Looking Smart versus Playing Dumb in Common-Value Auctions[1]

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Abstract. I compare the value of information acquired secretly to information acquired openly prior to a first-price common-value auction. Novel information (information which is independent of the other bidder's private information) is more valuable when learned openly, but redundant information (information the other bidder already has) is more valuable when learned covertly. In a dynamic game where a bidder can credibly signal he's well-informed without disclosing the content of his information, always signaling having novel information, and never signaling having redundant information, is consistent with (but not uniquely predicted by) equilibrium play. Full revelation of any information possessed by the seller increases expected revenue and is uniquely predicted in equilibrium.

Keywords: first price auctions; common values; endogenous information; signaling
JEL Classification: D44 (Auctions)

1 Introduction

For years, auctions have been used to allocate natural resources, such as the mineral rights to offshore oil and natural gas deposits – resources whose ex-post value is the same to any buyer, but highly uncertain at the time of the auction. Many of these auctions have been for *drainage tracts* – land adjacent to another tract already in use, where the lessee of this neighboring tract could be expected to have more accurate information than his competitors about the likely output of the land being auctioned.

Milgrom and Weber (1982a) analyze the value of information using the "drainage tract model" introduced by Wilson (1967) – a common-value, first-price (sealed-bid) auction in which one bidder has private information while the others have access only to the same public information. They find that the informed bidder gains more by improving his information about the value of the land openly (so that his competitors know he is better informed); while the uninformed bidders only gain by acquiring private information secretly. A plausible conclusion, then, might be that more generally, bidders at an informational disadvantage benefit from secrecy when acquiring new information, while bidders at an informational advantage do not.

This conclusion, however, turns out to be incorrect. When the model is extended to allow for two bidders with distinct private information, I find that it is not whose information is more accurate, but the nature of the incremental information itself, that determines whether secrecy is beneficial. In particular, regardless of whose information is "better" at the start,

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- a bidder gets more benefit from *novel* information (information which is independent of his opponent's private information) when it was learned *openly*
- a bidder gets more benefit from *redundant* information (information which his opponent already has) when it was learned *secretly*

Thus, a bidder with novel information benefits from "looking smart" (being known to possess it), while a bidder with redundant information benefits from "playing dumb" (being thought not to have it). (There is no conflict between these results and those in Milgrom and Weber (1982a); when my opponent is uninformed, *any* additional information I learn is novel; when he is perfectly informed, any information is redundant.)

Armed with these comparative statics, I then analyze equilibrium play in a game where a bidder who gains "unexpected" information prior to the auction has a chance to signal his opponent that he's done so. I find that always signaling upon receiving novel information, and never signaling upon receiving redundant information, is consistent with (but not uniquely predicted by) equilibrium play.

I also examine the incentives of the seller to reveal whatever information he has about the value of the item up for sale. When bidders are symmetric, this is known to increase the seller's ex-ante revenue (the Linkage Principle shown in Milgrom and Weber (1982b)). Despite the asymmetry among bidders here, the same result holds; in addition, due to unraveling, the seller will fully reveal any information he is known to have in equilibrium.

2 Related Literature

My results are closely connected to those in Milgrom and Weber (1982a), who analyze auctions where only one bidder has private information; I generalize many of their results to the case of two bidders, both of whom are privately informed. (Lemma 1 and Theorems 1, 2, and 5 of this paper are the same results as Theorems (1 and 2), 4, 5, and (6 and 7) of Milgrom and Weber (1982a), respectively, but in a model where *both* bidders, rather than just one, have private information to start with.)

Equilibrium in the drainage tract auction was analyzed in Engelbrecht-Wiggans, Milgrom and Weber (1983), who proved the striking result (verified empirically by Hendricks, Porter and Wilson (1994)) that the equilibrium bids of the informed and uninformed bidders would have the same probability distribution. Parreiras (2006) considers more general common-value auctions with affiliated signals, and shows that this same symmetry occurs when the bidders' signals are independent, a result I rely on. (Hafalir and Krishna (2006) show the same symmetry holds in a two-bidder asymmetric private-value auction with resale, as the possibility of resale induces common values when bids are sufficiently close.)

Aside from Milgrom and Weber (1982a), nearly all papers on information acquisition in auctions assume either covert or observable information-gathering, rather than comparing the two. Closest to this paper, Larson (2009) considers observable information in the case of two-bidder, second-price common-value auctions, using small private-value perturbations to solve the problem of equilibrium multiplicity. He finds that when each incremental signal is equally informative, a bidder gains more from redundant (he uses the term "duplicative") information than from novel ("expansive") information when he is already much better-informed than his rival, but gains more from novel information otherwise. He also finds that when there is sufficient asymmetry in the bidders' information, the seller gains more by supplying the less-informed bidder with a new signal than from exposing one of the better-informed bidder's signals.[3]

Hernando-Veciana (2009) also studies observable information-gathering, but in a setting with additively separable common and private values. He shows that when incremental information is about the common-value component, a sealed-bid second-price auction induces more information gathering than an ascending (open) auction; when incremental information is about the private-value component, the reverse is true. Persico (2000) considers covert information-gathering, and shows that first-price auctions induce more information gathering than second-price auctions when signals and values are affiliated. Rezende (2005) considers covert information-gathering that occurs in the middle of a private-value ascending auction, and shows that dynamic ascending auctions yield greater expected revenue than a "one-shot" (sealed-bid) second-price auction as the number of bidders gets large. Hausch and Li (1993) endogenize both entry and information gathering (with neither one being observable) in a simple common-values setting, and show that a first-price auction induces both more entry and demand for more precise information than a second-price auction.

Benoit and Dubra (2006) consider the incentive to reveal one's private information after it is received. In a variety of auction models (though not in a pure common values one), they demonstrate an "unraveling"-type result wherein a player will reveal all of his information in equilibrium, even though this lowers his ex-ante expected payoff. (In my signaling model, a bidder with "unexpectedly precise" information faces the question of whether to reveal *having* the better information, not whether to reveal its content.) Hernando-Veciana and Troge (2005) show that, in a setting with common and private values, a bidder who is very well-informed about the common component may benefit ex-ante by revealing that part of his information.

(Papers related to the seller's incentives to disclose information are discussed at the start of Section 6.)

3 Model

I consider a two-bidder setting with pure common values where the ex-post value of the object is additively separable into two components,

$$V = V_1 + V_2 \tag{1}$$

with $V_1, V_2 \geq 0$. Information about V_1 comes in the form of two related signals X and Y; similarly with V_2 and two related signals Z and W. The joint probability distributions of (V_1, X, Y) and (V_2, Z, W) are common knowledge, and have bounded support. In addition, I make a crucial assumption which is maintained throughout the paper:[4]

Assumption 1. (V_1, X, Y) *is statistically independent of* (V_2, Z, W).

[3] Mares and Harstad (2003) use a different common-values model to show that even with symmetric bidders, the seller can sometimes gain in a second-price auction by revealing new private information to one of them. This highlights a difference between first- and second-price auctions: as I show below, in a *first*-price auction with independent signals, each bidders' ex-ante payoff is a functions of only his own private information. This means that in a first-price auction, publicly learning redundant information has no value, and the seller could never gain by giving new private information to one of the bidders. Campbell and Levin (2000) examine a setting similar to mine – a first-price common-value auction – with signals which take discrete values. They show one case where *both* bidders benefit ex-ante from the better-informed bidder knowing less.

[4] Additive valuations and independent signals is of course a restrictive environment. Parreiras' (2006) characterization of equilibrium under general affiliated signals suggests that equilibrium strategies (and therefore payoffs) should not change discontinuously when a small amount of statistical dependence is introduced; by continuity, results that hold strictly under these assumptions should therefore still hold when they are relaxed slightly. Bulow, Huang and Klemperer (1999), Bulow and Klemperer (2002), and Larson (2009) make similar assumptions of additive separability and independence.

I begin by characterizing equilibrium play and payoffs in a first-price (sealed-bid) auction. Let $S_1 \subseteq \{X, Y\}$ and $S_2 \subseteq \{Z, W\}$, and assume it is common knowledge that each bidder i knows the realization of the signals in S_i. Define player i's **type** t_i as

$$t_i \equiv E(V_i|S_i) \tag{2}$$

Let F_{t_i} be the distribution of t_i, and assume that if F_{t_i} is nondegenerate, it is massless.[5]

Lemma 1. *The first-price auction has a unique equilibrium, in which*

1. *The probability distributions of the two bidders' equilibrium bids are identical*
2. *Player i's expected payoff given type $\hat{t}_i$ is*

$$\pi_i(\hat{t}_i) = \int_0^{\hat{t}_i} F_{t_i}(s)ds \tag{3}$$

3. *Player i's ex-ante expected payoff is*

$$\pi_i = \int_0^{\infty} F_{t_i}(s)(1 - F_{t_i}(s))ds \tag{4}$$

4. *For the case where F_{t_i} is nondegenerate,[6] player i's equilibrium bid given type $\hat{t}_i$ is*

$$\beta_i(\hat{t}_i) = E(t_i|t_i < \hat{t}_i) + E(t_j|t_j < F_{t_j}^{-1}(F_{t_i}(\hat{t}_i))) \tag{5}$$

Outline of Proof. Symmetry follows from the first-order conditions characterizing the two players' equilibrium strategies. If β_i is player i's equilibrium strategy and G_i the distribution of $\beta_i(t_i)$, then at any bid b in the range of both players' equilibrium strategies, the first-order conditions of the two bidders' payoff functions yield

$$\frac{G_1'(b)}{G_1(b)} = \frac{1}{\beta_1^{-1}(b) + \beta_2^{-1}(b) - b} = \frac{G_2'(b)}{G_2(b)} \tag{6}$$

from which symmetry can be proved. Equations 3, 4, and 5 follow from the envelope theorem, along with the fact that, given symmetry, bidder i with type $\hat{t}_i$ wins with probability $F_{t_i}(\hat{t}_i)$. $\square$

Equations 3 and 4 imply that each player's expected payoff depends only on the informativeness of his own signal, not that of his opponent. That is, if I am player 1, my expected profit is the same in equilibrium against an opponent who knows Z, an opponent who knows $\{Z, W\}$, and an opponent who knows nothing. Below, this will help to isolate the effect of new information on bidders' payoffs.

With a minor change,[7] Lemma 1 holds as well for the "hybrid auction" considered in Parreiras (2006), where the winning bidder pays a weighted-average $\epsilon b_{(1)} + (1 - \epsilon) b_{(2)}$ of the two bids with

[5]If F_{t_i} has an atom above the bottom of its support, the procedure in Engelbrecht-Wiggans et. al. (1983) can be used to "expand" the atom using an additional, uninformative signal. The results extend without a problem, but require additional notation; I present the case of massless distributions for expositional clarity. F_{t_i} degenerate for one bidder corresponds to the drainage tract model, and my results agree with those in Engelbrecht-Wiggans et. al. (1983) and Milgrom and Weber (1982a). If F_{t_i} is degenerate for both bidders, both bid $E(V)$.

[6]When F_{t_i} is degenerate, bidder i bids $E(V_i) + E(t_j|t_j < F_{t_j}^{-1}(x_i))$, where x_i is an uninformative randomizing signal drawn from the uniform distribution on $[0, 1]$.

[7]The right-hand side of Equation 5 now refers to player i's expected payment upon winning, not his bid.

$0 < \epsilon \leq 1$; the informational results below therefore hold for these auctions as well. The second-price auction (or the case $\epsilon = 0$) is difficult to analyze because with pure common values, it admits a severe multiplicity of equilibria, including some "collusive" equilibria where one bidder always wins the auction at a very low price.[8] Hybrid auctions with $\epsilon \in (0, 1]$ each have a unique equilibrium; the limit of these equilibrium strategies as $\epsilon \searrow 0$ is *an* equilibrium of the second-price auction. If we assume that this is the equilibrium played, then the results extend to the second-price auction as well (although Theorem 2 only holds weakly for the second-price auction), and therefore to ascending auctions, which are strategically equivalent when there are two bidders.

4 Static Information Results

Below, I consider whether information about V is more valuable to a bidder when it is gained covertly (in secret) or openly (so that the bidder's access to that information is common knowledge). In brief, information which is independent of your opponent's private information is always more valuable when it is common knowledge that you have learned it; but information which is a subset of your opponent's private information is on average more valuable when it is gained in secret. (In the next section, I use these static results to analyze the equilibria of a dynamic game in which a player gaining "unexpected" information prior to the auction has the option to signal his opponent that he has received this information.)

4.1 Novel Information

First, I examine the effect of player 1 sharpening the information he has about V_1. The additional information about V_1 is independent of bidder 2's information; I therefore refer to it as "novel" information.[9] As before, let $S_i \subseteq \{X, Y, Z, W\}$ refer to the signals observed by player i prior to the auction.

Theorem 1. *Let $S_2 = \{Z, W\}$ be common knowledge. At a realization (x, y) of (X, Y), let*

- *$A(x, y)$ be player 1's expected payoff in the auction where $S_1 = \{X\}$ is common knowledge*
- *$B(x, y)$ be player 1's expected payoff if player 2 bid as in the previous auction, but player 1 observed X and Y and bid optimally*
- *$C(x, y)$ be player 1's expected payoff in the auction where $S_1 = \{X, Y\}$ is common knowledge*

Then $C(x, y) \geq B(x, y) \geq A(x, y)$ for every (x, y).

If in addition the support of $E(V_1|X)$ is convex and the distribution of $E(V_1|X = x, Y)$ is massless for almost every x, then $C(x, y) > B(x, y) > A(x, y)$ for almost every (x, y).

$B(x, y) \geq A(x, y)$ is the familiar result that information gained secretly cannot be harmful, since at worst it could be ignored. $C(x, y) \geq B(x, y)$ reflects the fact that when player 1 is known to be better-informed in this way, player 2 faces a more severe winner's curse, and therefore bids more cautiously.

Theorem 1 means that when a player gains access to novel information he was not expected to have, he has no reason to conceal knowing it; on the contrary, in addition to the benefit of actually being better-informed, he benefits from *appearing* to be better-informed. In a later section, I examine equilibrium behavior in a multi-stage game where player 1 has the ability to signal to

[8] Larson (2009) discusses this problem, and offers one solution to it.
[9] Larson (2009) uses the term "expansive."

player 2 that he has observed y. (Since player 1 gains from appearing better informed even when he is not, it is important that this signal be verifiable in some way.)

4.2 Redundant Information

Next, I examine what happens when player 1 gains access to part of player 2's private information.[10] I first define a technical condition for the signal Z to be useful in the absence of the signal W:

Definition 1. *Z is* **meaningful on its own** *if it is nondegenerate and the probability distribution of $E(V_2|Z = z, W)$ is different for each realization z of Z.*[11]

Theorem 2. *Let $S_2 = \{Z, W\}$ be common knowledge. Let*

- *D be player 1's ex-ante expected payoff when $S_1 = \{X\}$ is common knowledge*
- *E be player 1's ex-ante expected payoff if player 2 bid as in the previous auction, but player 1 learned X and Z and bid optimally*
- *F be player 1's ex-ante expected payoff when $S_1 = \{X, Z\}$ is common knowledge*

Then $E \geq F = D$. If Z is meaningful on its own, then $E > F$.

When player 2 already knows Z, player 1 prefers to observe it covertly rather than openly. This is in contrast to Theorem 1, which said that when nobody knows Y, player 1 prefers to observe it openly. Theorem 2 follows from the fact that in expectation, player 1 does not gain anything from Z becoming common knowledge – Lemma 1 implies that both D and F depend only on the distribution of $E(V_1|X)$. Unlike Theorem 1, however, Theorem 2 is only an ex-ante result; in some settings, there are signal realizations (x, z) where player 1 does better when he learns Z openly.

Example 1. *Suppose $V_1 = X \sim U[0, X^*]$ and $V_2 = Z \sim U[0, 1]$.*

- *If $X^* \leq 1$, bidder 1 always prefers to learn Z covertly*
- *If $X^* > 1$, bidder 1 prefers to learn Z publicly if and only if*

$$z \geq \frac{1}{X^* - 1}\left(2x - \frac{x^2}{X^*}\right) \tag{7}$$

The right-hand side of equation 7 is increasing in x; when x is low (below $X^* - \sqrt{X^*}$) and z is sufficiently high, player 1 does better by learning z publicly; the rest of the time, he does better by learning it secretly.[12]

In the next section, I consider equilibrium play in a game where player 1 has the option to signal player 2 that he has learned Z prior to the auction.

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[10] Larson (2009) refers to this as "duplicative information."

[11] Similarly, Y is meaningful on its own if it is nondegenerate and the distribution of $E(V_1|Y = y, X)$ is different for each realization y. An example of how information can be valuable without being meaningful on its own: suppose $V_2 = 5 + zw$, $W \perp Z$, $W \sim U[-5, 5]$, and Z takes the values $\{-1, 1\}$ with equal probability.

[12] The intuition here: when $X^* > 1$ and z is high enough relative to x, bidder 2's equilibrium bid $\frac{1+X^*}{2}z$ is above the value of the object $x + z$; so even when bidder 1 knows V and anticipates his opponent's bid, he cannot earn a positive payoff. Learning z publicly leaves player 1 as the informed bidder in a drainage-tract auction, earning strictly positive expected payoff as long as $x > 0$.

5 Dynamic Results

Theorem 2 established that when bidder 2 already knows Z, bidder 1, on average, would prefer to learn Z in secret; Example 1, however, showed that for some signal realizations in some settings, bidder 1 would prefer to learn Z publicly. Of course, in these latter cases, bidder 1 could simply announce the realization of Z, demonstrating to 2 that he knew it. However, once this is acknowledged as a possibility, we would need to consider what bidder 2 would infer about X should such an announcement be made; and indeed, whether he might infer something about X should no such announcement be made.

Similarly, Theorem 1 established that when bidder 1 learns Y, he always prefers for his opponent to know he has learned it. One might therefore wonder whether he could credibly alert bidder 2 that this had occurred, without revealing the realization of Y. However, bidder 1 would benefit from convincing his opponent he knew Y even when he did not. In addition, even if such a signal were credible, we might wonder whether bidder 1 always revealing knowing Y was the only outcome we could expect.

I attempt to answer these questions by analyzing equilibrium play in a pair of dynamic games in which, prior to the auction, player 1 has the opportunity to credibly (verifiably) signal to his opponent that he has received additional information.

5.1 Novel Information

Consider the following dynamic game Γ_1:

1. Bidder 2 observes (Z, W). Bidder 1 observes X, and with probability $p > 0$ observes Y as well. (Whether 1 learns Y is independent of (V_1, X, Y, V_2, Z, W).)
2. *If and only if* player 1 observed Y, he can choose to send a message indicating that he has observed Y.
3. The auction is held.

The entire setup (including p) is common knowledge; the equilibrium concept is perfect Bayesian equilibrium. Note that it is important the signal be verifiable; player 1 must have a way to credibly demonstrate he's learned Y without revealing its value. Like many signaling games, there are multiple equilibria:[13]

Theorem 3. *Among the equilibria of the game Γ_1 are:*

- *An equilibrium in which bidder 1 sends the signal whenever he learns Y*
- *An equilibrium in which bidder 1 never sends the signal*
- *Under a further regularity condition,[14] an equilibrium in which bidder 1 mixes between signaling and not signaling at every realization (x, y) of (X, Y)*

In terms of ex-ante expected payoffs, the equilibrium in which the signal is never sent weakly Pareto-dominates all other equilibria.

However, conditional on having observed Y, bidder 1 would strictly prefer to be in a different equilibrium.

[13] A special case examined in an earlier version of this paper – $p = 0$ and X uninformative – has a continuum of equilibria, with the probability a signal is sent when bidder 1 learns Y varying continuously from 0 to 1.

[14] Specifically, if f_{XY}/f_X is bounded away from 0 on the support of f_X, where f_X is the distribution of $E(V_1|X)$ and f_{XY} the distribution of $E(V_1|X, Y)$.

In fact, provided that bidder 2 correctly anticipates bidder 1's signaling policy, bidder 2 is indifferent among all possible signaling policies, equilibrium or not; and bidder 1 weakly prefers the ex-ante expected payoffs from never signaling to any other policy, equilibrium or not. (Since never signaling is supported as an equilibrium, bidder 1 cannot gain further from the ability to commit ex-ante to a non-equilibrium signaling policy.)

However, conditional on having observed Y, bidder 1 does strictly better in the always-signal than in the never-signal equilibrium. This introduces a sort of dynamic inconsistency for player 1. Before knowing whether he'll learn Y, he would commit to keep it a secret; once he learns Y, he'd prefer to let this be known, but may be constrained by the inference 2 would make about the value of (x, y). (If 1 could send a signal after learning he was to know $\{X, Y\}$ but before observing the realization – so that 2 could not infer anything about (x, y) – he would always choose to do so, even though ex ante he would commit not to.)

5.2 Redundant Information

Next, I examine player 1's incentive to reveal knowing Z when this was expected to occur with some probability. Consider the following game, Γ_2:

1. Player 2 observes (Z, W). Player 1 observes X, and with probability $p > 0$, observes Z as well.
2. If and only if player 1 observed Z, he can choose to send a message indicating that he has observed Z.
3. The auction occurs as before.

Unlike in Γ_1, verifiability of the message is not important, since bidder 1 can simply announce the value z to show that he has learned it. We still must consider what bidder 2 would infer about X based on whether the message is or is not sent.

Theorem 4. *There is an equilibrium of Γ_2 in which player 1 never reveals learning Z.*

If Z is meaningful on its own, there is **no** *equilibrium of Γ_2 in which player 1 always signals that he has learned Z.*

Like Γ_1, however, Γ_2 may have other equilibria as well. To give an idea of the range of possible outcomes, consider the following example.

Example 2. *Let $V_1 = X \sim U[0, X^*]$ and $V_2 = Z \sim U[0, 1]$, consider the limit case $p = 0$, and assume bidder 1 wins ties.*[15]

- *If $X^* < 1$, then there is no equilibrium in which bidder 1 signals with positive probability.*
- *On the other hand, let $X^* = 2$. Choose any (not necessarily continuous) function $s : [0, 1] \to [0, 1]$. Then there exists an equilibrium of Γ_2 in which bidder 1, after learning $(X, Z) = (x, z)$, signals that he's learned Z if and only if*

  $$\tfrac{1-s(z)}{2} z \quad \leq \quad x \quad \leq \quad \tfrac{1+s(z)}{2} z$$

  That is, for each z, there's an interval of x, centered around $\frac{1}{2}z$, on which the signal is sent; and the width of this interval can be set independently for each z.[16]

[15] Without this tiebreaking rule, there is an "openness problem": bidder 1 will have no best-response in cases where he does not send the signal and wants to outbid his opponent.

[16] Up to signaling policies differing on a measure-zero subset of X for each z, these are the only equilibria. Since

6 The Seller's Problem

Finally, I consider the seller's incentive to publicize whatever information he may have. Milgrom and Weber (1982b) established the Linkage Principle – that in a symmetric, affiliated setting, the seller maximizes ex-ante expected revenues by always revealing whatever information he has. Perry and Reny (1999) and Krishna (2002) give examples of how the Linkage Principle can fail when bidders are asymmetric. Milgrom and Weber (1982a) show that it does hold for the drainage tract model: revelation of the seller's information increases expected revenue provided the seller's information, the informed bidder's information, and the value of the object are all affiliated (or the seller's information is a subset of the informed bidder's). Theorem 5 below extends those results to my setting; given my assumptions of additive separability and independence and Lemma 1, the proofs are virtually identical to those in Milgrom and Weber (1982a). Campbell and Levin (2000) find that in a setting similar to mine – first-price auctions with two asymmetric bidders and pure common values, but with a particular structure of discrete signals – the seller likewise always gains by revealing information.

Theorem 5. *Let $S_1 = \{X\}$ and $S_2 = \{Z, W\}$ be common knowledge.*

- *Always revealing Z gives the seller greater expected revenue than never revealing it, strictly so if Z is meaningful on its own.*
- *If (X, Y, V_1) are affiliated, then always revealing Y gives the seller greater expected revenue than never revealing it.*

Common-value auctions without a reserve price are a constant-sum game between the buyers and the seller. Revealing Z does not affect bidder 1's expected payoff (by Lemma 1), and reduces bidder 2's payoff on average, so it must increase revenue. Revealing Y does not affect bidder 2; its effect on bidder 1 depends on whether X and Y are informational substitutes or complements, as defined in Milgrom and Weber (1982a). Affiliation of (X, Y, V_1) is sufficient for X and Y to be informational substitutes, which in turn leads to revelation of Y hurting bidder 1 and therefore helping the seller.

Finally, consider the dynamic game where the bidders expect the seller to know something. Here, I get sharp results, due to an "unraveling" effect similar to Benoit and Dubra (2006): a seller always prefers to reveal the best realizations of his information; once these are being revealed in equilibrium, he prefers to reveal the next-best realizations; and so on.

Theorem 6. *Let $S_1 = \{X\}$ and $S_2 = \{Z, W\}$ be common knowledge, and let Y and Z each be meaningful on its own.*

- *If the seller is expected to know Z for sure, then he always reveals its value in equilibrium.*
- *If (X, Y, V_1) are affiliated and the seller is expected to know Y for sure, then he always reveals its value in equilibrium.*

Thus, seller revelation is both optimal ex-ante and uniquely predicted in equilibrium.

$p = 0$, it's meaningless to compare equilibria in terms of ex-ante expected profits; but then can be compared in terms of expected payoffs conditional on bidder 1 having observed Z. As the set of (x, z) at which the signal is sent grows, expected payoffs of both bidders go up, so the equilibrium where bidder 1 signals whenever $x \leq z$ Pareto-dominates all others.

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Appendix – Proof of Lemma 1

My setup is a special case of that in Parreiras (2006), with independent signals and $\epsilon = 1$; so equilibrium is unique and players' bids are increasing in their signals. When types are independent, $Q(x) = x$ is the unique solution to the differential equation defining the tying function, establishing symmetry.

(For intuition about the symmetry result, suppose that equilibrium bid strategies β_i are strictly increasing in types and have the same range. Let G_j be the cumulative distribution of player j's bids; player i's problem given type t_i can be written as

$$\max_b \int_0^b \left[t_i + \beta_j^{-1}(s) - b\right] dG_j(s)$$

Differentiating with respect to b gives the first-order condition

$$\left[t_i + \beta_j^{-1}(b) - b\right] G_j'(b) - G_j(b) = 0$$

At i's equilibrium strategy, $t_i = \beta_i^{-1}(b)$, so

$$\frac{1}{\beta_i^{-1}(b) + \beta_j^{-1}(b) - b} = \frac{G_j'(b)}{G_j(b)} = \frac{d}{db} \log G_j(b)$$

Since the left-hand side is the same for both players, $\frac{d}{db} \log G_i(b) = \frac{d}{db} \log G_j(b)$ for any b in the range of both players' bid strategies; since the bid strategies have the same range, $G_i = G_j$.[17])

If player i has type $\hat{t}_i$ and bids b, his expected payoff is

$$u_i(\hat{t}_i, b) \equiv \int_0^b \left(\hat{t}_i + t_j(x) - b\right) dG_j(x)$$

where G_j is the distribution of player j's bids and $t_j(x)$ is the expected value of V_j, conditional on player j's bid being x. The envelope theorem then tells us that

$$\pi_i(\hat{t}_i) = \pi_i(0) + \int_0^{\hat{t}_i} \left[\frac{\partial}{\partial s} \left(\int_0^b (s + t_j(x) - b)\, dG_j(x) \right) \Bigg|_{b=\beta_i(s)} \right] ds$$

Differentiating, the term in square-brackets is simply

$$\int_0^{\beta_i(s)} dG_j(x) = G_j(\beta_i(s))$$

[17] The first-order condition also shows why this result is particular to two-player auctions. With three bidders, the first-order condition would be

$$E\left(V | \beta_i(s_i) = b = \max\{\beta_j(s_j), \beta_k(s_k)\}\right) - b = \frac{G_{-i}(b)}{G'_{-i}(b)}$$

with G_{-i} the distribution of the higher of the other two players' bids. Symmetry of bid distributions would require (letting $x_i = F_i(s_i)$)

$$E(V|x_1 < x_2 = x_3 = x) = E(V|x_2 < x_1 = x_3 = x) = E(V|x_3 < x_1 = x_2 = x)$$

which will not hold if the players are asymmetric.

By symmetry, this is equal to $G_i(\beta_i(s))$; since β_i is increasing, this is $F_{t_i}(s)$. Since player i wins with probability 0 at the lowest realization of his private information, his expected payoff at his lowest signal is 0, so his expected payoff at $\hat{t}_i$ is

$$\pi_i(\hat{t}_i) = \int_0^{\hat{t}_i} F_{t_i}(s)ds$$

Player i's ex-ante expected payoff is

$$\pi_i = E_{\hat{t}_i}\left\{\int_0^{\hat{t}_i} F_{t_i}(s)ds\right\} = \int_0^{\infty}\left\{\int_0^{\hat{t}_i} F_{t_i}(s)ds\right\} dF_{t_i}(\hat{t}_i)$$

Integrating by parts with $u = \int_0^{\hat{t}_i} F_{t_i}(s)ds$ and $dv = dF_{t_i}(\hat{t}_i)$ gives

$$\begin{aligned} \pi_i &= F_{t_i}(\hat{t}_i)\int_0^{\hat{t}_i} F_{t_i}(s)ds\Big|_{\hat{t}_i=0}^{\infty} - \int_0^{\infty} F_{t_i}(\hat{t}_i)F_{t_i}(\hat{t}_i)d\hat{t}_i \\ &= \int_0^{\infty} F_{t_i}(s)ds - \int_0^{\infty} F_{t_i}^2(s)ds \\ &= \int_0^{\infty} F_{t_i}(s)(1-F_{t_i}(s))ds \end{aligned}$$

At a given type $\hat{t}_i$, bidder i wins whenever $F_{t_j}(t_j) < F_{t_i}(\hat{t}_i)$, winning an object worth, on average, $\hat{t}_i + E\left(t_j|F_{t_j}(t_j) < F_{t_i}(\hat{t}_i)\right)$ with probability $F_{t_i}(\hat{t}_i)$; combined with his expected payoff (calculated above), this gives

$$F_{t_i}(\hat{t}_i)\left(\hat{t}_i + E\left(t_j|F_{t_j}(t_j) < F_{t_i}(\hat{t}_i)\right) - \beta_i(\hat{t}_i)\right) = \int_0^{\hat{t}_i} F_{t_i}(s)ds$$

or

$$\beta_i(\hat{t}_i) = \hat{t}_i + E\left(t_j|F_{t_j}(t_j) < F_{t_i}(\hat{t}_i)\right) - \frac{\int_0^{\hat{t}_i} F_{t_i}(s)ds}{F_{t_i}(\hat{t}_i)}$$

Integrating $\int_0^{\hat{t}_i} F_{t_i}(s)ds$ by parts and simplifying gives the final part of the lemma.

Proof of Theorem 1

Let $\underline{t}_X \equiv \min_x E(V_1|X=x)$, $\overline{t}_X \equiv \max_x E(V_1|X=x)$, $\underline{t}_{XY} \equiv \min_{x,y} E(V_1|(X,Y)=(x,y))$, and $\overline{t}_{XY} \equiv \max_{x,y} E(V_1|(X,Y)=(x,y))$. Fix a realization $(\hat{x},\hat{y})$, and let $\hat{t} = E(V_1|(X,Y)=(\hat{x},\hat{y}))$.

First, consider the case $\hat{t} \in \left(\underline{t}_X, \overline{t}_X\right)$. $B(\hat{x},\hat{y}) \geq A(\hat{x},\hat{y})$, since bidder 1 could ignore $\hat{y}$ and bid according to $\hat{x}$. If the support of $E(V_1|X)$ is convex, then bidder 1 has a unique (strict) best-response at each interior point: with true type $\hat{t}$, by bidding like type t, his payoff would be

$$F_{t_1}(t)\left(\hat{t} + E(t_2|F_{t_2}(t_2) < F_{t_1}(t)) - E(t_1|t_1 < t) - E(t_2|F_{t_2}(t_2) < F_{t_1}(t))\right) = \int_0^t \left(\hat{t} - s\right) dF_{t_1}(s)$$

So if $E(V_1|(X,Y)=(\hat{x},\hat{y})) \neq E(V_1|X=\hat{x})$, bidder 1 bids differently knowing $(\hat{x},\hat{y})$ then only knowing $\hat{x}$, and does strictly better ($B(\hat{x},\hat{y}) > A(\hat{x},\hat{y})$); if the distribution of $E(V_1|X=\hat{x},Y)$ is massless, this occurs with probability 1.

Define two random variables $t_X = E(V_1|X)$ and $t_{XY} = E(V_1|X,Y)$, and let F_X and F_{XY} be their respective probability distributions. Note that for any $k \in (0,1)$,

$$E\left(t_{XY}|F_{t_{XY}}(t_{XY}) < k\right) \leq E\left(t_X|F_{t_X}(t_X) < k\right) \tag{8}$$

Both are expectations of $E(V_1)$, taken over regions with total probability k; but the expectation on the left considers only points (x,y) where $E(V_1|(X,Y)=(x,y)) \leq F_{XY}^{-1}(k)$, while the expectation on the right replaces some of these with points (x,y) where $E(V_1|(X,Y)=(x,y)) > F_{XY}^{-1}(k)$. When $E(V_1|X)$ has convex support and $E(V_1|X=x,Y)$ has massless distribution, the non-overlapping parts of the two regions have positive measure, and equation 8 holds strictly.

Continuing to assume that $\hat{t} \in (\underline{t}_X, \bar{t}_X)$, let t_2^* be the type that bidder 1 ties with when he bids optimally in the second situation; then

$$\begin{aligned} B(\hat{x},\hat{y}) &= F_{t_2}(t_2^*)\left(\hat{t} + E(t_2|t_2 < t_2^*) - E(t_X|F_X(t_X) < F_2(t_2^*)) - E(t_2|t_2 < t_2^*)\right) \\ &= F_{t_2}(t_2^*)\left(\hat{t} - E(t_X|F_X(t_X) < F_2(t_2^*))\right) \end{aligned}$$

Similarly,

$$C(\hat{x},\hat{y}) \geq F_{t_2}(t_2^*)\left(\hat{t} - E(t_{XY}|F_{XY}(t_{XY}) < F_2(t_2^*))\right)$$

since the right-hand side is what bidder 1 would get in the third situation if he bid to tie with the same type t_2^*. Equation 8 therefore establishes that $C(\hat{x},\hat{y}) \geq B(\hat{x},\hat{y})$, with strict inequality almost everywhere under the added assumptions.

For $\hat{t} \in (\underline{t}_{XY}, \underline{t}_X)$, $B(\hat{x},\hat{y}) = 0$. If $E(V_1|X = \hat{x}) > \underline{t}_X$, however, $A(\hat{x},\hat{y}) < 0$. (My best-response knowing $(\hat{x},\hat{y})$ is to bid to always lose, but knowing only $\hat{x}$, I would have bid to sometimes win.) By equation 3, $C(\hat{x},\hat{y}) > 0$.

For $\hat{t} \in (\bar{t}_X, \bar{t}_{XY})$, in situation 2, bidder 1 bids to always win; if $E(V_1|X = \hat{x}) \neq \bar{t}_X$, he bids too low knowing only $\hat{x}$, so $B(\hat{x},\hat{y}) > A(\hat{x},\hat{y})$. For $\hat{t} = \bar{t}_{XY}$, $C(\hat{x},\hat{y}) = B(\hat{x},\hat{y})$, because in both instances, bidder 1 bids to always win, paying bidder 2's maximum bid, which is $E(V)$ in either auction. However, the slope of $B(x,y)$, as a function of t_{XY}, is 1 on the region $(\bar{t}_X, \bar{t}_{XY})$, since bidder 1 always wins in that region; by equation 3, the slope of $C(x,y)$ is less than 1 below $\bar{t}_{XY}$. So $C(\hat{x},\hat{y}) > B(\hat{x},\hat{y})$ for $\hat{t} \in (\bar{t}_X, \bar{t}_{XY})$.

Proof of Theorem 2

$D = F$ follows from Lemma 1: player 1's expected payoff is $\pi_1 = \int_0^\infty F_{t_1}(s)(1-F_{t_1}(s))ds$ in either game. $E \geq D$ because information gathered "secretly" cannot have negative value, since it could simply be disregarded.

To show that $E > D$ when Z is meaningful on its own, I need only show that knowing Z changes player 1's optimal bid with positive probability. Recall that player 1's equilibrium bid when he knows only X is the solution to

$$\max_b \int_0^b (t_1 + t_2(s) - b)dG_2(s)$$

where $t_2(s)$ is the type at which player 2 bids s and G_2 is the distribution of 2's bids. When player 1 also observes Z, we can write his maximization problem as

$$\max_b \int_0^b (t_1 + t_2(s) - b)dG_1(s|Z = z)$$

Differentiating both with respect to b and setting the first-order condition equal to 0 implies that *if* the same bid b^* is optimal in both cases,

$$\frac{g_2(b^*)}{G_2(b^*)} = \frac{1}{t_1 + t_2(b^*) - b^*} = \frac{g_2(b^*|Z = z)}{G_2(b^*|Z = z)}$$

Since b^* varies with t_1, if knowing z leaves player 1's best-response unchanged almost everywhere, then

$$\frac{d}{db}\log G_2(b|Z = z) = \frac{g_2(b|Z = z)}{G_2(b|Z = z)} = \frac{g_2(b)}{G_2(b)} = \frac{d}{db}\log G_2(b)$$

almost everywhere, and so $G_2(b|Z = z) = G_2(b)$. But since player 2's bid is strictly increasing in t_2, if Z is meaningful on its own, different values of z give different distributions of t_2, and therefore different bid distributions, so this equality cannot hold for multiple z. That is, nearly all realizations of z must change bidder 1's optimal bid over some range of x, so learning z must strictly increase player 1's ex-ante expected payoff, or $E > D$, when Z is meaningful on its own.

Proof of Example 1

By Lemma 1, bidder 2's bidding function is

$$\beta_2(z) = \frac{1}{2}z\,(1 + X^*)$$

By learning Z secretly, bidder 1 anticipates his opponent's bid, and can outbid him by an arbitrarily small amount, giving him a payoff arbitrarily close to

$$\max\left\{0,\ x + z - \frac{1}{2}z\,(1 + X^*)\right\} = \max\left\{0,\ x + \frac{1}{2}z - \frac{1}{2}zX^*\right\} \tag{9}$$

On the other hand, when Z is learned publicly, it becomes common knowledge, leaving bidder 1 (knowing X) bidding against an uninformed opponent and getting expected payoff of

$$\int_0^x \frac{s}{X^*}ds = \frac{1}{2}\frac{x^2}{X^*} \tag{10}$$

which is at most $\frac{1}{2}x$. When $X^* \leq 1$, equation 9 is at least x, and bidder 1 always does better observing Z secretly. When $X^* > 1$, we can simplify the condition

$$\frac{1}{2}\frac{x^2}{X^*} \geq x + \frac{1}{2}z - \frac{1}{2}zX^*$$

to find the region (high z and low x) where learning Z publicly is preferred.

Proof of Theorem 3

Always Signaling

In the fully separating equilibrium, the posterior distribution of t_1 is F_{XY} when a signal is sent, and F_X otherwise. After observing $(X, Y) = (x, y)$, bidder 1 expects a payoff of $C(x, y)$ (as defined earlier) from signaling and then bidding optimally, and $B(x, y)$ from withholding the signal and bidding optimally; I showed above that $C(x, y) \geq B(x, y)$ at all (x, y), so withholding the signal is never a profitable deviation.

Never Signaling

Suppose that player 1 never sends the stage II message, and player 2's belief upon receiving the message is that player 1's realized type is the highest possible, that is,

$$(x, y) = \arg\max_{x,y} E(V_1|(X, Y) = (x, y))$$

If player 1 sent the message given this type, then in stage III, he would have no private information; as in the drainage tract model, he would get expected payoff of 0. For a given action, expected payoffs are nondecreasing in type; if player 1 sent the message at a lower type and bid optimally, he would still get expected payoff of 0. Thus, sending the message is never a profitable deviation given these off-equilibrium-path beliefs.

Always Mixing

Assume that f_{XY}/f_X is bounded away from 0 wherever $f_X > 0$. Choose

$$k \in \left(0, \frac{p}{1-p} \inf_{\{s: f_X(s)>0\}} \frac{f_{XY}(s)}{f_X(s)}\right)$$

and define

$$m(s)=\begin{cases}\frac{k}{1+k}\left(\frac{1-p}{p}\frac{f_X(s)}{f_{XY}(s)}+1\right) & if \quad f_X(s)>0\\ \frac{k}{1+k} & if \quad f_X(s)=0\end{cases}$$

I claim that player 1 revealing that he's learned y with probability $m(t_1)$ at each type t_1, along with the induced auction equilibria, is an equilibrium.

To see this, note first that

$$p\cdot f_{XY}(s)\cdot m(s)=\frac{k}{1+k}\left((1-p)f_X(s)+pf_{XY}(s)\right)=\frac{k}{1+k}f_{t_1}(s)$$

where f_{t_1} is the prior distribution of t_1 (unconditional of whether or not Y is observed). If we let $f_{t_1}^M$ be the posterior probability distribution of t_1 when player 1 sends a message that he observed Y, then Bayes' Law states that

$$f_{t_1}^M(s)=\frac{p\cdot f_{XY}(s)\cdot m(s)}{\int_0^\infty p\cdot f_{XY}(s')\cdot m(s')ds'}=\frac{\frac{k}{1+k}f_{t_1}(s)}{\frac{k}{1+k}\int_0^\infty f_{t_1}(s')ds'}=f_{t_1}(s)$$

Similarly,

$$(1-p)f_X(s)+pf_{XY}(s)(1-m(s))=f_{t_1}(s)-pf_{XY}(s)m(s)=\frac{1}{1+k}f_{t_1}(s)$$

and so if $f_{t_1}^{NM}$ is the posterior distribution of t_1 after no message is sent,

$$f_{t_1}^{NM}(s)=\frac{\frac{1}{1+k}f_{t_1}(s)}{\frac{1}{1+k}\int_0^\infty f_{t_1}(s')ds'}=f_{t_1}(s)$$

Thus, when player 1 plays the mixed strategy $m(t_1)$, his distribution of types when he sends a message is the same as his distribution of types when he does not. Thus, equilibrium bids and payoffs for each player and each type are the same whether a message is sent or not, making player 1 indifferent and therefore willing to mix whether he sends the signal.

Ranking of Equilibria

Let $r:X\times Y\to[0,1]$ be any signaling policy (equilibrium or not), that is, suppose that at realization (x,y), bidder 1 will signal with probability $r(x,y)$ that he has observed Y. I will show player 1's expected payoff given r is weakly lower than under the policy of never signaling.

Let $\delta(\cdot,\cdot)$ denote the joint density of (X,Y), and define

$$q\equiv p\int\int_{X\times Y}r(x,y)\delta(x,y)dxdy$$

as the unconditional probability of a signal being sent.

Bayes' Law defines the posterior distribution of t_1 conditional on a signal not being sent, and the posterior distribution conditional on a signal provided that $q>0$. (If $q=0$, ex-ante expected payoffs are the same as if signals were never sent, making the question moot.) Denote these distributions F_S and F_{NS}, respectively. By iterated expectations, the unconditional distribution of t_1 is

$$F_U(t)=qF_S(t)+(1-q)F_{NS}(t)$$

By Lemma 1, if bidder 1 never signals, he gets an expected payoff of

$$\int_0^\infty F_U(t)(1-F_U(t))dt \tag{11}$$

and when bidder 1 uses r, he gets an ex-ante expected payoff of

$$q\int_0^\infty F_S(t)(1-F_S(t))dt+(1-q)\int_0^\infty F_{NS}(t)(1-F_{NS}(t))dt \tag{12}$$

Define $\psi(s) \equiv s(1-s)$. We can rewrite equation 12 as

$$\int_0^\infty \left[q\psi(F_S(t)) + (1-q)\psi(F_{NS}(t))\right] dt$$

and equation 11 as

$$\int_0^\infty \psi(qF_S(t) + (1-q)F_{NS}(t))dt$$

Since ψ is concave, the latter integrand is weakly higher at every t by Jensen's Inequality.

As for player 2, Lemma 1 established that player 2's expected payoff is $\int_0^{t_2} F_{t_2}(s)ds$, regardless of the signaling policy being used and whether a signal was sent.

Proof of Theorem 4

Never revealing is always an equilibrium

As in the proof of Theorem 3 earlier, we can sustain an equilibrium where 1 never reveals knowing Z by giving player 2 the off-equilibrium path belief that if he receives a signal, player 1's type is the highest possible. As before, this would give player 1 an expected payoff of 0 from sending a message, so it could not be a profitable deviation.

(Note that in the final-stage auction in game Γ_2, the players' signals are not independent, since with some probability both players have observed Z. Thus, none of the earlier results hold. However, Parreiras guarantees existence and uniqueness of an equilibrium in this auction, which is all that is required, since any deviation would be to a payoff of 0.)

Always revealing is not an equilibrium

In an equilibrium where player 1 always reveals when he's learned Y, the auction following a signal gives bidder 1 an expected payoff of F (as defined in Theorem 2), while a deviation to always withholding the signal would give expected payoff of E (conditional on having observed Z). When Z is meaningful on its own, Theorem 2 guarantees the $E > F$, making this a profitable deviation for at least some realizations (x, z).

Proof of Example 2

Note that when bidder 1 learns Z, both he and bidder 2 observe the same realization z; so the equilibrium of the signaling game is independent at each value z.

First, consider the case $X^* < 1$. Suppose that at some realization z, bidder 1 signals with positive probability. Then at that z when a signal is sent, the final round is a drainage tract auction, with the posterior distribution of X specified by Bayes' Law. By Lemma 1, bidder 2's bid in this auction is always at least z, and so bidder 1's expected payoff from sending the signal at (x, z) is at most x. But when no signal is sent, by Lemma 1, bidder 2 bids $\beta_2(z) = \frac{1}{2}z\,(1 + X^*)$, so by sending no signal, bidder 1 would get expected payoff

$$\max\left\{0,\ x + z - \frac{1}{2}z\,(1 + X^*)\right\}$$

which is strictly greater than x as long as $z > 0$. So for any $z > 0$, bidder 1 must signal with probability 0 in equilibrium.

Now consider the case $X^* = 2$. At a given value z, let $s = s(z)$. Note that bidder 1's expected payoff at x if he does not signal is

$$\max\{0,\ x + z - \beta_2(z)\} = \max\left\{0,\ x + z - \frac{3}{2}z\right\} = \max\left\{0,\ x - \frac{1}{2}z\right\}$$

When he does send the signal, bidder 2 will infer that X is drawn uniformly from the interval $\left[\frac{1-s}{2}z, \frac{1+s}{2}z\right]$, and so bidder 1's expected payoff in the ensuing auction would be

$$\int_{\frac{1-s}{2}z}^{x} \frac{t - \frac{1-s}{2}z}{sz} dt = \frac{1}{2sz}\left(x - \frac{1-s}{2}z\right)^2$$

for $x \in \left[\frac{1-s}{2}z, \frac{1+s}{2}z\right]$. Within this range, $\frac{1}{2sz}\left(x - \frac{1-s}{2}z\right)^2 - \left(x - \frac{1}{2}z\right)$ is strictly decreasing, and equal to 0 at $x = \frac{1+s}{2}z$, so for $x \in \left[\frac{1-s}{2}z, \frac{1+s}{2}z\right]$, bidder 1 never gains by withholding the signal. For $x < \frac{1-s}{2}z$, bidder 1 would get 0 payoff from sending the signal, since his best move in the subsequent auction would be to imitate type $x = \frac{1-s}{2}z$ and bid to always lose. For $x > \frac{1+s}{2}z$, bidder 1 would bid as type $x = \frac{1+s}{2}z$ if he signaled, always winning at a price of $\frac{1}{2}z + z$ for a payoff of $x - \frac{1}{2}z$, no better than if he did not signal.

As for equilibrium comparisons, consider a particular z, and let $s = s(z)$. Bidder 2 gets a payoff of $x + z - \beta_2(z) < 0$ when $x < \frac{1-s}{2}z$, and an expected payoff of 0 the rest of the time, so increasing s across the board helps bidder 2. Bidder 1's expected payoff at a given x can be written as

$$\max\left\{0,\ x - \frac{1}{2}z,\ \int_{\frac{1-s}{2}z}^{\min\left\{x, \frac{1+s}{2}z\right\}} \frac{t - \frac{1-s}{2}z}{sz} dt\right\}$$

(where the last term is taken to be 0 when $x < \frac{1-s}{2}z$); the last term can be shown to be increasing in s at every x.

Proof of Theorem 5

First, consider the effect of revelation of Z on bidder 2's ex-ante expected payoff. Let F_{t_2} denote the distribution of $E(V_2|Z, W)$, and $F_{t_2|z}$ the distribution of $E(V_2|Z = z, W)$. Once Z has been revealed to have realization z, bidder 2's expected payoff is

$$\int_0^\infty F_{t_2|z}(s)\left(1 - F_{t_2|z}(s)\right) ds$$

and so bidder 2's ex-ante expected payoff, knowing that Z will be revealed, is

$$E_z\left\{\int_0^\infty F_{t_2|z}(s)\left(1 - F_{t_2|z}(s)\right) ds\right\} = \int_0^\infty E_z\left\{F_{t_2|z}(s)\left(1 - F_{t_2|z}(s)\right)\right\} ds$$

On the other hand, by iterated expectations, bidder 2's ex-ante expected payoff when Z is not revealed is

$$\int_0^\infty F_{t_2}(s)\left(1 - F_{t_2}(s)\right) ds = \int_0^\infty E_z\left\{F_{t_2|z}(s)\right\}\left(1 - E_z\left\{F_{t_2|z}(s)\right\}\right) ds$$

Since the function $x(1-x)$ is concave, the latter is weakly greater at each s, and strictly greater wherever $F_{t_2|z}(s)$ varies with z, which is guaranteed to happen at some s when Z is meaningful on its own.

By Lemma 1, bidder 1's expected payoff is $\int_0^\infty F_{t_1}(s)\left(1 - F_{t_1}(s)\right) ds$ whether Z is revealed or not; since the seller's revenue is simply $E(V)$ minus the two bidders' ex-ante expected payoffs, it is strictly higher when Z is revealed.

When Y is revealed, bidder 2's expected payoff is similarly unaffected; the change in bidder 1's ex-ante expected payoff is

$$E_y\left\{\int_0^\infty F_{t_1|y}(s)\left(1 - F_{t_1|y}(s)\right) ds\right\} - \int_0^\infty F_X(s)\left(1 - F_X(s)\right) ds$$

Milgrom and Weber (1982a) show that when (X, Y, V_1) are affiliated, X and Y are informational substitutes and this difference is negative; so revealing Y raises expected revenue.

Proof of Theorem 6

First, suppose the seller is expected to know Z and Z is meaningful on its own. Suppose in some equilibrium, there were multiple values of Z which the seller did not always reveal. In the ensuing auction, Z would still be nondegenerate (although from a different posterior distribution), and would still be meaningful on its own, so Theorem 5 would still hold for the auction in this continuation game, and revealing Z would strictly increase the seller's expected payoff.

Second, suppose the seller is expected to know Y but did not reveal certain realizations. If (conditional on Y not being revealed) X and Y are not independent, then the publicity effect of revealing Y would strictly favor the seller. If X and Y were independent, then $E(Y)$ would be priced into both bidders' bids, and so publicizing realizations $y > E(Y)$ would increase expected revenue.

Bibliography

1. Benoit, Jean-Pierre, and Juan Dubra (2006), "Information Revelation in Auctions," Games and Economic Behavior 57, 181-205

2. Bulow, Jeremy, Ming Huang, and Paul Klemperer (1999), "Toeholds and Takeovers," Journal of Political Economy 107, 427-454

3. Bulow, Jeremy, and Paul Klemperer (2002), "Prices and the Winner's Curse," RAND Journal of Economics 33, 1-21

4. Campbell, Colin, and Dan Levin (2000), "Can the Seller Benefit from an Insider in Common-Value Auctions?" Journal of Economic Theory 91, 106-120

5. Engelbrecht-Wiggans, Richard, Paul Milgrom, and Robert Weber (1983), "Competitive Bidding and Proprietary Information," Journal of Mathematical Economics 11, 161-169

6. Hafalir, Isa, and Vijay Krishna (2008), "Asymmetric Auctions with Resale," American Economic Review 98, 87-112

7. Hausch, D., and L. Li (1993), "A Common Value Auction Model with Endogenous Entry and Information Acquisition," Economic Theory 3, 315-334

8. Hendricks, Kenneth, Robert Porter, and Charles Wilson (1994), "Auctions for Oil and Gas Leases with an Informed Bidder and a Random Reservation Price," Econometrica 62, 1415-1444

9. Hernando-Veciana, Ángel (2009), "Information acquisition in auctions: Sealed bids vs. open bids", Games and Economic Behavior 65, 372-405

10. Hernando-Veciana, Ángel, and Michael Tröge (2005), "The Insider's Curse", Discussion Paper WP-AD 2005-08, IVIE

11. Krishna, Vijay (2002), *Auction Theory*, Academic Press

12. Larson, Nathan (2009), "Private value perturbations and informational advantage in common value auctions", Games and Economic Behavior 65, 430-460

13. Mares, Vlad, and Ronald Harstad (2003), "Private Information Revelation in Common-Value Auctions," Journal of Economic Theory 109, 264-282

14. Milgrom, Paul, and Robert Weber (1982a), "The Value of Information in a Sealed-Bid Auction," Journal of Mathematical Economics 10, 105-114

15. Milgrom, Paul, and Robert Weber (1982b), "A Theory of Auctions and Competitive Bidding," Econometrica 50, 1089-1122

16. Parreiras, Sergio (2006), "Affiliated Common Value Auctions with Differential Information: The Two Bidder Case", Contributions to Theoretical Economics 6, Iss. 1, Art. 12

17. Perry, Motty, and Philip Reny (1999), "On the Failure of the Linkage Principle in Multi-Unit Auctions," Econometrica 67, 895-900

18. Persico, Nicola (2000), "Information Revelation in Auctions," Econometrica 68, 135-148

19. Rezende, Leonardo (2005), "Mid-Auction Information Acquisition," mimeo.

20. Wilson, Robert (1967), "Competitive Bidding with Asymmetric Information," Management Science 13, 816-820