Q_2}{1-Q_1-Q_2} \\ p^*(1-Q_2) & = & \frac{1-Q_2}{1-Q_1-Q_2} & = & 1+\frac{Q_1}{1-Q_1-Q_2} \end{array}$$

Along with the best-response functions for the other patentholders in T_1^N and T_2^N, this leads to

$$P_1^N(1-Q_1) \quad = \quad n_1+\frac{Q_2}{1-Q_1-Q_2} \quad and \quad P_2^N(1-Q_2) \quad = \quad n_2+\frac{Q_1}{1-Q_1-Q_2}$$

in equilibrium, along with the usual conditions $P^E\left(1-\sum_{k\in\mathcal{K}}Q_k\right)=n_E$ and $P_k^N(1-Q_k)=n_k$ for $k\in\mathcal{K}-\{1,2\}$. Thus, the equilibrium effect of the pool is equivalent to increases in n_1 and n_2, each of which reduces producer surplus.

As for the second, a pool containing one nonessential patent blocking each technology has the same effect as an increase (by one) of n_E and decreases (by one) of each n_k. Using the expressions calculated earlier for the welfare effects of each parameter, we can calculate that

$$\begin{array}{rcl} \frac{1}{\delta}\left(\frac{\partial W}{\partial n_E}-\sum\frac{\partial W}{\partial n_k}\right) & = & -\frac{1}{1+P^E}\sum_{k\in\mathcal{K}}\sum_{k'\neq k}Q_{k'}(P_k^N+P^E)\frac{Q_k}{1-Q_k+P_k^NQ_k} \\ & & -\sum\sum_{k'<k}(Q_k-Q_{k'})(P_{k'}^N-P_k^N)\frac{Q_{k'}}{1-Q_{k'}+P_{k'}^NQ_{k'}}\frac{Q_k}{1-Q_k+P_k^NQ_k} \end{array}$$

Each unordered pair $\{k,k'\}$ contributes two terms to the first double-summation and one term to the second; collecting these three terms

$$\begin{array}{l} -\frac{1}{1+P^E}Q_{k'}(P_k^N+P^E)\frac{Q_k}{1-Q_k+P_k^NQ_k}-\frac{1}{1+P^E}Q_k(P_{k'}^N+P^E)\frac{Q_{k'}}{1-Q_{k'}+P_{k'}^NQ_{k'}} \\ -(Q_k-Q_{k'})(P_{k'}^N-P_k^N)\frac{Q_{k'}}{1-Q_{k'}+P_{k'}^NQ_{k'}}\frac{Q_k}{1-Q_k+P_k^NQ_k} \end{array}$$

they can be shown to be negative, so summing up over all $\{k,k'\}$ pairs gives $\frac{\partial W}{\partial n_E}-\sum\frac{\partial W}{\partial n_k}<0$, making the pool welfare-destroying.

As for consumer surplus, we calculated earlier that

$$\frac{\partial CS}{\partial n_E} \quad = \quad -\frac{\delta}{1+P^E}\sum_{k\in\mathcal{K}}\frac{1-Q_k}{Q_0}\frac{Q_k}{1-Q_k+P_k^NQ_k} \quad and \quad \frac{\partial CS}{\partial n_k} \quad = \quad -\frac{\delta}{1+P^E}\frac{Q_k}{1-Q_k+P_k^NQ_k}$$

Since $1-Q_k=Q_0+\sum_{k'\neq k}Q_{k'}>Q_0$, $\frac{\partial CS}{\partial n_E}-\sum_{k'\in\mathcal{K}}\frac{\partial CS}{\partial n_{k'}}<0$, so such a pool lowers consumer surplus.

A.7 Theorem 5 (Compulsory Individual Licensing)

We assume, as in Lerner and Tirole and Brenner, that in the presence of a patent pool with compulsory individual licensing, pricing occurs in two stages:

1. The patent pool, and patentholders outside the pool, name prices
2. The members of the patent pool name prices for their individual patents

Producers then choose a technology, licensing pooled patents as needed, either individually or through the pool, whichever is cheaper.

Claim 7. *Suppose that each patent in the pool is worthless without every patent in the pool. (That is, all patents in the pool are essential, or all patents in the pool are nonessential and block the same technology.) Then if the pool's price in Stage* 1 *is a static best-response to the prices of the patents outside the pool, then Stage* 2 *changes nothing.*

The logic is as follows. In stage 2, each member of the pool, by naming a sufficiently high price, can "sabotage" individual licensing, that is, ensure that nobody will license *any* of the pooled patents individually. Thus, if pool members name prices which sum to less than the price of the pool, it must be that each of them is earning weakly higher profits than they would without individual licensing. Since the price of the pool was the unique maximizer of the pooled patentholders' combined profits, this is only possible if the sum of the individual prices of the pooled patents is the same as the price of the pool; so total prices, and each patentholder's revenue, is the same as without individual licensing. Anticipating this in stage 1, the pool has no reason to play anything other than a static best-response, and the same prices emerge as in equilibrium without individual licensing of pooled patents.

A.8 Theorem 6 (Stackelberg Pools)

A Stackelberg pool names a price p^* to maximize profits $p^* Q_A$ after all other patentholders have best-responded. At these equilibrium prices, let $\hat{n} = P^E Q_0 - n_0$. Aggregate prices correspond to the solution to Equation 5, with $n_E = \hat{n} + n_0$. A regular pool corresponds to the case of $n_E = 1 + n_0$. Setting n^* to be the lowest whole number greater than or equal to $\hat{n} + n_0$, and the fact that welfare and all outsider payoffs are decreasing in n_E, will complete the proof once we show that $\hat{n} > 1$.

By optimality, the outsider patentholders $j \in \mathcal{J}^E$ set prices $p_j = \frac{1}{Q_0}$, so $P^E = p^* + \frac{n_0}{Q_0}$, so $\hat{n} = P^E Q_0 - n_0 = p^* Q_0 + n_0 - n_0 = p^* Q_0$. Since $\hat{n} + n_0 = P^E Q_0$, $p^* = \frac{\hat{n}}{\hat{n}+n_0} P^E_{\hat{n}+n_0}$, where P^E_x is the static equilibrium price when $n_E = x$; since P^E is increasing in n_E, $\frac{\hat{n}}{\hat{n}+n_0} P^E_{\hat{n}+n_0}$ is strictly increasing in $\hat{n}$, from 0 at $\hat{n} = 0$ to infinity as $\hat{n} \to \infty$; so instead of choosing a price, envision the pool choosing $\hat{n}$ and then getting payoff $\frac{\hat{n}}{\hat{n}+n_0} P^E_{\hat{n}+n_0} Q_A$.

Now, increasing $\hat{n}$ has two effects: a direct one (it increases the pool's price, which can increase or decrease profits), and a strategic one (it causes other patentholders to adjust their prices). Both $\frac{1}{n_E} P^E$ and P^N_k are decreasing in n_E, so all outside patentholders (both essential and nonessential) demand lower prices as $\hat{n}$ increases; so the strategic effect always favors raising $\hat{n}$. At $\hat{n} < 1$, the pool's price is below the static best-response, so raising it is strictly beneficial; at $\hat{n} = 1$, the pool's price is exactly its static best-response, so the direct effect of increasing $\hat{n}$ is second-order and the strategic effect dominates. Thus, the Stackelberg pool maximizes profits by setting $\hat{n} > 1$.

A.9 Examples 1, 2, 3, and 4

For Examples 1, 2, and 4 (in the text) and Example 3 (in the Appendix), equilibrium prices and marketshares are shown; the reader can verify that the marketshares satisfy Equation 4 and the prices and marketshares together satisfy Equation 5, which characterizes equilibrium.

In the case of the Stackelberg pool (Example 4), the pool sets P^E first, knowing that nonessential patentholders will best-respond. Equilibrium prices, then, are the same as the solutions to Equation 5 at the (non-integer) value of n_E which maximizes $\Pi_E = P^E(Q_1 + Q_2 + Q_3)$. In Example 4, this was calculated numerically to be $n_E \approx 2.129$. Since welfare is decreasing in n_E, this means that the Stackelberg pool is welfare-positive when $n_E \geq 3$ but welfare-negative when $n_E = 2$.

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