Online-Only Appendix for Daniel Quint, "Imperfect Competition with Complements and Substitutes"

B1 Example of the Pathology Ruled Out by Assumption 3

Theorem 2 establishes that if the marginal cost of one of the components of product 2 increases, the equilibrium prices of *every* product rise. It is tempting to assume that this must be good news for the makers of components of the other products. For products with just one component, this is indeed the case. For example, if $n_1 = 1$, the lone firm in $\mathcal{T}_1$ sees the price of every competing product go up; and if it chooses in equilibrium to raise its own price P_1, this can only be to further increase profits.

However, when $n_1 > 1$, it may not follow that an increase in a competing product's cost is good news. The increases in $P_2, \ldots, P_K$ certainly increase the profits of each firm in $\mathcal{T}_1$; but at least in principle, it is possible that these increases also change the shape of the residual demand curve Q_1 in such a way that the double-marginalization problem among the firms in $\mathcal{T}_1$ gets more severe, leading to P_1 increasing "too much" in equilibrium and actually lowering the profits of these firms.

To see an example of how this could happen (though not within the model of this paper), consider the following. Suppose there are two products, zero marginal costs, and $n_1 = 9$ and $n_2 = 1$. There are two populations of consumers: a measure 1 of "type-1" consumers, with $v_1 \sim U[0, 1]$ and $v_2 = 0$; and a measure $\frac{1}{10}$ of "type-2" consumers, with $v_1 \sim U[1, 3]$ and $v_2 = 10$. Suppose the price of product 2 increases from 5 to 10, and the firms in $\mathcal{T}_1$ jointly best-respond. I will show that this change leads to a decrease in profits for the firms in $\mathcal{T}_1$.

When $P_2 = 5$, all type-2 consumers buy product 2 regardless of P_1; so the residual demand for product 1 is the demand from the type-1 consumers, which is

$$Q_1 \quad = \quad 1 - P_1$$

for $0 \leq P_1 \leq 1$. With $n_1 = 9$ (and taking $P_2 = 5$ as exogenous), the combined equilibrium price of the nine firms in $\mathcal{T}_1$ is the price maximizing $P_1^9 Q_1$; this turns out to be $P_1 = \frac{9}{10}$, implying $Q_1 = \frac{1}{10}$, as well as $p_i = \frac{1}{10}$ and $\pi_i = \frac{1}{100}$ for each $i \in \mathcal{T}_1$.

Now suppose $P_2 = 10$, so that no consumer of either type buys product 2. The residual demand for product 1 shifts out, and is given by

$$Q_1 \quad = \quad \begin{cases} \frac{11}{10} - P_1 & for \quad P_1 \in [0, 1] \\ \frac{3 - P_1}{20} & for \quad P_1 \in [1, 3] \end{cases}$$

(If $P_1 < 1$, demand is $\frac{1}{10}$ higher than before, since all type-2 consumers buy; if $P_1 > 1$, demand increases from 0 before to whatever share of the type-2 consumers buy.) This time, the combined equilibrium price is $P_1 = 2.7$, giving $Q_1 = \frac{3}{200}$, $p_i = 0.3$ and $\pi_i = \frac{9}{2000}$, a little less than half as much as before.

So by increasing the demand for product 1 initially, the change in P_2 ended up reducing the equilibrium profits of the 9 firms selling its components once they adjusted their prices. The disappearance of the second product did two things: it increased the demand for product 1 at any price (good), but it also changed the nature of the double-marginalization problem, making it more severe (bad). This example was carefully chosen such that the second effect would dominate, leaving profits lower for firms in $\mathcal{T}_1$. (In some sense, the actual profit function $P_1 Q_1$ increased, but the "distorted" profit function $P_1^9 Q_1$ changed *more*, so that the greater loss due to double-marginalization outweighed the gain in profits from weaker competition.)

Of course, this example is not consistent with our setup – it uses two distinct types of consumers, violating the independence of v_1 and v_2 across consumers. It's not clear whether such an example could be found which is consistent with Assumption 1.

B2 Example Showing Log Demand can be Super- Or Submodular with Preferences Over Product Attributes

Suppose there are two products, and consumers differ in two dimensions: willingness to pay (for *either* product) and physical location. The products are at different locations but otherwise identical, and a linear transportation cost is incurred based on the distance between the consumer and the product. Let β_1^l denote consumer l's location, with product 1 located at 0 and product 2 located at 1; and let β_2^l denote consumer l's willingness to pay for either product, gross of transportation cost. This means consumer l gets benefit $-t\beta_1^l + \beta_2^l - P_1$ from product 1, and $-t(1-\beta_1^l) + \beta_2^l - P_2$ from product 2.[1] Any consumer gets payoff 0 from consuming neither. β_1^l and β_2^l are independent, and are both distributed uniformly between 0 and 1. As long as P_1 and P_2 are neither too big nor too far apart, the demand for product 1 is[2]

$$Q_1(P_1, P_2) \quad = \quad \frac{P_2 - P_1 + t}{2t} \cdot \frac{4 - 3P_1 - P_2 - t}{4}$$

This is easily shown to be log-concave in P_1, but it is neither log-supermodular nor log-submodular: the cross-partial $\frac{\partial^2 \log Q_1}{\partial P_1 \partial P_2}$ can be either positive or negative, depending on the values of t, P_1, and P_2.[3]

B3 Extension: Wholesale Pricing Through Bilateral Negotiations

If we think of each product representing a distinct vertical supply chain, the assumption that all firms set prices simultaneously, and that all firms producing components of the same product are inherently symmetric, may seem artificial. That is, it might seem strange to suppose that each of Ford's suppliers for tires, brake pads, and so on get to unilaterally

[1] This maps to the payoff formulation in the text if $x_1 = (-t, 1)$, $x_2 = (t, 1)$, $\xi_1 = 0$, $\xi_2 = -t$, and $\epsilon_k^l = 0$.

[2] Q_1 is the mass of consumers for whom $-t\beta_1^l + \beta_2^l - P_1 > \max\{-t(1-\beta_1^l) + \beta_2^l - P_2, 0\}$. When $|P_1 - P_2| < t$ and $P_1 + P_2 + t < 2$, this is the area of the region of $[0,1]^2$ above the line $\beta_2 = t\beta_1 + P_1$ and to the left of the vertical line $\beta_1 = \frac{t+P_2-P_1}{2t}$, which is the trapezoid with corners $(0, P_1)$, $(0, 1)$, $(\frac{P_2-P_1+t}{2t}, \frac{P_1+P_2+t}{2})$, and $(\frac{P_2-P_1+t}{2t}, 1)$, which has area $\frac{P_2-P_1+t}{2t} \cdot \frac{4-3P_1-P_2-t}{4}$.

[3] If there were a monopolist firm selling each product with zero marginal costs, then at equilibrium prices, prices are strategic substitutes when $t > \frac{2+6\sqrt{3}}{13} \approx 0.953$, and strategic complements otherwise. Thus, in some sense, both cases are empirically relevant.

name prices, which are tacked on to the price of each Ford vehicle; and similarly strange to suppose that Ford cannot use the threat to change suppliers to force narrower margins on its suppliers. Next, I present an extension to the baseline model that addresses these concerns. Each product will be represented by a single dominant (downstream) firm, with full control over the final price of the product, and one or more upstream suppliers. Wholesale prices will be determined via simultaneous bilateral bargaining between the downstream firm and each of its suppliers. (A similar model – with two downstream firms and one upstream supplier for each – is used in Horn and Wolinsky (1988).) Competition across upstream firms to supply a particular downstream firm will still not be explicitly modeled; however, the extent of such competition can be incorporated, at least roughly, by manipulating the bargaining power ascribed to each upstream firm.

Continue to assume that there are K products labeled $1, 2, \ldots, K$. For each product k, suppose firm i_k^0 is the retail seller, and firms $i_k^1, i_k^2, \ldots, i_k^{m_k}$ are suppliers of components to that retailer. For $j \in \{0, 1, \ldots, m_k\}$, let c_k^j denote the marginal cost of firm i_k^j; for $j \in \{1, \ldots, m_k\}$, let p_k^j denote the price charged by firm i_k^j for its components; and let P_k denote the retail price, set by firm i_k^0. We will assume that each wholesale price p_k^j is determined via Nash bargaining between that wholesale supplier and the retailer, so that p_k^j is set as the maximizer of the usual Nash product $(\pi_k^j)^{\phi_k^j}(\pi_k^0)^{1-\phi_k^j}$, where ϕ_k^j is the bargaining power of the wholesale firm i_k^j.

Ideally, we would model this bargaining process as taking place under correct beliefs about the impact the resulting price p_k^j will have on second-stage competition, that is, π_k^j and π_k^0 would be the actual second-stage payoffs of the two firms given the results of first-round negotiations. Unfortunately, given the complex way in which wholesale prices affect subsequent downstream competition, this "full" model is too difficult to work with. Instead, we consider a simpler model, in the spirit of conjectural variations. We will assume that during Nash bargaining over p_k^j, competitors' downstream prices $P_{k'}$ ($k' \neq k$) are assumed to be fixed (at their equilibrium levels), and it is assumed that P_k will increase linearly with p_k^j, with slope $\alpha_k^j \in [0, 1]$.[4] Once wholesale prices have all been determined (simultaneously) in

[4]This allows bargaining under the assumption that P_k is fixed ($\alpha_k^j = 0$), that downstream markup $P_k - p_k^j$

this way, downstream prices P_k are then set to maximize retailer profits given those wholesale prices.

Under these assumptions, prices $(\overline{P}_1, \ldots, \overline{P}_K)$ and $(\overline{p}_k^j)_{k,j}$ constitute an equilibrium if and only if they simultaneously satisfy

$$\overline{P}_k = \arg\max_{P_k} \left\{ \left(P_k - c_k^0 - \sum_{j>0} \overline{p}_k^j \right) Q_k(P_k, \overline{P}_{-k}) \right\}$$

$$\overline{p}_k^j = \arg\max_{p_k^j} \left\{ \begin{array}{l} \left[(p_k^j - c_k^j)\, Q_k\left(\overline{P}_k + \alpha_k^j(p_k^j - \overline{p}_k^j), \overline{P}_{-k}\right) \right]^{\phi_k^j} \times \\ \left[\left(\overline{P}_k + \alpha_k^j(p_k^j - \overline{p}_k^j) - c_k^0 - p_k^j - \sum_{j' \neq j} \overline{p}_k^{j'} \right) Q_k\left(\overline{P}_k + \alpha_k^j(p_k^j - \overline{p}_k^j), \overline{P}_{-k}\right) \right]^{1-\phi_k^j} \end{array} \right\}$$

for each $k \in \mathcal{K}$ and each $j \in \{1, 2, \ldots, m_k\}$. (The former is the profit-maximization problem of downstream firm i_k^0, and the latter is the Nash product $(\pi_k^j)^{\phi_k^j}(\pi_k^0)^{1-\phi_k^j}$, where profits are evaluated at the price $P_k = \overline{P}_k + \alpha_k^j(p_k^j - \overline{p}_k^j)$ and all prices besides P_k and p_k^j are assumed fixed at their equilibrium values.) While these conditions look complex, they lead to a surprisingly straightforward characterization of equilibrium prices, which are a natural analog to the baseline model:

Lemma B1. *Suppose that $\phi_k^j \in (0,1)$ and $\alpha_k^j \in [0,1]$ for every $k \in \mathcal{K}$ and every $j \in \{1, 2, \ldots, m_k\}$. Under Assumption 1...*

1. *The game described above – simultaneous Bertrand-Nash competition in the retail market and bilateral Nash bargaining in the upstream markets, with exogenously fixed (ϕ_k^j) and (α_k^j) – has a unique equilibrium*

2. *Let $\beta_k^j \equiv \frac{\phi_k^j}{\alpha_k^j + (1-\alpha_k^j)(1-\phi_k^j)}$ and $n_k = 1 + \sum_{j=1}^{m_k} \beta_k^j$. Then...*

   (i) Equilibrium retail prices $(\overline{P}_1, \overline{P}_2, \ldots, \overline{P}_K)$ are those described in Lemma 1 – that

is fixed ($\alpha_k^j = 1$), or any intermediate level of responsiveness. In principle, we could attach similar conjectures to the effect of p_k^j on competitor downstream prices $P_{k'}$, and then use fixed-point arguments to look for a point where those conjectures were all correct, indicating an equilibrium of the "full" model. In practice, however, this fixed point would shift with model primitives, and the full model would therefore not give us simple comparative statics; as Theorem B1 below shows, assuming that the α are fixed leads to strong comparative statics results under this more "partial" model.

is, the equilibrium of the K-player game with payoffs $u_k = n_k \log(P_k - C_k) + \log Q_k(P_k, P_{-k})$

(ii) Equilibrium wholesale prices are $\overline{p}_k^j = c_k^j + \frac{\beta_k^j}{n_k}(\overline{P}_k - C_k)$, and the equilibrium markup of the retailer is $\overline{P}_k - \sum_{j>0} \overline{p}_k^j = c_k^0 + \frac{1}{n_k}(\overline{P}_k - C_k)$

Proof of Lemma B1 First, we show that that the two maximization problems defining $\overline{P}_k$ and $\overline{p}_k^j$ above are both log-concave, so that equilibrium is equivalent to a set of first-order conditions. The derivatives the log of the first maximand is $\frac{1}{P_k - c_k^0 - \sum_{j>0} \overline{p}_k^j} + \frac{\partial \log Q_k}{\partial P_k}(P_k, \overline{P}_{-k})$, which is decreasing in P_k. The derivative of the log of the second is

$$\phi_k^j \left[\frac{1}{p_k^j - c_k^j} + \alpha_k^j \frac{\partial \log Q_k}{\partial P_k}(\overline{P}_k + \alpha_k^j (p_k^j - \overline{p}_k^j), \overline{P}_{-k}) \right]$$
$$+ (1 - \phi_k^j) \left[- \frac{1 - \alpha_k^j}{\overline{P}_k + \alpha_k^j (p_k^j - \overline{p}_k^j) - c_k^0 - p_k^j - \sum_{j' \neq j} \overline{p}_k^{j'}} + \alpha_k^j \frac{\partial \log Q_k}{\partial P_k}(\overline{P}_k + \alpha_k^j (p_k^j - \overline{p}_k^j), \overline{P}_{-k}) \right]$$

which is decreasing in p_k^j. So both maximization problems are log-concave, and equilibrium is therefore equivalent to the first-order conditions

$$\frac{1}{\overline{P}_k - c_k^0 - \sum_{j>0} \overline{p}_k^j} = - \frac{\partial \log Q_k}{\partial P_k}(\overline{P}_k, \overline{P}_{-k})$$

$$\frac{\phi_k^j}{\overline{p}_k^j - c_k^j} - \frac{(1 - \phi_k^j)(1 - \alpha_k^j)}{\overline{P}_k - c_k^0 - \sum_{j>0} \overline{p}_k^j} = - \alpha_k^j \frac{\partial \log Q_k}{\partial P_k}(\overline{P}_k, \overline{P}_{-k})$$

Using the first to substitute for the middle term of the second gives

$$\frac{\phi_k^j}{\overline{p}_k^j - c_k^j} = - \left((1 - \phi_k^j)(1 - \alpha_k^j) + \alpha_k^j \right) \frac{\partial \log Q_k}{\partial P_k}(\overline{P}_k, \overline{P}_{-k})$$

Defining $\beta_k^j = \frac{\phi_k^j}{\alpha_k^j + (1 - \phi_k^j)(1 - \alpha_k^j)}$ as in the text, we get

$$\overline{p}_k^j - c_k^j = \frac{\beta_k^j}{-\partial \log Q_k / \partial P_k}$$

and $\overline{P}_k - c_k^0 - \sum_{j>0} \overline{p}_k^j = \frac{1}{-\partial \log Q_k / \partial P_k}$. Summing over $j \geq 0$, the wholesale prices cancel and we get

$$\overline{P}_k - C_k = \frac{1 + \sum_{j>0} \beta_k^j}{-\partial \log Q_k / \partial P_k}$$

Defining $n_k = 1 + \sum_{j>0} \beta_k^j$, this is $\frac{\overline{P}_k - C_k}{n_k} = \left(-\frac{\partial \log Q_k}{\partial P_K}\right)^{-1}$, which is equivalent to (A3), so equilibrium values of $(\overline{P}_1, \ldots, \overline{P}_K)$ are unique and as characterized in Lemma 1; and $\overline{p}_k^j - c_k^j = \beta_k^j \Big/ \left(-\frac{\partial \log Q_k}{\partial P_k}\right) = \beta_k^j \frac{\overline{P}_k - C_k}{n_k}$, completing the proof. □

As a result of the similarity between the equilibrium characterizations of this model and the baseline model, most of the previous results apply here as well:

Theorem B1. *Under Assumptions 1 and 2,*

1. *An increase in the marginal cost c_k^j of firm i_k^j leads to higher retail prices for all products, lower demand Q_k for product k, and lower profits for all firms $i_k^{j'}$. Under Assumption 3, it also leads to greater demand for all other products $Q_{k'}$ and higher profits for all firms $i_{k'}^{j'}$ ($k' \neq k$).*

2. *A quality increase to product k leads to a higher retail price P_k, higher demand Q_k, and higher profits for each firm i_k^j ($j \in \{0, 1, \ldots, m_k\}$), and lower prices $P_{k'}$ for all other products $k' \neq k$. Under Assumption 3, it also leads to lower demand $Q_{k'}$ and lower profits for firms $i_{k'}^{j'}$ for $k' \neq k$. The introduction of a new product has these same effects on all existing products.*

3. *A merger between a retail firm i_k^0 and one of its suppliers i_k^j, provided it does not affect the bargaining position ($\phi_{k'}^{j'}$ and $\alpha_{k'}^{j'}$) of other firms, leads to lower retail prices for all products, greater demand for product k, and higher profits for non-merging firms $i_k^{j'}$ ($j' \neq 0, j$). Under Assumption 3, it also leads to lower demand for all other products $Q_{k'}$ and lower profits for all firms $i_{k'}^{j'}$ ($k' \neq k$).*

4. *A merger between two complementary suppliers i_k^j and $i_k^{j'}$ ($j \neq j' \neq 0$) has the same effects, provided the bargaining power of the merged firm is such that $\beta_k^{j,j'}$ post-merger is no greater than the pre-merger sum $\beta_k^j + \beta_k^{j'}$.*

In light of Lemma B1, the proofs are identical to those of Theorems 2, 3, and 4. The baseline model can therefore be thought of as a reduced form for a more complex model involving bilateral negotiation between upstream suppliers and downstream retailers, who then compete in prices.

B4 Extension: Essential Components

On Assumption 4

Since an increase in P^E affects demand in exactly the same way as equal increases in each of the prices P_k^N, it's straightforward to show that if $\frac{\partial \log Q_A}{\partial P^E}$ is decreasing in P_k^N for every $k \in \mathcal{K}$, it's decreasing in P^E. Let F^* be the CDF of $\max_{k\in\mathcal{K}}\{v_k^l - P_k^N\}$. Since $F^*(t) = \prod_{j\in\mathcal{K}} F_j(t+P_j^N)$, differentiating and rearranging gives

$$\frac{f^*(t)}{1-F^*(t)} = X(t)\left[\frac{F_k(t+P_k^N)\sum_{j\neq k}\frac{f_j(t+P_j^N)}{F_j(t+P_j^N)}}{1-F_k(t+P_k^N)X(t)} + \frac{f_k(t+P_k^N)}{1-F_k(t+P_k^N)X(t)}\right] \tag{B1}$$

where $X(t) = \prod_{j\neq k} F_j(t+P_j^N)$. If (B1) is increasing in P_k^N, then $1-F^*(t)$ is log-submodular in (t, P_k^N); then, similar to the proof of Theorem 1, the distribution F^{**} of $\max_{k\in\mathcal{K}}\{v_k^l - P_k^N\} - v_0^l$ inherits the same property, which would make $Q_A = 1 - F^{**}(P^E)$ log-submodular in P^E and P_k^N. Thus, a sufficient condition for Assumption 4 is for (B1) to be increasing in P_k^N.

Now, $X(t)$ is a constant with respect to P_k^N. Examining the first term in the square brackets, the numerator is increasing in P_k^N and the denominator is decreasing, so the first term is increasing. Examining the second term, the denominator is decreasing in P_k^N, but the numerator could be increasing or decreasing. Further, we've already assumed that $\frac{f_k}{1-F_k}$ is increasing, so without the extra $X(t)$ term in the denominator, the entire second term would be increasing and we'd be done. But Assumption 1 does not guarantee that $\frac{f_k(t+P_k^N)}{1-F_k(t+P_k^N)\prod_{j\neq k}F_j(t+P_j^N)}$ is increasing in P_k^N. So we explicitly assume what we need – that $\frac{\partial \log Q_A}{\partial P^E}$ is increasing in each P_k^N, which also implies increasing in P^E – and move on.

Equilibrium Characterization With Essential Components

In addition to the notation defined in the text, let $n_E = |\mathcal{T}^E|$ be the number of essential components and $C^E = \sum_{i \in \mathcal{T}^E} c_i$ their combined marginal costs, and likewise let $n_k = |\mathcal{T}_k^N|$ be the number of non-essential components required for product k and $C_k^N = \sum_{i \in \mathcal{T}_k^N} c_i$ their combined marginal cost. As noted in the text, let $Q_A = \sum_{k \in \mathcal{K}} Q_k$.

Lemma B2. *Under Assumptions 1, 2 and 4, the simultaneous-move pricing game with essential components has a unique equilibrium. Firm i sets equilibrium price*

$$p_i = \begin{cases} c_i + \frac{1}{n_k}\left(\overline{P}_k^N - C_k^N\right) & if \quad i \in \mathcal{T}_k^N \\ c_i + \frac{1}{n_E}\left(\overline{P}^E - C^E\right) & if \quad i \in \mathcal{T}^E \end{cases}$$

where $\left(\overline{P}_1^N, \ldots, \overline{P}_K^N, \overline{P}^E\right)$ is the unique equilibrium of a different, $K+1$ player game with players $\mathcal{K} \cup \{E\}$ and payoff functions[5]

$$u_k(\cdot) = \begin{cases} n_k \log(P_k^N - C_k^N) + \log Q_k(\cdot) & for \quad k \in \mathcal{K} \\ n_E \log(P^E - C^E) + \log Q_A(\cdot) & for \quad k = E \end{cases}$$

Further, this latter game is a supermodular game in $\left(P_1^N, \ldots, P_K^N, -P^E\right)$, indexed by C_k^N and n_k ($k \in \mathcal{K}$) and $-C^E$ and $-n_E$.

Proof. As in the proof of Lemma 1, the best-responses of all the firms in $\mathcal{T}_k^N$ collectively solve $\frac{n_k}{P_k^N - C_k^N} = -\frac{\partial \log Q_k}{\partial P_k^N}$, and, by the same logic, the best-responses of the firms in $\mathcal{T}^E$ collectively solve

$$\frac{n_E}{P^E - C^E} = -\frac{\partial \log Q_A}{\partial P^E} \tag{B2}$$

These are the first-order conditions to the problems $\max_{P_k^N} \left\{n_k \log(P_k^N - C_k^N) + \log Q_k\right\}$ and $\max_{P^E} \left\{n_E \log(P^E - C^E) + \log Q_A\right\}$. Under Assumption 1, as before, the former is concave; under Assumption 4, the latter is concave as well; so solutions to these $K+1$ problems correspond to the first-order conditions, and so the equilibria of this latter $K+1$-player game

[5]Again, to be complete, u_k is defined as $-\infty$ for $P_k^N \leq C_k^N$, and u_E as $-\infty$ for $P^E \leq C^E$.

correspond to equilibrium prices in the original pricing game. As before, the former problem has increasing differences in $(P_k^N, P_{k'}^N)$ for $k' \neq k$. Since an increase in P^E is equivalent to the same decrease in P_0 (the imaginary price of the outside option) and $\frac{n_k}{P_k^N - C_k^N} + \frac{\partial \log Q_k}{\partial P_k^N}$ is increasing in the prices of every alternative to k, it is decreasing in P^E; we explicitly assume $\frac{n_E}{P^E - C^E} + \frac{\partial \log Q_A}{\partial P^E}$ is decreasing in P_k^N, which together make the new game a supermodular game when the sign of P^E is reversed.

To show equilibrium existence, first note that by supermodularity, the best-response for player E is bounded above by his best-response to $P_1^N = \ldots = P_K^N = 0$. The best-response for player $k \neq E$ is bounded above by his best-response to $P^E = 0$ and $P_{k'}^N = \infty$ for $k' \neq k$, which, as argued in the proof of Lemma 1, is finite. Player E's best-response to the upper bounds on each P_k^N gives a lower bound above C^E, and player k's best-response to the upper bound on P^E, along with $P_{k'}^N = 0$ for $k' \neq k$, gives a lower bound above C_k^N. So we have a continuous, supermodular game on a bounded strategy space, so equilibrium existence is guaranteed.

The uniqueness proof is likewise similar to that in Lemma 1. Suppose there were two equilibria, $\left(\overline{P}_1^N, \ldots, \overline{P}_K^N, \overline{P}^E\right)$ and $\left(\tilde{P}_1^N, \ldots, \tilde{P}_K^N, \tilde{P}^E\right)$. We treat two cases separately.

First, suppose

$$\left|\tilde{P}^E - \overline{P}^E\right| \geq \max_{k \in \mathcal{K}} \left|\tilde{P}_k^N - \overline{P}_k^N\right|$$

and assume without loss that $\tilde{P}^E > \overline{P}^E$. This means the overall price $P^E + P_k^N$ of each product k is weakly higher at the second equilibrium, so $\frac{\partial Q_A}{\partial P^E}$ is weakly lower. Since $\tilde{P}^E > \overline{P}^E$, $\frac{n_E}{\tilde{P}^E - C^E} < \frac{n_E}{\overline{P}^E - C^E}$, so the first-order condition (B2) cannot hold at both equilibria.

For the second case, $\left|\tilde{P}^E - \overline{P}^E\right| < \max_{k \in \mathcal{K}} \left|\tilde{P}_k^N - \overline{P}_k^N\right|$, fix $k \in \arg\max_{j \in \mathcal{K}} \left|\tilde{P}_j^N - \overline{P}_j^N\right|$, and assume without loss that $\tilde{P}_k^N > \overline{P}_k^N$. By assumption, $\tilde{P}_k^N - \overline{P}_k^N > \tilde{P}^E - \overline{P}^E$, so the price of product k is strictly higher at the second equilibrium; and $\tilde{P}_k^N - \overline{P}_k^N \geq \tilde{P}_{k'}^N - \overline{P}_{k'}^N$, so it's gone up by at least as much as any other price. By the same logic as in the proof of Lemma 1, this means $\frac{\partial \log Q_k}{\partial P_k^N}$ is lower at the second equilibrium; $\tilde{P}_k^N > \overline{P}_k^N$ implies $\frac{n_k}{\tilde{P}_k^N - C_k^N} < \frac{n_k}{\overline{P}_k^N - C_k^N}$, so the first-order condition $\frac{n_k}{P_k^N - C_k^N} = -\frac{\partial \log Q_k}{\partial P_k^N}$ cannot hold at both equilibria.

Finally, note that $\frac{n_k}{P_k^N - C_k^N} + \frac{\partial \log Q_k}{\partial P_k^N}$ is increasing in n_k and C_k^N and $\frac{n_E}{P^E - C^E} + \frac{\partial \log Q_A}{\partial P^E}$ is increasing in n_E and C^E, so the $K+1$-player supermodular game characterizing equilibrium prices is indexed by n_k, C_k^N, $-n_E$, and $-C^E$.

Proof of Theorem 5

Part 1. Since the $K+1$-player supermodular game in Lemma B2 is indexed by C_k^N, the drop in price c_i for $i \in \mathcal{T}_k^N$ leads to an increase in P^E and a decrease in $P_{k'}^N$ for all $k' \in \mathcal{K}$. Following the logic of the uniqueness proof in Lemma B2, if $\max\left\{\Delta P^E, \max_{k' \neq k}\left|\Delta P_{k'}^N\right|\right\} \geq \left|\Delta P_k^N\right|$, then the first-order condition for either P^E or $\arg\max_{k' \neq k}\left|\Delta P_{k'}^N\right|$ cannot hold both before and after the change; which means that both $-\Delta P_k^N > \Delta P^E$ and $\left|\Delta P_k^N\right| > \left|\Delta P_{k'}^N\right|$, meaning $\Delta P_k < 0$ and $|\Delta P_k| > |\Delta P_{k'}|$. Quint (2014) offers examples of mergers (using logit demand, which satisfies all our assumptions) where a competing product's price $P_{k'} = P_{k'}^N + P^E$ can go up or down, and the total welfare effect can be positive or negative.

Since P_k fell, and fell by more than any other price, $\frac{g_k(P_k)}{1-G_k(P_k)}$ (the hazard rate of the distribution G_k defined in the proof of Theorem 1) must be lower than before, so $p_j - c_j$ must be higher for any $j \in \mathcal{T}_k^N - \{i\}$ for the first-order condition to still hold. Since P_k fell, and fell more than any other price, Q_k must rise, so firms $j \in \mathcal{T}_k^N - \{i\}$ have higher prices and higher demand, hence higher profits. As for a merger, the same logic holds, since the $K+1$-player supermodular game in Lemma B2 is indexed by n_k, which is effectively decreased by 1 by a merger. As before, a new product is like a reduction in costs C_k^N from $+\infty$ to a new finite level. As in the proof of Theorem 2, a right-shift in the distribution of v_k^l can be seen as a reduction in C_k^N when the "quality-adjusted" price $\tilde{P}_k^N = P_k^N - \Delta v$ is used, so the results are the same.

Part 2. Since the supermodular game in Lemma B2 is indexed by $-C^E$, lower costs mean lower P^E and higher P_k^N for every k, and by the same logic as before, $\left|\Delta P^E\right| > \Delta P_k^N$ for every k; the results follow via the same steps as part 1. Similarly, since the supermodular game is indexed by $-n_E$, the same results follow for a merger between two firms in $\mathcal{T}^E$.