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Online Appendix for

Boerner and Quint, "Medieval Matching Markets"

This online appendix is organized in the following way.

- Section A1 defines the explanatory variables used in the paper.
- Section A2 gives two additional examples referenced in the theory section (p. 18 and 25).
- Section A3 shows probit results as an alternate specification for the main empirical results of the paper.
- Section A4 offers additional empirical results.

  Tables 10 and 11 give some additional specifications on the determinants of the existence of brokerage regulations: Table 10 uses a balanced panel created by replacing missing population entries with a population of 500, and Table 11 splits the effects of city-specific variables into pre-1500 and post-1500 effects.

  Table 12 offers additional specifications of the determinants of fee structure (unit versus value-based fees) for all regulations with fixed fees, only those containing the "matchmaking design" combination of rules, and a separate specification examining the determinants of whether or not to include the private business constraint conditional on the other "matchmaking design" rules being present.
- Sections A5 to A10 give proofs of the theoretical results in the paper.

  Section A5 gives some preliminary results used in many of the proofs.

  Section A6 gives the proof of Theorem 1.

  Section A7 gives the proof of Theorem 2.

  Section A8 gives the proof of Theorem 3.

  Section A9 gives the proof of Theorem 4.

  Section A10 gives the proof of Theorem 5.
- Section A11 gives an overview of the simulations mentioned in the text on the social value of brokerage in different settings and some simulation results.
- Section A12 lists the archival sources used.

A1 Description of Explanatory Variables

VARIABLE	*DESCRIPTION*
Free Imperial Towns	A binary variable taking the value 1 if the town is either an "imperial" or "free" city and 0 otherwise. Imperial cities were directly ruled by a local consulate representing (some of) their citizens but were formally under the legal protection of the German Kaiser (who had normally only limited influence on the political decision making of a town). The same holds for free cities, but they liberated themselves from the regimen of a bishop. (See Heining 1983, Johannek 2000.) For the identification of free and imperial cities, see Johanek (2000).
Bishop	A binary variable taking the value 1 if a town is ruled by a bishop and 0 otherwise. Bishop cities are documented in Bautier et al. (1977-1999).
Territorial Towns	A binary variable taking the value 1 for towns that were neither free, imperial, nor bishop towns. All other towns were controlled by local (territorial) dukes (see Isenmann 1988). Note this variable is not included in the regression since the political town characteristics are already sufficiently specified.
Hanseatic	A binary variable that is 1 if the city joined the Hanseatic League and 0 otherwise. The information on the participation can be found in Dollinger (1966).
University	A binary variable that is 1 if a city hosts a university and 0 otherwise. A history of the foundation and evolution of university towns can be found in de Ridder-Symoens and Rüegg (1992-2011).
Roman	A binary variable that is 1 if a town was founded by Romans and 0 otherwise. The information about Roman towns can be found in Putzger (1956) and Bedon (2001).
Water	A binary variable that is 1 if a town has a navigable port and 0 otherwise. The information of navigable ports (river or sea) can be found in Putzger (1956), p. 70.
Sea	A binary variable that is 1 if a town has a port with access to the sea and 0 otherwise. This information can be found in Putzger (1956), p.70.
Post Black Death	A binary variable that is 1 for the 1350-1400 period, i.e., after the outbreak and main spread of the Black Death, and 0 otherwise.
Thirty Years War	This variable is 1 for the 1650-1700 period if a town has been involved the Thirty Years War and 0 otherwise.
Brokerage Neighbour	The number of neighbouring towns (measured in logarithm) in a radius of 100 kilometers that had brokerage regulations during the previous fifty years period.
Year	This variable identifies the specific year a brokerage regulation has been dated. In the empirical analysis on the existence of brokerage, it assigns the year to the 50-year interval in which it is located.
Longitude	This variable describes the longitude at which a city is located. (see google maps)
Latitude	This variable describes the latitude at which a city is located. (see google maps)
Log(Population)	The logarithm of the city population size, based on data collected by Bairoch (1988) and interpolated when necessary
Population Quintile	These are five binary variables dividing the sample observations into five equally weighted quintiles, ordered by the city population size (quintile 1 being the smallest population size). Each variable is 1 if the observation can be assigned to the respective quintile and 0 otherwise.

A2 Additional examples referenced in text

Examples 3 and 4 (referenced in Section 3) are two settings where brokerage with either form of fee yields a lower total surplus than random matching, illustrating that the results in Theorems 1 and 2 need not hold in balanced markets (roughly the same numbers of buyers and sellers).

Example 3 (Costs Dominate). *Let* $N_b/N_s = 1$*, and let* $U(v, q_s) = v$ *(uniform quality). Suppose that for some small* $\epsilon > 0$*,* $v_b \sim U[50+\epsilon, 100+\epsilon]$*, and* $c_s \sim U[50-\epsilon, 100-\epsilon]$*. A broker maximizing unit fees will pair up buyers and sellers such that everyone trades, by matching buyer* $x+\epsilon$ *with seller* $x-\epsilon$*, which means each trade will generate a surplus of only* 2ϵ*. Percentage fees leads to the same outcome. The efficient outcome is for buyers with valuations above* 75 *to buy from sellers with costs below* 75*: for* $\epsilon \approx 0$*, there are half as many trades, but the average trade generates a surplus of* 25*. Under random matching, again, half of the traders trade, and the average surplus per trade is* $17\frac{1}{3}$*.*

Example 4 (Quality Dominates). *Let* $N_b/N_s = 1$*, let* $U(v, q) = vq$*, and suppose all sellers' costs are* $c_s = 79$*; let* v_b *take the two values* 8 *and* 10 *with equal probability and* q_s *take the values* 8 *and* 10 *with equal probability. Under either unit or percentage fees, brokers will pair up* 8*s with* 10*s and* 10*s with* 8*s so that everyone trades and each trade generates a surplus of* $80-79=1$*. It would be far more efficient for only the high-type buyers and high-quality sellers to trade: there would be half as many trades, but each trade would generate a surplus of* $100-79=21$*. Under random matching, three quarters of traders would trade, and the average trade would generate a surplus of* $7\frac{2}{3}$*.*

Example 5 (referenced on page 26) is a variation on Example 4 and illustrates that the result in Theorem 5 (that increasing the heterogeneity of traders on the long side of the market increases the social value of brokerage) need not hold in balanced markets.

Example 5. *Let* $N_b/N_s = 1$*, let* $U(v, q) = vq$*, and suppose all sellers' costs are* $c_s = 79$*. Buyer types* v_b *take the three values* $\{6, 8, 10\}$ *with equal probability, and seller quality* q_s *takes the three values* $\{7, 8, 9\}$ *with equal probability. The only buyers able to trade are those with* $v_b = 10$*; therefore, under either unit or percentage fees, the broker pairs all the buyers with* $v_b = 10$ *to sellers with* $q_s = 9$*; brokerage is far more efficient than random matching.*

Now suppose we make sellers more heterogeneous by changing the support of q_s *from* $\{7, 8, 9\}$ *to* $\{6, 8, 10\}$*. Now the broker can pair 8s with 10s and 10s with 8s, doubling the number of trades and increasing commissions under either unit or percentage fees. However, now each trade generates very little surplus, so brokerage is now less efficient than random matching.*

A3 Alternate Specification for Main Results

Tables 6 and 7 give additional specifications for Table 2 in the text: Table 6 gives results for probit regressions on additional combinations of explanatory variables, including two on a smaller balanced panel; Table 7 shows additional fixed effects results, including two on the smaller balanced panel. Table 8 replicates the first four columns of Table 5 in the text alongside equivalent probit regressions. Table 9 replicates the last four columns of Table 5 in the text alongside equivalent probit regressions.

Table 6: Regression Results: Existence of Brokerage Regulations, 1200-1700

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Dependent Variable:	Existence of Brokerage								
Sample:	unbalanced							balanced	
Regression Type:	probit								
Free Imperial City	0.18***	0.12***	0.10***	0.09**	0.09**	0.09**	0.08**	0.10**	0.12**
	[0.05]	[0.04]	[0.03]	[0.04]	[0.03]	[0.04]	[0.03]	[0.05]	[0.06]
Bishop	0.07	-0.03	-0.04	-0.04	-0.04	-0.04	-0.02	-0.06	-0.05
	[0.04]	[0.04]	[0.03]	[0.04]	[0.03]	[0.04]	[0.04]	[0.04]	[0.06]
University	0.25***	0.13**	0.11**	0.12**	0.11**	0.12***	0.13***	0.14***	-0.16**
	[0.06]	[0.06]	[0.06]	[0.06]	[0.06]	[0.06]	[0.05]	[0.07]	[0.07]
Hanseatic	-0.02	-0.04	-0.05*	-0.06*	-0.05*	-0.06*	-0.06**	-0.01	-0.01
	[0.04]	[0.03]	[0.03]	[0.03]	[0.03]	[0.03]	[0.03]	[0.07]	[0.08]
Roman city	0.07	0.03	0.07	0.09*	0.07	0.09	0.10*	0.04	0.05
	[0.05]	[0.04]	[0.04]	[0.05]	[0.05]	[0.05]	[0.05]	[0.05]	[0.06]
Log (Population)		0.17***	0.13***	0.13***	0.13***	0.13***		0.17***	0.15***
		[0.03]	[0.03]	[0.03]	[0.03]	[0.02]		[0.04]	[0.03]
Population Quintile 2							0.04		
							[0.03]		
Population Quintile 3							0.09**		
							[0.04]		
Population Quintile 4							0.16***		
							[0.04]		
Population Quintile 5							0.28***		
							[0.05]		
Water (any port)			0.02	0.01	0.02	0.01	0.00	0.02	-0.04
			[0.02]	[0.03]	[0.02]	[0.03]	[0.03]	[0.04]	[0.05]
Sea port			0.20***	0.23***	0.19***	0.22***	0.25***	0.29***	0.35***
			[0.06]	[0.07]	[0.06]	[0.07]	[0.07]	[0.08]	[0.10]
Number trade routes			0.02**	0.02***	0.02**	0.02***	0.03***	0.02*	0.02**
			[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
Neighbours w brokerage (log, within 100 km)				0.05**		0.05**	0.06**		0.08*
				[0.03]		[0.02]	[0.02]		[0.05]
Post Black Death					0.09***	0.06***	0.05**	0.12**	0.07*
					[0.03]	[0.02]	[0.02]	[0.05]	[0.04]
Thirty Years War					-0.05**	-0.05***	-0.04***	-0.06**	- 0.06***
					[0.02]	[0.01]	[0.01]	[0.03]	[0.02]
Year	0.00	-0.00	-0.00	-0.00**	-0.00	0.00	-0.00	0.00	-0.00
	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
Longitude	-0.01**	-0.01***	-0.01***	-0.00	-0.01***	-0.00	-0.00	-0.01***	0.01
	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.01]
Latitude	0.01	0.00	-0.01	-0.01	-0.01	-0.01	-0.01	-0.04***	-0.04***
	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.02]
Observations	2310	1823	1823	1697	1823	1697	1697	1246	1122
No. of City Clusters	231	225	225	225	225	225	225	125	125
No. of Century Clusters	5	5	5	5	5	5	5	5	5
R-squared	0.14	0.24	0.28	0.29	0.29	0.29	0.28	0.26	0.26

Notes: ***p < 0.01, **p < 0.05, *p < 0.1. Probit regression results, with robust standard errors in brackets, clustered by city and century. Columns (1)-(7) use an unbalanced panel due to missing population size variable; columns (8) and (9) report results with a reduced balanced panel. Column (1) measures institutional effects; column (2) adds population size; column (3) adds trade-geographic effects; column (4) adds the effect of nearby cities having brokerage; column (5) instead adds historical effects; column (6) incorporates both; and column (7) replaces population size with population quintiles. Columns (8) and (9) use a balanced panel to replicate (5) and (6).

Table 7: Regression Results: Existence of Brokerage Regulations, 1200-1700, Part II

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Dependent Variable:	Existence of Brokerage								
Sample:	unbalanced							balanced	
Regression Type:	LPM	LPM FE	LPM FE	LPM FE	LPM FE	LPM FE	LPM FE	LPM FE	LPM FE
Free Imperial City	0.10*	0.19***	0.26***	0.26***	0.23***	0.33***	0.23***	0.31***	0.28***
	[0.04]	[0.05]	[0.05]	[0.05]	[0.07]	[0.12]	[0.07]	[0.07]	[0.09]
Bishop	-0.02	-0.19**	-0.20***	-0.20***	-0.24***	-0.21**	-0.24**	-0.16**	-0.21***
	[0.06]	[0.08]	[0.08]	[0.08]	[0.09]	[0.08]	[0.04]	[0.07]	[0.07]
University	0.13*	0.10	0.07	0.07	0.04	0.05	0.05	0.09	0.07
	[0.06]	[0.07]	[0.07]	[0.07]	[0.07]	[0.06]	[0.08]	[0.08]	[0.08]
Hanseatic	-0.06								
	[0.04]								
Roman city	0.07								
	[0.05]								
Log (Population)	0.18***		0.15***	0.15***	0.14***	0.14***		0.11***	0.11***
	[0.03]		[0.03]	[0.04]	[0.03]	[0.04]		[0.03]	[0.04]
Population Quintile 2							0.02		
							[0.02]		
Population Quintile 3							0.04**		
							[0.02]		
Population Quintile 4							0.07*		
							[0.04]		
Population Quintile 5							0.14***		
							[0.05]		
Water (any port)	0.00								
	[0.03]								
Sea port	0.20**								
	[0.05]								
Number trade routes	0.02								
	[0.03]								
Neighbours w brokerage	0.05				0.05**	0.05**	0.06**		0.05*
(log, within 100 km)	[0.03]				[0.03]	[0.02]	[0.03]		[0.03]
Post Black Death	0.06**			0.08**	0.06*	0.07**	0.06*	0.09**	0.07*
	[0.02]			[0.03]	[0.03]	[0.03]	[0.03]	[0.04]	[0.04]
Thirty Years War	-0.05**			-0.04	-0.04	-0.04	-0.04	-0.06	-0.05
	[0.01]			[0.03]	[0.03]	[0.03]	[0.03]	[0.04]	[0.05]
Year	-0.00	0.00**	0.00	-0.00	-0.00	-0.00	-0.00	-0.00	-0.00
	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
Longitude	-0.00								
	[0.00]								
Latitude	0.01								
	[0.01]								
Observations	2310	2310	1823	1823	1697	1697	1697	1246	1122
No. of City Clusters	231	225	225	225	225	225	225	125	125
No. of Century Clusters	5	.	.	.	.	.	.	.	.
R-squared	0.26	0.48	0.50	0.50	0.54	0.53	053	0.48	0.52

Notes: ***$p < 0.01$, **$p < 0.05$, *$p < 0.1$. Linear probability model estimates, with robust standard errors in brackets, clustered by city in all columns and also by century in column (1). Results in columns (2)-(9) include city and century fixed effects. Columns (1)-(7) use an unbalanced panel; columns (8)-(9), a balanced panel. Column (1) includes all variables. Columns (2)-(9) incorporate groups of variables not captured by fixed effects. Column (2) uses time-varying institutional effects only; column (3) adds population size; column (4) adds historical effects; and column (5) adds the effect of neighbouring cities. Column (6) replaces the time-varying instituional effects with their one-period lag; column (7) replaces population size with the corresponding population quintiles. Columns (8) and (9) replicate (4) and (5) on a balanced sample.

Table 8: Regression Results – Determinants of Fee Structure, 1200-1700

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Dependent Variable:	1 if unit fees, 0 if value fees (sample is brokerage rules with fixed fees)							
Regression:	Probit	Probit	Probit	Probit	LPM	LPM FE	LPM FE	LPM FE
Finance	-0.42**	-0.45***	-0.46***	-0.47***	-0.47***	-0.43***	-0.43***	-0.43***
	[0.17]	[0.16]	[0.15]	[0.17]	[0.09]	[0.07]	[0.07]	[0.07]
Property	-0.36	-0.36	-0.37	-0.37	-0.40**	-0.33***	-0.32***	-0.33***
	[0.27]	[0.28]	[0.27]	[0.29]	[0.14]	[0.07]	[0.07]	[0.07]
Horses	-0.49**	-0.48**	-0.49**	-0.49**	-0.52**	-0.54***	-0.53***	-0.54***
	[0.22]	[0.23]	[0.23]	[0.22]	[0.14]	[0.10]	[0.10]	[0.10]
Wine and Beer	0.40***	0.40***	0.40***	0.39***	0.32**	0.35***	0.35***	0.35***
	[0.07]	[0.05]	[0.05]	[0.05]	[0.10]	[0.07]	[0.07]	[0.07]
Grain	0.32***	0.32***	0.32***	0.32***	0.24	0.24**	0.24**	0.24**
	[0.07]	[0.06]	[0.06]	[0.07]	[0.12]	[0.10]	[0.10]	[0.11]
Fish	0.38***	0.38***	0.38***	0.37***	0.33*	0.32***	0.32***	0.32***
	[0.06]	[0.04]	[0.04]	[0.05]	[0.14]	[0.09]	[0.09]	[0.09]
Cattle and meat	0.31***	0.30***	0.30***	0.30***	0.22	0.24**	0.24***	0.29***
	[0.07]	[0.06]	[0.07]	[0.07]	[0.12]	[0.11]	[0.11]	[0.09]
Oil and fat	0.40***	0.40***	0.40***	0.40***	0.34**	0.34***	0.34***	0.34***
	[0.06]	[0.04]	[0.04]	[0.05]	[0.12]	[0.06]	[0.06]	[0.06]
Construction material	0.34***	0.33***	0.33***	0.33***	0.24	0.24**	0.24**	0.23**
	[0.08]	[0.07]	[0.07]	[0.07]	[0.12]	[0.10]	[0.10]	[0.10]
Metal	0.29***	0.29***	0.28***	0.28***	0.13	0.09	0.09	0.09
	[0.05]	[0.06]	[0.05]	[0.06]	[0.12]	[0.10]	[0.10]	[0.10]
Spices	0.23***	0.22***	0.22***	0.22***	0.04	0.02	0.02	0.02
	[0.04]	[0.04]	[0.04]	[0.04]	[0.06]	[0.08]	[0.08]	[0.07]
Raw textile	0.03	0.02	0.01	0.01	-0.19	-0.20**	-0.19*	-0.20*
	[0.17]	[0.16]	[0.16]	[0.17]	[0.12]	[0.11]	[0.11]	[0.11]
Fur, skin and leather	0.22***	0.23***	0.22***	0.22***	0.08	0.04	0.04	0.03
	[0.08]	[0.07]	[0.08]	[0.08]	[0.10]	[0.10]	[0.10]	[0.10]
Cloth	-0.05	-0.06	-0.08	-0.11	-0.28*	-0.28**	-0.29**	-0.29**
	[0.17]	[0.16]	[0.17]	[0.21]	[0.11]	[0.11]	[0.11]	[0.11]
Time and (geography)	YES	YES	YES	YES	YES	YES	YES	YES
Institutions		YES	YES	YES	YES		YES	YES
Population			YES	YES	YES			YES
Trade geography				YES	YES			
Observations	683	683	680	680	680	683	683	680
City Clusters	57	57	57	57	57	57	57	57
Century Clusters	5	5	5	5	5	.	.	.
(Pseudo) R-squared	0.38	0.38	0.39	0.39	0.41	0.53	0.53	0.53

Notes: ***p < 0.01, **p < 0.05, *p < 0.1. Robust standard errors clustered by city code (and by century for columns (1)- (5)) are in brackets. Columns (1)-(4) give marginal results for probit regressions, column (5) the linear probability, and columns (6)-(8) the linear probability regression results with city and century fixed effects for unit (as opposed to value) fees for all products. Column (1) controls for time and geographical coordinates, column (2) incorporates the institutional variables, column (3) adds population, and column (4) incorporates trade geography. Column (5) controls for all variables; columns (6)-(8) control for time, the remaining time varying institutional variables, and population in the fixed effects specifications.

Table 9: Regression Results – Determinants of Fee Structure, 1200-1700

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Dependent Variable:	1 if unit fees, 0 if value fees – smaller sample of "matchmaking" observations							
Regression:	Probit	Probit	Probit	Probit	LPM	LPM FE	LPM FE	LPM FE
Finance	-0.54**	-0.70***	-0.71***	-0.71***	-0.46**	-0.44***	-0.42***	-0.41***
	[0.26]	[0.17]	[0.16]	[0.21]	[0.10]	[0.13]	[0.13]	[0.13]
Property	.	.	.	.	-0.45*	-0.44***	-0.42***	-0.42***
					[0.18]	[0.13]	[0.13]	[0.13]
Horses	.	.	.	.	-0.63**	-0.63***	-0.64***	-0.64***
					[0.14]	[0.14]	[0.15]	[0.15]
Wine and Beer	0.28***	0.27***	0.27***	0.25***	0.30**	0.29**	0.30***	0.30***
	[0.04]	[0.04]	[0.04]	[0.03]	[0.08]	[0.11]	[0.11]	[0.11]
Grain	0.23***	0.22***	0.22***	0.19***	0.27	0.27**	0.28**	0.28**
	[0.07]	[0.05]	[0.05]	[0.02]	[0.15]	[0.13]	[0.13]	[0.13]
Fish	0.26***	0.25***	0.25***	0.22***	0.31	0.31**	0.32**	0.32***
	[0.05]	[0.03]	[0.03]	[0.03]	[0.17]	[0.15]	[0.15]	[0.15]
Cattle and meat	0.19***	0.10	0.09	0.06	0.15	0.14	0.15	0.12
	[0.06]	[0.10]	[0.11]	[0.09]	[0.15]	[0.16]	[0.16]	[0.16]
Oil and fat	0.27***	0.26***	0.26***	0.22***	0.33*	0.29**	0.29**	0.29**
	[0.04]	[0.03]	[0.03]	[0.03]	[0.13]	[0.12]	[0.12]	[0.12]
Construction material	0.29***	0.27***	0.27***	0.23***	0.37**	0.33**	0.34**	0.34**
	[0.05]	[0.04]	[0.04]	[0.03]	[0.15]	[0.15]	[0.15]	[0.15]
Metal	0.17**	0.16**	0.16**	0.15**	0.14	0.09	0.10	0.10
	[0.08]	[0.08]	[0.08]	[0.06]	[0.15]	[0.14]	[0.14]	[0.14]
Spices	0.09	0.09	0.08	0.10*	0.02	0.00	0.00	0.00
	[0.06]	[0.06]	[0.06]	[0.06]	[0.10]	[0.12]	[0.12]	[0.12]
Raw textile	-0.17	-0.20	-0.21	-0.17	-0.24	-0.28	-0.28	-0.28
	[0.33]	[0.34]	[0.33]	[0.35]	[0.21]	[0.21]	[0.21]	[0.21]
Fur, skin and leather	0.05	0.05	0.05	0.08	0.01	-0.04	-0.03	-0.03
	[0.15]	[0.14]	[0.13]	[0.13]	[0.13]	[0.15]	[0.15]	[0.15]
Cloth	-0.40	-0.45*	-0.45*	-0.53*	-0.45*	-0.50***	-0.49***	-0.49***
	[0.29]	[0.27]	[0.26]	[0.30]	[0.18]	[0.17]	[0.17]	[0.17]
Time and (geography)	YES	YES	YES	YES	YES	YES	YES	YES
Institutions		YES	YES	YES	YES		YES	YES
Population				YES				YES
Trade geography			YES	YES				
Observations	326	326	326	325	362	362	362	361
City Clusters	38	38	38	38	38	38	38	26
Century Clusters	5	5	5	5	5	.	.	.
(Pseudo) R-squared	0.43	0.48	0.54	0.54	0.33	0.63	0.64	0.64

Notes: ***p < 0.01, **p < 0.05, *p < 0.1. Robust standard errors clustered by city code (and by century for columns (1)- (4)) are in brackets. Columns (1)-(4) give marginal results for probit regressions, column (5) the linear probability regression results, and columns (6)-(8) the linear probability results with city and century fixed effects for unit (as opposed to value) fees for all products. Column (1) controls for time and geographical coordinates, column (2) incorporates the institutional variables, column (3) adds population, and column (4) incorporates trade geography. Column (5) controls for all variables; columns (6)-(8) control for time, the remaining time varying institutional variables, and population in the fixed effects specifications. Results for the product categories property and horses are omitted in the probit regressions: for property and horses, all observations were of matching with value fees.

A4 Additional Empirical Results

Table 10: Regression Results: Existence of Brokerage Regulations, 1200-1700, Part III

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Dependent Variable:				Existence Brokerage				
Regression Type:	Probit	Probit	Probit	Probit	LPM	LPM FE	LPM FE	LPM FE
Free Imperial City	0.10***	0.09***	0.08***	0.09***	0.11***	0.18***	0.18***	0.19***
	[0.03]	[0.03]	[0.03]	[0.03]	[0.04]	[0.05]	[0.05]	[0.06]
Bishop	-0.02	-0.03	-0.03	-0.02	-0.00	-0.18**	-0.18**	-0.23**
	[0.03]	[0.02]	[0.02]	[0.03]	[0.05]	[0.08]	[0.08]	[0.09]
University	0.10**	0.08*	0.09**	0.10**	0.15**	0.11	0.11	0.08
	[0.05]	[0.04]	[0.04]	[0.04]	[0.06]	[0.07]	[0.07]	[0.07]
Hanseatic	-0.02	-0.03	-0.03	-0.04	-0.06			
	[0.02]	[0.02]	[0.02]	[0.02]	[0.06]			
Roman City	0.03	0.04	0.04	0.05	0.07			
	[0.03]	[0.04]	[0.04]	[0.04]	[0.05]			
Log(Population)	0.11***	0.08***	0.08***	0.09***	0.12***	0.07***	0.07***	0.06***
	[0.02]	[0.02]	[0.02]	[0.01]	[0.02]	[0.02]	[0.02]	[0.04]
Water (any port)		0.02	0.02	0.02	0.00			
		[0.02]	[0.02]	[0.02]	[0.02]			
Sea port		0.14***	0.14***	0.17***	0.18***			
		[0.05]	[0.05]	[0.06]	[0.05]			
Number trade routes		0.01**	0.01**	0.02***	0.02***			
		[0.01]	[0.01]	[0.01]	[0.01]			
Log(Number of neighbouring cities with brokerage, 100km)				0.04*	0.05*			0.04**
				[0.02]	[0.02]			[0.02]
After Plague			0.06***	0.08**	0.04**		0.05**	0.04
			[0.02]	[0.03]	[0.01]		[0.02]	[0.03]
30 Years War			-0.03**	-0.03***	-0.05**		-0.05*	-0.05*
			[0.02]	[0.01]	[0.01]		[0.03]	[0.03]
Year	-0.00	-0.00	0.00	-0.00	-0.00*	0.00*	0.00	-0.00
	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
Longitude	-0.00***	-0.00***	-0.00***	-0.00	-0.00			
	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]			
Latitude	-0.00	-0.01	-0.01	-0.01	-0.01			
	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]			
Observations	2310	2310	2310	2079	2079	2310	2310	2079
No. of City Clusters	231	231	231	231	231	231	231	231
No. of Century Clusters	5	5	5	5	5			
R-squared	0.26	0.31	0.31	0.31	0.24	0.48	0.49	0.53

Notes: ***p < 0.01, **p < 0.05, *p < 0.1. Robust standard errors in brackets, clustered by city in all columns and by century in columns (1)-(5). Results in column (1) report estimates for a linear dependent regression. (2)-(9) report linear dependent regression estimates with city and century fixed effects. All regressions are based on an extended population sample where missing observations are replaced by an arbitrary 500 inhabitants to create a balanced panel. Columns (1)-(4) report marginal results for probit regressions adding first institutions and population, next incorporating trade geography variables, then adding historical event variables and finally incorporating brokerage neighbouring effects. Column (5) depicts results for a linear probability model incorporating all variables and columns (6)-(8) for a linear probability model with city and century fixed effects first adding institutions and population, then adding historical event, and finally brokerage neighbouring effects. All regressions control for time and (in case of no fixed effects) longitude and latitude.

Table 11: Regression Results: Existence of Brokerage Regulations, 1200-1700, Part IV

	(1)	(2)	(3)
Dependent Variable:	Existence Brokerage		
Regression Type:	Probit	LPM	LPM FE
Free Imperial City pre 1500	0.06**	0.07	0.20***
	[0.03]	[0.03]	[0.07]
Free Imperial City post 1500	0.14***	0.14**	0.25***
	[0.04]	[0.04]	[0.07]
Bishop pre 1500	-0.02	-0.00	-0.23**
	[0.05]	[0.08]	[0.09]
Bishop post 1500	-0.05	-0.04	-0.24***
	[0.03]	[0.06]	[0.09]
University pre 1500	0.09*	0.14*	0.06
	[0.05]	[0.06]	[0.11]
University post 1500	0.14**	0.13	0.05
	[0.06]	[0.06]	[0.08]
Hansa pre 1500	-0.02	-0.02	
	[0.02]	[0.03]	
Hansa post 1500	-0.08	-0.10	
	[0.02]	[0.04]	
Roman city	0.09*	0.07	
	[0.02]	[0.05]	
Log (Population)	0.13***	0.18***	0.14***
	[0.02]	[0.03]	[0.04]
Water (any port)	0.01	0.00	
	[0.03]	[0.03]	
Sea Port	0.22***	0.20**	
	[0.07]	[0.05]	
Number trade routes	0.02***	0.02*	
	[0.01]	[0.01]	
Log(Number of neighbouring	0.05**	0.05	0.05*
cities with brokerage, 100km)	[0.02]	[0.03]	[0.03]
After Plague	0.06**	0.05*	0.06*
	[0.02]	[0.02]	[0.03]
30 Years War	-0.05***	-0.04**	-0.04
	[0.01]	[0.01]	[0.03]
Year	-0.00	-0.00	-0.00
	[0.00]	[0.00]	[0.00]
Longitude	-0.00	-0.00	
	[0.00]	[0.00]	
Latitude	-0.01	-0.01	
	[0.01]	[0.01]	
Observations	1697	1697	1697
No. of City Clusters	225	225	225
No. of Century Clusters	5	5	
R-squared	0.30	0.33	0.54

Notes: ***p < 0.01, **p < 0.05, *p < 0.1. Robust standard errors in brackets, clustered by city in all columns and by century in columns (1)-(2). Results in column (1) report, marginal results for a probit regression, column (2) the estimates for a linear dependent regression, column (3) report linear dependent regression estimates with city and century fixed effects. All regressions incorporate additional institutional variables (beyond the standard specification): whereas institutional effects are divided into a period pre and post 1500.

Table 12: Regression Results: Fee Structure, Fee Structure with Matchmaking Design, and Matchmaking Design With and Without Private Business Constraint (Reporting Control Variables)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Dependent Variable	unit vs value fees			matching with unit vs value fees			matching vs matching without Private business constraint		
Regression type	Probit	LPM	LPM FE	Probit	LPM	LPM FE	Probit	LPM	LPM FE
Finance	-0.47***	-0.47***	-0.47***	-0.71***	-0.46**	-0.41***	0.04	0.01	0.05
	[0.14]	[0.09]	[0.07]	[0.21]	[0.10]	[0.13]	[0.07]	[0.08]	[0.06]
Property	-0.37*	-0.40**	-0.33***	.	-0.45*	-0.42***	-0.07	-0.08	0.03
	[0.29]	[0.14]	[0.07]	.	[0.18]	[0.13]	[0.10]	[0.08]	[0.07]
Horses	-0.49**	-0.52**	-0.53***	.	-0.63***	-0.64***	-0.15	-0.10	0.05
	[0.22]	[0.14]	[0.10]	.	[0.14]	[0.15]	[0.19]	[[0.15]	[0.08]
Wine and beer	0.39***	0.32***	0.35***	0.25***	0.30***	0.30***	0.03	0.02	0.14***
	[0.05]	[[0.10]	[[0.07]	[0.03]	[0.08]	[0.11]	[0.10]	[[0.11]	[0.06]
Grain	0.32***	0.24	0.24**	0.19***	00.27	0.28**	-0.01	-0.02	-0.00
	[0.07]	[0.12]	[0.11]	[0.02]	[0.15]	[0.13]	[0.10]	[0.09]	[0.06]
Fish	0.37***	0.33*	0.32***	0.22***	0.31	0.32**	0.02	0.02	0.04
	[0.05]	[0.14]	[0.09]	[0.03]	[0.17]	[0.15]	[0.07]	[0.06]	[0.04]
Cattle and meat	0.30***	0.22	0.24***	0.06	0.15	0.16	-0.09	-0.07	-0.05
	[0.07]	[0.12]	[0.11]	[0.09]	[[0.15]	[0.15]	[0.19]	[0.17]	[0.12]
Oil and fat	0.40***	0.34**	034***	0.22***	0.33*	0.29**	-0.16**	-0.12*	-0.06
	[0.05]	[0.12]	[0.06]	[0.02]	[0.13]	[0.12]	[0.06]	[0.05]	[0.06]
Construction material	0.33***	0.24	0.23**	0.23***	0.37*	0.34**	-0.08	-0.06	-0.02
	[0.07]	[0.12]	[0.10]	[0.03]	[0.15]	[0.15]	[0.09]	[0.07]	[0.05]
Metal	0.28***	0.13	0.09	0.15***	0.14	0.10	-0.01	-0.02	0.00
	[0.06]	[0.11]	[0.10]	[0.06]	[0.15]	[0.14]	[0.09]	[0.07]	[0.05]
Spices	0.22***	0.04	0.02	0.10	0.02	0.00	-0.07	-0.04	0.03
	[0.05]	[0.06]	[0.07]	[0.06]	[0.10]	[0.12]	[0.13]	[0.10]	[0.06]
Raw textile	0.01	-0.19	-0.20*	--0.17	-0.24	-0.28	0.11*	0.10	0.09
	[0.17]	[0.12]	[0.11]	[0.34]	[0.21]	[0.21]	[0.06]	[0.08]	[0.07]
Fur skin, and leather	0.22***	0.08	0.03	0.08	0.01	-0.03	-0.01	-0.03	0.00
	[0.08]	[0.10]	[0.10]	[0.13]	[0.13]	[0.15]	[0.06]	[0.04]	[0.05]
Cloth	-0.11	--0.28*	-0.29***	-0.53*	-0.45*	-0.49***	0.01	0.03	0.03
	[0.21]	[0.11]	[0.11]	[0.30]	[0.18]	[0.17]	[0.05]	[0.05]	[0.07]
Free Imperial cities	-0.04	-0.03	-0.00	-0.18**	-0.08*	-0.17*	0.21***	0.26**	-0.00
	[0.10]	[0.07]	[0.12]	[0.07]	[0.04]	[0.09]	[0.08]	[0.08]	[0.14]
Bishop	0.04	0.01	-0.97***	-0.26***	-0.15	0.10	0.41***	0.47***	0.06
	[0.08]	[0.04]	[0.19]	[0.06]	[0.08]	[0.21]	[0.08]	[0.07]	[0.23]
University	0.17**	0.12	0.10*	-0.04	0.02	-0.12	0.11*	0.11	-0.26
	[0.07]	[0.06]	[0.05]	[0.13]	[0.04]	[0.09]	[0.06]	[0.11]	[0.18]
Hansa	0.06	-0.07		0.20	0.13		0.08	0.12	
	[0.10]	[0.06]		[0.14]	[0.12]		[0.12]	[0.15]	
Roman	-0.15	-0.08		0.12	0.07		0.00	-0.08	
	[0.16]	[0.08]		[0.18]	[0.06]		[0.10]	[0.10]	
Water (any port)	-0.01	-0.01		0.86***	0.37		-0.07	- 0.15	
	[0.27]	[0.18]		[0.11]	[0.20]		[0.12]	[0.17]	
Sea port	-0.04	-0.03		-0.39*	-0.18		-0.21	--0.18	
	[0.25]	[0.16]		[0.23]	[0.16]		[0.16]	[0.19]	
Number trade routes	-0.01	-0.01		-0.07**	-0.04	-	-0.03	-0.03	
	[0.04]	[0.03]		[0.03]	[0.02]		[0.03]	[0.04]	
Population	-0.00	-0.00	0.00	-0.00	0.00	0.00	-0.00	-0.00	-0.00
	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
Year	-0.00	-0.00	-0.00	-0.00***	-0.00*	-0.00	-0.00*	-0.00	0.00
	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]	[0.00]
Latitude	-0.07	--0.04		-0.05	-0.04		0.09***	0.08	
	[0.05]	[0.03]		[0.05]	[0.04]		[0.03]	[0.04]	
Longitude	-0.01	-0.01		0.00	0.00		0.01	0.01	
	[0.02]	[0.01]		[0.03]	[0.02]		[0.01]	[0.01]	
Observations	680	680	680	325	361	361	631	631	631
No. of City Clusters	57	57	57	38	38	38	56	56	56
No. of Century Clusters	5	5		5	5		5	5	
R-squared	0.39	0.41	0.57	0.54	0.58	0.64	0.37	0.37	0.66

Notes: ***p < 0.01, **p < 0.05, *p < 0.1. Robust standard errors clustered by city code and century (columns (1),(2), (4), (5) (7), (8)) in brackets. Column (1)-(3) report the estimates for the fee structure including the control variables, starting with marginal results for a probit regression in Column (1), a linear probability model in Column (2) and linear probability model with town and century fixed effects in Column (3). Analogously, Columns (4)-(6) report the fee structure with the matching design with control variables, and finally columns (7)-(9) report the comparison of the matching design with the matching design without private business constraints, again starting with a probit specification, and a linear probability model without and with fixed effects.

A5 Preliminaries for Theorem Proofs

A5.1 Nash bargaining solution

We assume that when a buyer and seller are compatible, they trade at the Nash price, which is the solution to

$$\max_p \left(U(v_b, q_s) - p\right)^{\phi} \left(p - \delta(p) - c_s\right)^{1-\phi}$$

In the case of unit fees, $\delta(p)$ is a constant u; we can take logs and solve

$$p \;=\; \arg\max_p \left(\phi \ln(U(v_b, q_s) - p) + (1-\phi)\ln(p - u - c_s)\right)$$

The first-order condition

$$\frac{\phi}{U(v_b, q_s) - p} \;=\; \frac{1-\phi}{p - u - c_s}$$

leads to the unique solution

$$p \;=\; (1-\phi)U(v_b, q_s) + \phi c_s + \phi u$$

In the case of percentage fees, $\delta(p) = f \cdot p$ is a fixed fraction of the price paid. The Nash problem then becomes

$$p \;=\; \arg\max_p \left(\phi \ln(U(v_b, q_s) - p) + (1-\phi)\ln((1-f)p - c_s)\right)$$

with first-order condition

$$\frac{\phi}{U(v_b, q_s) - p} \;=\; \frac{(1-\phi)(1-f)}{(1-f)p - c_s}$$

giving solution

$$p \;=\; (1-\phi)U(v_b, q_s) + \frac{\phi}{1-f} c_s$$

Below, we let $P(b, s)$ refer to the Nash price.

A5.2 Notation and terminology

Following standard terminology from the two-sided matching literature, a *matching* refers to a mapping

$$M \;:\; \mathcal{B} \cup \mathcal{S} \;\to\; \mathcal{B} \cup \mathcal{S}$$

satisfying the conditions that (i) for any $b \in \mathcal{B}$, $M(b) \in \mathcal{S} \cup \{b\}$; (ii) for any $s \in \mathcal{S}$, $M(s) \in \mathcal{B} \cup \{s\}$; and (iii) for any $(b, s) \in \mathcal{B} \times \mathcal{S}$, $M(b) = s$ if and only if $M(s) = b$; that is, each buyer matches either with him- or herself or with a seller who also matches with him or her; "matching with yourself" means you don't trade.

In the text, we defined a buyer and seller as being *compatible* if they generate enough surplus through trade to strictly cover the broker's fee. Fixing the type and level of fees, we'll say a matching M is *feasible* if for every b such that $M(b) \in \mathcal{S}$, b and $M(b)$ are compatible. Let $\mathcal{M}$ denote the set of feasible matchings. We assume the broker can select any matching in $\mathcal{M}$, and chooses the one that maximizes his or her commissions For $M \in \mathcal{M}$, let $B(M) = \{b : M(b) \in \mathcal{S}\}$ be the set of buyers who trade; $S(M) = \{s : M(s) \in \mathcal{B}\}$ the set of sellers who trade; let $T(M) = \|B(M)\| = \|S(M)\|$ the number of trades; and $P(M) = \sum_{b \in B(M)} P(b, M(b))$ the sum of the prices traded at.

A5.3 Broker's preferences over matchings

Under unit fees, the broker's problem is

$$\max_{M \in \mathcal{M}} \left\{ \sum_{b \in B(M)} u \cdot \left(1 - e^{-K(U(v_b, q_{M(b)}) - c_{M(b)} - u)}\right) \right\}$$

and under percentage fees, it is

$$\max_{M \in \mathcal{M}} \left\{ \sum_{b \in B(M)} f \cdot P(b, M(b)) \cdot \left(1 - e^{-K(U(v_b, q_{M(b)}) - c_{M(b)} - f \cdot P(b, M(b)))}\right) \right\}$$

where $P(b,s) = (1-\phi)v_b q_s + \frac{\phi}{1-f} c_s$ is the Nash price when buyer b trades with seller s. Since we're taking the limit $K \to \infty$, the e^{-K} terms vanish and therefore only matter in the case where the seller would otherwise be indifferent between two matchings, i.e., to determine preferences among two matchings with the same number of trades (under unit fees) or the same sum of prices (under percentage fees). This leads to the following:

Claim 1. *Pick two feasible matchings $M, M' \in \mathcal{M}$. Calculate the joint surplus $U(v_b, q_{M(b)}) - c_{M(b)} - \delta$ earned by each buyer-seller pair matched under M, and sort them in ascending order, letting d_1 denote the smallest, d_2 the second-smallest, and so on; likewise for M' with $d'_1 \leq d'_2$ and so on.*

1. *A broker charging unit fees prefers M to M' if any of the following holds:*
   - *$T(M) > T(M')$ (M gives more trades than M')*
   - *$T(M) = T(M')$, and $d_1 > d'_1$*
   - *$T(M) = T(M')$, and for some $k > 1$, $d_k > d'_k$ and $d_i = d'_i$ for every $i < k$*
2. *A broker charging percentage fees prefers M to M' if any of the following holds:*
   - *$P(M) > P(M')$ (the sum of prices is higher under M than under M')*
   - *$P(M) = P(M')$, and $d_1 > d'_1$*
   - *$P(M) = P(M')$ and for some $k > 1$, $d_k > d'_k$ and $d_i = d'_i$ for every $i < k$*

Thus, under unit fees, the broker first maximizes the number of transactions and then has lexicographic preferences among matchings with the same number of trades to maximize the smallest gains-from-trade of all the transactions, then the second-smallest, then the third-smallest, and so on. Likewise, under percentage fees, the broker first maximizes total prices paid, and if two matchings give the same combined prices, the broker then has lexicographic preferences to maximize the smallest gains-from-trade, then the second-smallest, and so on.

Proof of Claim 1. Under unit fees, the difference in expected commissions between M and M' can be written as

$$(T(M) - T(M'))u + u \left[\sum_{b \in B(M')} e^{-K(U(v_b, q_{M'(b)}) - c_{M'(b)} - u)} - \sum_{b \in B(M)} e^{-K(U(v_b, q_{M(b)}) - c_{M(b)} - u)} \right]$$

Since (in order for M and M' to be feasible) $U(v_b, q_s) - c_s - u > 0$ for each term in both sums, every term in both sums goes to 0 as $K \to \infty$; therefore, this is obviously positive if $T(M) > T(M')$. When $T(M) = T(M') = T$, we can rewrite this difference as

$$u\left[\sum_{i=1}^{T} e^{-Kd'_i} - \sum_{i=1}^{T} e^{-Kd_i}\right] = ue^{-Kd'_1}\left[1 + \sum_{i=2}^{T} e^{-K(d'_i - d'_1)} - \sum_{i=1}^{T} e^{-K(d_i - d'_1)}\right]$$

If $d'_1 < d_1$ (and therefore $d'_1 < d_i$ for every i), every term in the second sum vanishes, making the entire expression positive. If $d_i = d'_i$ for every $i < k$, we can rewrite the difference as

$$u\left[\sum_{i=k}^{T} e^{-Kd'_i} - \sum_{i=k}^{T} e^{-Kd_i}\right] = ue^{-Kd'_k}\left[1 + \sum_{i=k+1}^{T} e^{-K(d'_k - d'_1)} - \sum_{i=k}^{T} e^{-K(d_i - d'_k)}\right]$$

and again, if $d'_k < d_k \leq d_i$ for every i in the second sum, the second sum vanishes as K grows, and the entire expression is positive.

Under percentage fees, all the e^{-Kd_i} terms likewise vanish, making the broker strictly prefer M to M' when the sum of prices paid is higher. When the sum of prices paid is the same, the difference in commissions is then

$$\left[\sum_{i=1}^{T} f \cdot P'_i \cdot e^{-Kd'_i} - \sum_{i=1}^{T} f \cdot P_i \cdot e^{-Kd_i}\right]$$

where P_i is the price paid in the transaction corresponding to d_i (and P'_i to d'_i). Rewriting this as

$$f\left[\sum_{i=1}^{T} e^{\ln(P'_i)} e^{-Kd'_i} - \sum_{i=1}^{T} e^{\ln(P_i)} e^{-Kd_i}\right] = f\left[\sum_{i=1}^{T} e^{-K\left(d'_i - \frac{1}{K}\ln(P'_i)\right)} - \sum_{i=1}^{T} e^{-K\left(d_i - \frac{1}{K}\ln(P_i)\right)}\right]$$

makes it clear that as $K \to \infty$, even the price paid in each transaction vanishes in importance next to d_i, and the preferences (among matchings with the same total prices) are the same as in the unit fees case. □

A6 Theorem 1—when brokerage outperforms random matching

For ease of notation, rescale v_b and t_s such that F_b and F_s both have support $[0, 1]$. (This is without loss of generality since the functions $U(\cdot, \cdot)$, $q(\cdot)$, and $c(\cdot)$ are so loosely defined.) Let $I(v_b, t_s)$ be an indicator function for whether a buyer and seller type are compatible, i.e., can generate sufficient surplus net of broker's commissions to trade. Note that when a buyer with type v_b and a seller with type t_s are paired up, the social surplus generated is $I(v_b, t_s)[U(v_b, q(t_s)) - c(t_s)]$, since the surplus of $U - c$ is realized if the two are able to trade. (The broker's fee $\delta(p)$ is lost to the buyer and seller but gained by the broker; therefore, it is not subtracted from social surplus.)

A6.1 Case 1: many sellers, costs dominate

Consider the case where there are many sellers for each buyer, that is, the case in which $\frac{N_b}{N_s}$ is small, and suppose costs dominate. Under unit fees, the broker's incentive is to pair up as many compatible buyer-seller pairs as possible; this means if there are sufficiently many sellers per buyer, every buyer will trade. Further, the broker's incentive is to use the "best" sellers, i.e., those that create the highest surplus, as this minimizes the probability a trade fails; when costs dominate, the "best" sellers are the low-cost ones. Thus, when costs dominate, if there is a seller type $\underline{t}$ such that (a) every buyer is compatible with all sellers with types below $\underline{t}$ and (b) there are more sellers with types below $\underline{t}$ than there are buyers, then under unit fees, every buyer will buy from a seller with type below $\underline{t}$. By making $\frac{N_b}{N_s}$ large enough, we can make this true for $\underline{t}$ low enough that each type of buyer v_b creates more expected surplus under unit fees than under random matching.

If costs dominate, then the match surplus $U(v_b, q(t)) - c(t)$ is decreasing in t for every v_b; since all buyer types are compatible with at least some seller types (by assumption), all buyer types are compatible with sellers of the lowest type $t_s = 0$. Define

$$x(v_b) \quad = \quad \frac{U(v_b, q(0)) - c(0)}{\int_0^1 I(v_b, t)[U(v_b, q(t)) - c(t)]dF_s(t)}$$

The numerator is the surplus generated when buyer v_b is paired with seller $t_s = 0$; the denominator is the expected surplus when buyer v_b is paired with a random seller. As long as seller types are not all the same, the fraction must be strictly greater than 1 for every v_b. Both the numerator and denominator are continuous in v_b (and the denominator is nonzero), so $x(v_b)$ is continuous; thus, it achieves a minimum over $[0, 1]$, meaning

$$x \quad \equiv \quad \inf_{v_b} x(v_b) \quad = \quad \min_{v_b} x(v_b) \quad > \quad 1$$

Thus, now $x > 1$, and for every v_b,

$$U(v_b, q(0)) - c(0) \quad \geq \quad x \int_0^1 I(v_b, t)[U(v_b, q(t)) - c(t)]dF_s(t)$$

By continuity, for t_s sufficiently small,

$$I(v_b, t_s)[U(v_b, q(t_s)) - c(t_s)] \quad > \quad \sqrt{x} \int_0^1 I(v_b, t)[U(v_b, q(t)) - c(t)]dF_s(t)$$

for every v_b; define t^* such that this holds at $t_s = t^*$ for every v_b. Note that this requires every buyer to be compatible with all sellers with types below t^*.

Now set $k_* = F_s(t^*)$ so that if $\frac{N_b}{N_s} < k_*$, the expected number of sellers with types below t^*, $N_s F_t(t^*)$, is strictly greater than N_b the number of buyers. As (N_b, N_s) become large, the probability goes to 1 that there are at least as many sellers with types below t^* as there are buyers; thus, for sufficiently large markets, this probability is greater than $\frac{1}{\sqrt{x}}$. As noted above, this means that with probability at least $\frac{1}{\sqrt{x}}$, every buyer will be paired with a seller with type below t^*; thus, for any buyer type v_b, the expected surplus generated by such a buyer is at least

$$\frac{1}{\sqrt{x}}[U(v_b, q(t^*)) - c(t^*)] \quad > \quad \int_0^1 I(v_b, t)[U(v_b, q(t)) - c(t)]dF_s(t)$$

Thus, every buyer generates strictly more expected surplus under unit fees than under random matching; adding across buyers gives the result.

A6.2 Case 2: many sellers, quality dominates

When quality dominates, $U(v_b, q(t)) - c(t)$ is increasing in t, and every buyer is compatible with sellers with types close to 1. We can let

$$x = \min_{v_b} \frac{U(v_b, q(1)) - c(1)}{\int_0^1 I(v_b, t)[U(v_b, q(t)) - c(t)]dF_s(t)}$$

and define t^* so that

$$I(v_b, t^*)[U(v_b, q(t^*)) - c(t^*)] > \sqrt{x} \int_0^1 I(v_b, t)[U(v_b, q(t)) - c(t)]dF_s(t)$$

for every v_b. Let $k_* = 1 - F_s(t^*)$; if $\frac{N_b}{N_s} < k_*$, then the expected number of sellers with types above t^*, $N_s(1 - F_s(t^*))$, is greater than N_b. Once again, as N_b and N_s become large, the probability goes to 1 (and is therefore above $\frac{1}{\sqrt{x}}$) that there will be more sellers with types above t^* than total buyers.

Under percentage fees, the broker has an incentive to use the highest-quality (highest-type) sellers available, as price (therefore commission) is increasing in seller type; and under unit fees, the broker has an incentive to use the highest-type sellers as well, since they generate greater trade surplus and thus are less likely to fail to agree. Thus, under either type of fees, if there are more sellers with types above t^* than there are buyers, every buyer will trade with a seller with type above t^*. Thus, under percentage fees, the expected surplus generated by a buyer with type v_b is at least

$$\frac{1}{\sqrt{x}}[U(v_b, q(t^*)) - c(t^*)] > \int_0^1 I(v_b, t)[U(v_b, q(t)) - c(t)]dF_s(t)$$

and is higher than under random matching.

A6.3 Case 3: many buyers, costs dominate

When there are instead many buyers for each seller, we'll focus on the expected surplus generated by each seller. Note that any seller is compatible with buyers with types close to 1 and that with either unit or percentage fees, the broker will again want to (a) pair up every seller when possible and (b) use the highest-type buyers available. Let

$$x = \min_{t_s} \frac{U(1, q(t_s)) - c(t_s)}{\int_0^1 I(v, t_s)[U(v, q(t_s)) - c(t_s)]dF_b(v)}$$

Note that $x > 1$, and now pick v^* such that

$$U(v^*, q(t_s)) - c(t_s) > \sqrt{x} \int_0^1 I(v, t_s)[U(v, q(t_s)) - c(t_s)]dF_b(v)$$

for every t_s; i.e., every seller type t_s generates greater surplus by matching with any buyer with $v_b > v^*$ than by pairing up at random. Let $k^* = \frac{1}{1-F_b(v^*)}$. If $\frac{N_b}{N_s} > k^*$, then the expected number of buyers with types above v^*, $N_b(1 - F_b(v^*))$, is greater than N_s. As (N_b, N_s) become large, the probability is above $\frac{1}{\sqrt{x}}$ that there are more buyers with types above v^* than sellers and therefore that every seller is paired with a buyer above v^*; under brokerage with either type of fees, a seller with type t_s therefore generates an expected surplus of at least

$$\frac{1}{\sqrt{x}}[U(v^*, q(t_s)) - c(t_s)] \quad > \quad \int_0^1 I(v, t_s)[U(v, q(t_s)) - c(t_s)] dF_b(v)$$

and therefore more surplus than under random matching.

A6.4 Case 4: many buyers, quality dominates

When quality dominates and the broker earns either unit or percentage fees, the incentive is still to use the highest-type buyers available; the analysis is identical to Case 3.

A7 Theorem 2 – first-best in big unbalanced markets

To go from Theorem 1 to Theorem 2, we note the following result:

Claim 2. *Define a trader as* universal *if he or she is compatible with every trader on the other side of the market.*

1. *Suppose costs dominate. If there are more universal buyers than total sellers* or *more universal sellers than total buyers, then unit fees lead to the exact matching that maximizes total surplus.*
2. *Suppose quality dominates. If there are more universal buyers than total sellers or more universal sellers than total buyers, then percentage fees lead to the exact matching that maximizes total surplus.*

Proof of Claim 2. First, suppose costs dominate. Under unit fees, as noted above, the broker's incentive is to pair up as many compatible buyer-seller pairs as possible and then maximize the surplus of the "smallest-surplus" pair, then the second-smallest, and so on. If there are at least N_s universal buyers, then every seller will trade, and the broker maximizes the surplus within each pair by using the highest-type buyers, who are the universal ones. Further, the broker maximizes the surplus of the "smallest-surplus" pair by pairing the weakest seller with the strongest buyer, the second-weakest with the second-strongest, and so on. (Since all buyers trading are universal, we don't have to worry about which buyers are compatible with which sellers.) When costs dominate, the "weakest" seller is the highest-cost, who is also the highest-quality; given the supermodularity of $U(\cdot, \cdot)$, the broker therefore pairs the highest-quality seller with the buyer willing to pay most for quality and so on, which maximizes total surplus. On the other hand, if there are at least N_b universal sellers, the broker chooses the N_b "strongest" sellers, who are the lowest-cost (and therefore the highest-surplus-generating), and again pairs "weakest" to "strongest," again pairing the highest-type buyer with the highest-quality seller trading and again maximizing total surplus.

Second, suppose quality dominates. If there are at least N_s universal buyers, then the broker maximizes commissions by pairing the N_s highest-type buyers to the N_s sellers, pairing them up assortatively (given the supermodularity of U and no worries about who is compatible with whom). Likewise, if there are at least N_b universal sellers, the broker uses the N_b highest-type sellers, who are all universal, and pairs them assortatively with the N_b buyers. In both cases, this is the matching that maximizes total surplus. □

We showed in the proof of Theorem 1 that for $\frac{N_b}{N_s} > k^*$, as the market becomes large, the probability goes to 1 that there are more than N_s universal buyers; in addition, for $\frac{N_b}{N_s} < k_*$, as the market becomes large, the probability goes to 1 that there are more than N_b universal sellers. Theorem 2 follows. □

A8 Theorem 3 – unit vs percentage – efficiency

A corollary of Claim 1 is that under the genericity assumption in the text (that any two traders have distinct types), when either costs or quality dominates, the broker always has a unique optimal matching. This is not vital on its own but simplifies the proofs below. We will let M^u refer to the broker's most preferred matching under unit fees and M^p the most preferred matching under percentage fees.

A8.1 Proof of Theorem 3 part 1

We need to show that when costs dominate and fees are small, M^u is more efficient than M^p. To show this, we will characterize the matching chosen by the broker under both types of fees. Consider the following algorithm for matching buyers and sellers.

Algorithm 1. *Let M^* denote the matching that results from the following algorithm, and $t^* = T(M^*)$ the number of trades.*

1. *Renumber the buyers from "best" to "worst" so that $v_1 > v_2 > \ldots > v_B$*
2. *Renumber the sellers from "best" to "worst" as well; since we are in the costs-dominate case, this means $c_1 < c_2 < \ldots < c_S$ and $q_1 \leq q_2 \leq \ldots \leq q_S$*
3. *For $i = 1$ to B...*
   - *If buyer i is compatible with any seller who is not yet paired with a buyer, pair buyer i with the* worst *(highest-numbered) unpaired seller he or she is compatible with.*
   - *If buyer i is not compatible with any seller who is not yet paired with a buyer, terminate the algorithm.*

We will show that in the costs-dominate case $M^p = M^*$ (this algorithm maximizes percentage fees), $T(M^u) = T(M^*)$ (this algorithm also maximizes the number of trades), and M^u is the most efficient matching with that many trades and is thus at least as efficient as M^p.

Claim 3. *Algorithm 1 maximizes the number of trades: for any $M' \in \mathcal{M}$, $T(M') \leq T(M^*)$.*

Proof of Claim 3. Suppose there existed a matching M' with $T(M') > T(M^*)$. Let B' and S' be the set of buyers and the set of sellers, respectively, who trade at M', and B^* and S^* the sets of buyers and sellers who trade at M^*. Note from the algorithm that $B^* = \{1, 2, \ldots, t^*\}$.

Note that if you start with a feasible matching $\widetilde{M}$ and replace one of the buyers in it with a *higher*-value buyer who doesn't trade at $\widetilde{M}$, the resulting matching is still feasible. Similarly, if you start with a feasible matching $\widetilde{M}$ and remove a trade, the resulting matching is feasible. Thus, if a feasible matching exists with more than t^* trades, one exists where the set of buyers who trade is exactly $\{1, 2, \ldots, t^* + 1\}$. Thus, assume without loss of generality that $B' = \{1, 2, \ldots, t^* + 1\}$.

Note also that (under either unit or percentage fees) with the sellers ranked best-to-worst, if a buyer is compatible with seller k, the buyer is compatible with any seller $k' < k$ and that if the buyer is not compatible with seller k, he or she is not compatible with any seller $k' > k$.

Now, by assumption, buyer $t^* + 1$ trades at M'. Let $s_{(1)}$ be the seller he or she buys from. If $s_{(1)} \notin S^*$, then seller $s_{(1)}$ would have still been available at step $t^* + 1$ of Algorithm 1 when M^* was determined. Since buyer $t^* + 1$ doesn't trade at M^*, the algorithm must have terminated then, which means no seller he or she was compatible with was available. Thus, $s_{(1)} \in S^*$. Let $b_{(1)} \in B^*$

be the buyer he or she trades with at M^*. Since $B^* \subset B'$, $b_{(1)}$ also trades at M'; let $s_{(2)}$ be the seller he or she buys from.

Now let's think about who $s_{(2)}$ could be. He or she cannot be worse (higher-cost) than $s_{(1)}$ and not in S^*, because then, buyer $b_{(1)}$ wouldn't have paired with $s_{(1)}$ under M^*, since there was someone worse available with whom he or she was compatible. $s_{(2)}$ also can't be better (lower-cost) than $s_{(1)}$ and not in S^*, because we know buyer t^*+1 is compatible with $s_{(1)}$ (since they trade at M'); thus, he or she would also be compatible with $s_{(2)}$, who would have been available when Algorithm 1 terminated at step t^*+1. Thus, $s_{(2)}$ must be in S^*; i.e., he or she must be a seller who trades at M^*. Let $b_{(2)}$ be the buyer he or she sells to at M^*. Since $B^* \subset B'$, $b_{(2)}$ trades at M'; let $s_{(3)}$ be the seller he or she buys from at M'.

Next, we consider who $s_{(3)}$ could be. If he or she were worse than $s_{(2)}$ and not in S^*, then the algorithm would not have paired $b_{(2)}$ with $s_{(2)}$, since $s_{(3)}$ was higher-cost, compatible, and available. If he or she were worse than $s_{(1)}$ but better than $s_{(2)}$ and not in S^*, then the algorithm wouldn't have paired $b_{(1)}$ with $s_{(1)}$, because buyer 1 is compatible with $s_{(2)}$ and therefore with $s_{(3)}$ and $s_{(3)}$ would have been available and higher-cost when $b_{(1)}$ was paired with $s_{(1)}$. Finally, if he or she were better than $s_{(1)}$ and not in S^*, he or she would have been available and compatible when buyer t^*+1 went unmatching in the algorithm, which is impossible. Thus, it must be that $s_{(3)} \in S^*$.

With identical logic, we can show that every seller who trades in M', must also trade in M^*; however, this implies $S' \subseteq S^*$, which is impossible since by assumption $|S'| > |S^*|$. The contradiction proves there cannot be a matching with more than t^* trades and therefore that M^* maximizes the number of trades. □

Claim 4. *The matching chosen under unit fees M^u matches buyer 1 to seller t^*, buyer 2 to seller t^*-1, buyer 3 to seller t^*-2, and so on; M^u therefore maximizes the total surplus among all matchings with t^* trades.*

Proof of Claim 4. We know that $T(M^u) = \max_{M \in \mathcal{M}} T(M) = t^*$. If any buyer outside of the t^* highest-value buyers was trading at M^u, the broker could increase the probability of that trade occurring by replacing that buyer with the unmatched higher-value buyer. Similarly, since (under the costs-dominate case) a lower-cost seller always generates more surplus when paired with any buyer, the broker could increase the probability of a trade occurring by replacing any seller not among the t^* cheapest with a cheaper seller. Thus, $B(M^u) = \{1, 2, \ldots, t^*\}$, and $S(M^u) = \{1, 2, \ldots, t^*\}$.

To show that given these traders, the broker chooses to match them reverse-assortatively (with the "best" buyer paired with the "worst" seller), suppose there were two buyers with $v_b > v_{b'}$ and two sellers with $c_s < c_{s'}$ and $q_s \leq q_{s'}$. (Thus, b is the "good" buyer, and s the "good" seller, despite s' being higher-quality.) We claim that the broker earns higher expected fees by pairing them reverse-assortatively (b to s' and b' to s) than assortatively (b to s and b' to s').

First, note that if both trades can occur when they are matched assortatively, both can occur when they are swapped: if the worse buyer is compatible with the worse seller, then he or she is also compatible with the better seller, and the better buyer is compatible with the worse seller.[44] Next,

$$e^{-K(U(v_b,q_s)-c_s)} + e^{-K(U(v_{b'},q_{s'})-c_{s'})} \quad > \quad e^{-K(U(v_{b'},q_s)-c_s)} + e^{-K(U(v_b,q_{s'})-c_{s'})}$$

as K becomes large, since $U(v_{b'}, q_{s'}) - c_{s'}$ is the smallest of the four combinations (and as K becomes large, the smallest-gain term dominates). From this, we can calculate

$$2 - e^{-K(U(v_b,q_s)-c_s-u)} - e^{-K(U(v_{b'},q_{s'})-c_{s'}-u)} \quad < \quad 2 - e^{-K(U(v_{b'},q_s)-c_s-u)} - e^{-K(U(v_b,q_{s'})-c_{s'}-u)}$$

[44]Formally, if $U(v_{b'}, q_{s'}) - c_{s'} > u$, then $(v_{b'}, q_s) - c_s > U(v_{b'}, q_{s'}) - c_{s'} > u$ and $U(v_b, q_{s'}) - c_{s'} > U(v_{b'}, q_{s'}) - c_{s'} > u$.

Thus, matching assortatively gives fewer expected trades than matching reverse-assortatively. Thus, for any matching between the best t^* buyers and the best t^* sellers that did not match them perfectly reverse-assortatively, the broker could increase his or her expected commission by swapping two assortatively-ranked buyer/seller pairs; therefore, M^u must match reverse-assortatively.

To see this is also maximally efficient, first note that if there is strict variation in quality across sellers, then $q_1 < q_2 < \ldots < q_{t^*}$; thus, matching reverse-assortatively on cost actually matches assortatively on quality and willingness-to-pay-for-quality, which is efficient. When the sellers all have identical quality, all matchings among these traders are equally efficient if we ignore the possibility of failed trades; the expected surplus lost to a trade failing to happen is

$$(U(v,q)-c)e^{-K(U(v,q)-c-u)} \quad = \quad e^{\ln(U(v,q)-c)}e^{-K(U(v,q)-c-u)} \quad = \quad e^{-K\left(-\frac{\ln(U(v,q)-c)}{K}+U(v,q)-c-u\right)}$$

As K becomes large, the $-\frac{\ln(U(v,q)-c)}{K}$ term is dominated by the other terms; thus, minimizing the expected surplus lost is the same as minimizing the expected number of failed trades, which the broker is already doing. Thus, M^u maximizes efficiency among all matchings with t^* trades. □

Claim 5. *Algorithm 1 maximizes percentage fees, and therefore, $M^p = M^*$.*

Proof of Claim 5. We first claim that for any given set of buyers who will trade, matching them to sellers according to Algorithm 1 (after sorting them best-to-worst) maximizes percentage fees. To see this, recall first that given a buyer i and seller j, the price paid is

$$(1-\phi)U(v_i,q_j)+\frac{\phi}{1-f}c_j$$

With costs dominating and sellers sorted best-to-worst, both q_j and c_j are increasing in j; thus, to maximize percentage fees, the broker will always select the "best" (highest value) buyers but the "worst sellers he can get away with"—he or she will never pair a buyer b with seller j if another seller $j' > j$ is compatible with b but unmatched.

Next, we noted above that if two buyers and two sellers can generate two trades when matched assortatively (better buyer with lower-cost seller), they can still generate two trades when matched the other way (better buyer with higher-cost/higher-quality seller); making such a switch also increases fees: if $v_b > v_{b'}$, $c_s < c_{s'}$ and $q_s \leq q_{s'}$, then the combined price of the two transactions is

$$(1-\phi)U(v_b,q_s)+(1-\phi)U(v_{b'},q_{s'})+\frac{\phi}{1-f}c_s+\frac{\phi}{1-f}c_{s'}$$

when they are matched assortatively and

$$(1-\phi)U(v_{b'},q_s)+(1-\phi)U(v_b,q_{s'})+\frac{\phi}{1-f}c_s+\frac{\phi}{1-f}c_{s'}$$

when they are matched reverse-assortatively. Since U is supermodular (and $v_b > v_{b'}$ but $q_s \leq q_{s'}$), the latter is weakly higher.

Finally, note that since buyers are ordered best-to-worst ($v_1 > v_2 > \ldots > v_B$), for each buyer $i \in \{1,2,\ldots,t^*\}$, there must be at least t^*+1-i sellers outside of the set $\{M^*(b_j)\}_{j<i}$ with whom i is compatible. (This is because Algorithm 1 matched buyer i and t^*-i subsequent worse buyers to sellers, even after all the sellers in $\{M^*(b_j)\}_{j<i}$ had already been assigned.)

Having established those preliminaries, take any matching M'. We will show that we can change it to M^* through a series of steps that all weakly increase percentage fees and therefore that M^* must maximize percentage fees. Recall that M^* maximized the number of trades; thus, $T(M') \leq T(M^*)$. We modify M' via the following steps:

1. If any buyer $b > t^*$ trades at M', replace him or her with a buyer from $\{1, \ldots, t^*\}$ who is not trading. Since each of these switches increases the type of a buyer without affecting anything else, this increases fees.

2. For $i = 1$ to t^*...

   (a) At this stage in the algorithm, note that each buyer $j < i$ is paired with seller $M^*(b_j)$. (This holds vacuously when $i = 1$; we will note at the end of this step that it still holds when we increment i.)

   (b) If buyer i is not already buying, add him or her and pair him or her with any seller who's not yet selling. (As noted above, there are at least $t^* + 1 - i$ sellers buyer i could buy from outside of $\{M^*(b_j)\}_{j<i}$; since no matching can have more than t^* trades, there are at most $t^* - i$ sellers outside of $\{M^*(b_j)\}_{j<i}$ currently paired with a buyer; therefore, there must be an unpaired seller buyer i could trade with.)

   (c) If (either after being added or because he or she was already trading) buyer i is paired with a seller other than $M^*(b_i)$ and seller $M^*(b_i)$ is not trading, replace the seller b_i is trading with $M^*(b_i)$. Since (by Algorithm 1) $M^*(b_i)$ was the "worst" seller outside of $\{M^*(b_j)\}_{j<i}$ that seller i could trade with, this increases fees.

   (d) If (either after being added or because he or she was already trading) buyer i is paired with a seller other than $M^*(b_i)$ and seller $M^*(b_i)$ is paired with a different buyer, then switch the two sellers. Note that the seller b_i is expected to trade with must be "better" (lower-cost/lower-quality) than $M^*(b_i)$, since the sellers $\{M^*(b_j)\}_{j<i}$ are all paired with buyers $j < i$ and $M^*(b_i)$ was defined as the worst seller outside of $\{M^*(b_j)\}_{j<i}$ that b_i can trade with. Thus, the switch must be feasible—we know b_i can trade with $M^*(b_i)$, whoever is currently matched to $M^*(b_i)$ is being asked to switch to a better seller—and that since this switch pairs the better of two buyers with the worse of two sellers rather than the better, it raises fees.

   (e) At this stage in the algorithm, note that each buyer $j \leq i$ is paired with seller $M^*(b_j)$ and that fees are weakly higher than before this iteration of the loop. Iterate i, and continue.

3. Let $\widetilde{M}$ denote the matching when the loop ends.

At $\widetilde{M}$, each buyer $i \in \{1, 2, \ldots, t^*\}$ is paired with seller $M^*(b_i)$. Since no feasible matching has more than t^* trades, no other traders are trading, and therefore, $\widetilde{M} = M^*$. In addition, since each step of the algorithm weakly increased fees, M^* earns weakly higher fees than M'. Thus, M^* must maximize percentage fees over all feasible matchings; thus, $M^p = M^*$. □

Finally, stringing together Claims 3, 4, and 5, we see that $M^p = M^*$, $T(M^u) = T(M^*)$, and M^u is the most efficient matching with $T(M^*)$ trades; thus, M^u must yield weakly higher surplus than M^p, which was part 1 of the theorem. □□

A8.2 Proof of Theorem 3 part 2

We need to show that when quality dominates, M^p is more efficient than M^u. We do this by showing (i) that M^p is the most efficient matching with $|M^p|$ trades and (ii) that (roughly) M^p has closer to the efficient number of trades than M^u.

We again order both buyers and sellers from best to worst; since now quality dominates, this means $v_1 > v_2 > \ldots > v_B$, $q_1 > q_2 > \ldots > q_S$, and $c_1 \geq c_2 \geq \ldots > c_S$. Let t^* denote the maximal number of trades over all feasible matchings. For $t \leq t^*$, let M_t^e be the most efficient matching with t trades and M_t^p the matching with t trades that maximizes percentage fees.

Claim 6. *Under both M_t^e and M_t^p, buyers $\{1, 2, \ldots, t\}$ and sellers $\{1, 2, \ldots, t\}$ trade.*

Proof of Claim 6. With quality dominating and buyers and sellers both indexed best-to-worst, the surplus created by a given trade, $U(v_i, q_j) - c_j$, is strictly decreasing in both i and j, as is the price at which the trade occurs, $(1-\phi)U(v_i, q_j) + \frac{\phi}{1-f}c_j$. Thus, if any buyer $i > t$ or any seller $j > t$ is trading, replacing him or her with a buyer $i' \leq t$ or a seller $j' \leq t$ who is not trading strictly increases both the total surplus and total percentage fees. □

Claim 7. *Consider two matchings M_1 and M_2 in which the same buyers and the same sellers trade. M_1 is more efficient than M_2 if and only if it generates higher percentage fees.*

Proof of Claim 7. Let $\widetilde{B}$ be the set of buyers who trade in either matching and $\widetilde{S}$ the set of sellers who trade. For $i \in \{1, 2\}$, the total surplus generated by matching M_i is

$$\sum_{b \in \widetilde{B}} \left(U(v_b, q_{M_i(b)}) - c_{M_i(b)} \right) \quad = \quad \sum_{b \in \widetilde{B}} U(v_b, q_{M_i(b)}) - \sum_{s \in \widetilde{S}} c_s$$

Total percentage fees are proportional to the sum of the prices of each transaction, which is

$$\sum_{b \in \widetilde{B}} \left((1-\phi)U(v_b, q_{M_i(b)}) + \frac{\phi}{1-f} c_{M_i(b)} \right) \quad = \quad (1-\phi) \sum_{b \in \widetilde{B}} U(v_b, q_{M_i(b)}) + \frac{\phi}{1-f} \sum_{s \in \widetilde{S}} c_s$$

Thus, M_1 generates both more total surplus and higher percentage fees than M_2 if and only if $\sum_{b \in \widetilde{B}} U(v_b, q_{M_1(b)}) > \sum_{b \in \widetilde{B}} U(v_b, q_{M_2(b)})$. □

Claim 8. *M_t^p maximizes total surplus among all matchings with t trades ($M_t^p = M_t^e$).*

Proof of Claim 8. This follows directly from Claims 6 and 7. Claim 6 establishes that the same set of traders trade under M_t^p and M_t^e; Claim 7 establishes that among all matchings where that set of traders trade, the one maximizing total surplus and the one maximizing percentage fees are the same. □

Claim 9. *Let $t < t'$. If M_t^p generates higher percentage fees than $M_{t'}^p$, then it is more efficient.*

Proof of Claim 9. Rewrite the price at which a trade occurs as

$$(1-\phi)U(v_b, q_s) + \frac{\phi}{1-f} c_s \quad = \quad (1-\phi)\left(U(v_b, q_s) - c_s\right) + \left(1 - \phi + \frac{\phi}{1-f}\right) c_s$$

and let $X = 1 - \phi + \frac{\phi}{1-f}$. If M^p_t gives higher fees than $M^p_{t'}$, then

$$\begin{aligned}
\sum_{b=1}^{t}\left[(1-\phi)\left(U(v_b, q_{M^p_t(b)}) - c_{M^p_t(b)}\right) + X c_{M^p_t(b)}\right] &> \sum_{b=1}^{t'}\left[(1-\phi)\left(U(v_b, q_{M^p_{t'}(b)}) - c_{M^p_{t'}(b)}\right) + X c_{M^p_{t'}(b)}\right] \\
&\downarrow \\
(1-\phi)\sum_{b=1}^{t}\left(U(v_b, q_{M^p_t(b)}) - c_{M^p_t(b)}\right) + X\sum_{s=1}^{t} c_s &> (1-\phi)\sum_{b=1}^{t'}\left(U(v_b, q_{M^p_{t'}(b)}) - c_{M^p_{t'}(b)}\right) + X\sum_{s=1}^{t'} c_s \\
&\downarrow \\
(1-\phi)\sum_{b=1}^{t}\left(U(v_b, q_{M^p_t(b)}) - c_{M^p_t(b)}\right) &> (1-\phi)\sum_{b=1}^{t'}\left(U(v_b, q_{M^p_{t'}(b)}) - c_{M^p_{t'}(b)}\right) + X\sum_{s=t+1}^{t'} c_s \\
&\downarrow \\
\sum_{b=1}^{t}\left(U(v_b, q_{M^p_t(b)}) - c_{M^p_t(b)}\right) &> \sum_{b=1}^{t'}\left(U(v_b, q_{M^p_{t'}(b)}) - c_{M^p_{t'}(b)}\right)
\end{aligned}$$

Therefore, if the broker prefers M^p_t to $M^p_{t'}$ despite it having fewer trades, it must be more efficient. □

To finish the proof of Theorem 3 part 2, let $t^p = T(M^p)$ and $t^u = T(M^u)$. Since $M^p_{t^u}$ is the most efficient matching with t^u trades (Claim 8), it gives a weakly higher surplus than M^u. Since M^u maximizes the number of trades over all feasible matchings, $t^p \leq t^u$; in addition, since $M^p = M^p_{t^p}$ gives higher percentage fees than $M^p_{t^u}$, it also gives a higher total surplus (Claim 9). □□

A9 Theorem 4 – unit versus percentage – stability

A9.1 Proof of Theorem 4 part 1

For the costs-dominate case, we need to prove that M^u is weakly more stable than M^p. Assume that buyers and sellers are numbered best-to-worst; i.e., $v_1 > v_2 > \ldots > v_B$, $c_1 < c_2 < \ldots < c_S$, and $q_1 \leq q_2 \leq \ldots \leq q_S$. Recall that for Theorem 4, we assume commissions are small; thus, we ignore them below.

Recall that ε-stability is defined by the pair of traders (b, s) with the greatest incentive to deviate from their chosen matches ($M^u(b)$ and $M^u(s)$ or $M^p(b)$ and $M^p(s)$) to match with each other instead, looking at the *smaller* of the two players' gains from that deviation. Thus, ε can be thought of as the smallest fixed cost of trading without the broker that would make the broker's chosen matching stable.

In the text, we defined $\mu(b, s)$ as the payoff to buyer b from trading with seller s (at the Nash price) and $\nu(b, s)$ the payoff to seller s. For ease of notation, for a given buyer b and seller s and a feasible matching M, let

$$D(b, s, M) \quad = \quad \min\{\mu(b, s) - \mu(b, M(b)),\ \nu(s, b) - \nu(s, M(s))\}$$

be the smaller of the two traders' change in payoffs if they each abandoned their match under M and instead traded with each other. This means a matching M is ε-feasible if and only if $D(b, s, M) \leq \varepsilon$ for every pair (b, s).

Claim 10. *The matching M^u is ε-stable if and only if*

$$\varepsilon \quad \geq \quad D(i, j, M^u)$$

for every $i < t^*$ and $j \leq t^* - i$.

Thus, the only deviations we need to consider for the stability of M^u are deviations between a buyer $i \in \{1, 2, \ldots, t^* - 1\}$ and a seller strictly better than the one buyer i is already trading with, seller $t^* + 1 - i$. (Buyer t^* is already paired with the best seller, so he or she cannot benefit from switching.)

Proof of Claim 10. Note first that no buyers and sellers who are not trading with anyone at M^u are compatible with each other, or M^u would have more trades. Next, note that under the Nash bargaining assumption, each trader's surplus is proportional to the total gains from trade and therefore increasing with the "quality" of his or her trading partner. Thus, since every trader who trades at M^u is trading with a partner who is "better" (higher valuation or lower cost) than any of the unmatched traders, no buyer or seller who trades under M^u can benefit from deviating to trade with a partner who does not trade under M^u or with a "worse" trader who does. All this leaves, then, are deviations between a buyer who trades under M^u and a seller better than the one he or she trades with. □

Claim 11. *For any buyer $i < t^*$ and seller $j < t^*$, $D(i, j, M^u) \leq D(i, j, M^p)$.*

That is, "good" traders (the pairs we worry about under M^u for stability) have a greater incentive to deviate from M^p than from M^u.

Proof of Claim 11. Since fees are small, $\mu(i, j)$ and $\nu(j, i)$ are the same under either matching; therefore, what we want to show is that good traders earn weakly lower payoffs under M^p than under M^u.

Begin with the buyers. Recall that $M^p = M^*$, the matching selected by Algorithm 1. In the proof of Claim 5, we pointed out that each buyer $i \leq t^*$ must be compatible with at least $t^* + 1 - i$ sellers who are not paired (under M^*) with better buyers $i' < i$. Since Algorithm 1 pairs each buyer with the worst compatible seller who isn't paired to a better buyer, this means that at M^p, each buyer $i < t^*$ trades with *at best* the $t^* + 1 - i$-th best seller. Under M^u, however, buyer $i < t^*$ trades with exactly the $t^* + 1 - i$-th best seller. Thus, each buyer $i < t^*$ weakly prefers his match under M^u to his match under M^p, meaning $\mu(i, s) - \mu(i, M^u(i)) \leq \mu(i, s) - \mu(i, M^p(i))$ for any $i < t^*$ and any s.

Next, consider the sellers. Seller $j < t^*$ matches with buyer $t^* + 1 - j$ under M^u, and all buyers better than that match with worse sellers. Since each buyer $i < t^*$ trades with a weakly worse seller under M^p than under M^u, all the buyers better than $t^* + 1 - j$ still trade with sellers worse than j under M^p; thus, j at best trades with $t^* + 1 - j$ under M^p. (He or she might alternatively trade with a worse buyer or not trade at all.) Thus, $\nu(j, M^p(j)) \leq \nu(j, M^u(j))$ for $j < t^*$, and therefore, $\nu(j, b) - \nu(j, M^p(j) \geq \nu(j, b) - \nu(j, M^u(j))$ for any b.

Together, then, these imply $D(i, j, M^u) \leq D(i, j, M^p)$, as claimed. □

To wrap up the proof of Theorem 4 part 1, let

$$(b^*, s^*) \quad = \quad \arg\max_{(b,s) \in \mathcal{B} \times \mathcal{S}} D(b, s, M^u)$$

be the buyer-seller pair with the greatest incentive to deviate from M^u. If $D(b^*, s^*, M^u) = 0$, then M^u is stable and therefore ε-stable for every ε. Otherwise, Claim 10 implies $b^* < t^*$ and $s^* \leq t^* - b^* < t^*$. Then fix a value of ε, and suppose M^p is ε-stable. By definition, this implies $\varepsilon \geq D(b, s, M^p)$ for every (b, s), or

$$\varepsilon \quad \geq \quad \max_{(b,s)\in\mathcal{B}\times\mathcal{S}} D(b, s, M^p) \quad \geq \quad D(b^*, s^*, M^p)$$

Claim 11 then implies $D(b^*, s^*, M^p) \geq D(b^*, s^*, M^u)$, and therefore,

$$\varepsilon \quad \geq \quad D(b^*, s^*, M^u) \quad = \quad \max_{(b,s)\in\mathcal{B}\times\mathcal{S}} D(b, s, M^u)$$

Thus, M^u is ε-stable as well, proving the result. □□

A9.2 Proof of Theorem 4 part 2

Finally, we need to show that when quality dominates, M^p is more stable than M^u.

Claim 12. *Let t^* be the maximal number of trades in any feasible matching. Under unit fees, when quality dominates, M^u pairs buyer 1 with seller t^*, buyer 2 with seller $t^* - 1$, buyer 3 with seller $t^* - 2$, and so on.*

Proof of Claim 12. The broker will obviously choose a matching with t^* trades; among those, he or she seeks to minimize the expected number of trades that fail to consummate,

$$F(M) \quad = \quad \sum_{b\in B(M)} e^{-K(U(v_b, q_{M(b)}) - c_{M(b)} - u)}$$

Since the gains from trade are increasing in v_b, if any of the buyers with the t^* highest valuations are not matched to sellers, the broker can decrease F by replacing that buyer with one of the t^* best who is not trading. Similarly, since (with sellers ranked best-to-worst) $U(v_b, q_j) - c_j$ is decreasing in j for every relevant v_b, if any of the t^* best sellers are not trading, the broker can likewise decrease F by replacing that seller with one of the t^* best who is not trading. Thus, under M^u, the set of buyers who trade will be $\{1, 2, \ldots, t^*\}$ and the set of sellers $\{1, 2, \ldots, t^*\}$.

What's left to show, then, is that given this set of traders, the broker optimizes by pairing them reverse-assortatively, with the best buyer buying from the worst (lowest-cost/lowest-quality) seller. To see this, consider two buyers and two sellers, with $v_b > v_{b'}$, $q_s > q_{s'}$, and $c_s \geq c_{s'}$. Since quality dominates, we know that $U(v, q_s) - c_s > U(v, q_{s'}) - c_{s'}$ for $v \in \{v_b, v_{b'}\}$, and therefore, $U(v_{b'}, q_{s'}) - c_{s'}$ is the smallest surplus that can be generated in a single trade among these four traders. This means that (a) if it's feasible to generate two trades by pairing these traders assortatively, it's also feasible to generate two trades by pairing them reverse-assortatively; and (b) pairing them reverse-assortatively minimizes the expected number of failed trades, since as we showed earlier, $\sum_i e^{-Kd_i}$ is always dominated by the smallest value of $\{d_i\}$.

Thus, for any matching in which some pair of buyers and sellers are matched assortatively, the broker can always feasibly increase expected fees by swapping them so that the better buyer buys from the worse seller. Thus, the only matching that is not sub-optimal is the one that is fully reverse assortative. □

Claim 13. *When quality dominates and fees are small, M^u is ε-stable if and only if*

$$\varepsilon \quad \geq \quad \min\{\mu(b_1, s_1) - \mu(b_1, s_{t^*}),\ \nu(s_1, b_1) - \nu(s_1, b_{t^*})\}$$

Proof of Claim 13. First, note that since buyers $\{1, 2, \ldots, t^*\}$ and sellers $\{1, 2, \ldots, t^*\}$ trade at M^u, there is no pair of traders (b, s) with either $b > t^*$ or $s > t^*$ who can both benefit from trading with each other. (If $b > t^*$ *and* $s > t^*$, then b and s can't be compatible, or else t^* would have been higher to begin with; if $b \leq t^*$ and $s > t^*$, buyer b would prefer to trade with $M^u(b)$ rather than s; and if $s \leq t^*$ and $b > t^*$, seller s would prefer $M^u(s)$ to b.)

Given that, among all buyers, buyer 1 has the most to gain from any possible deviation, since (a) he or she values an increase in quality the most and (b) he or she is currently paired with the worst seller trading. Similarly, seller 1 has the most to gain among the sellers, since (a) he or she benefits most from a high-value buyer and (b) he or she is currently paired with the lowest-value buyer trading. In addition, of course, both gain the most by switching to the "best" possible trading partner. Therefore, the binding constraint on whether M^u is ε-stable is if buyer 1 and seller 1 would both gain more than ε by switching and trading with each other. □

Claim 14. *When quality dominates, for every buyer b and every seller s,*

- *either $\mu(b, s) - \mu(b, M^p(b)) \leq \mu(b_1, s_1) - \mu(b_1, s_{t^*})$ or $\nu(s, b) \leq \nu(s, M^p(s))$ and*
- *either $\nu(s, b) - \nu(s, M^p(s)) \leq \nu(s_1, b_1) - \nu(s_1, b_{t^*})$ or $\mu(b, s) \leq \mu(b, M^p(s))$*

Proof of Claim 14. We know that $t^p = T(M^p)$ is weakly less than t^*, and we showed before (Claim 6) that under M^p, the buyers $\{1, 2, \ldots, t^p\}$ and sellers $\{1, 2, \ldots, t^p\}$ are the ones who trade. Consider a buyer $b > t^p$, who does not trade at M^p. He or she must not be compatible with any seller who does not trade at M^p, or a transaction (and its associated fees) could have been added without changing anything else in the matching. While he or she might be able to benefit from being paired with one of the sellers who is trading, that seller would not want to trade with him or her, as he or she is already trading with someone "better"; thus, $\nu(s, b) \leq \nu(s, M^p(s))$ for any seller s who trades under M^p.

On the other hand, consider a buyer b who is already paired up under M^p. This buyer's current seller is no worse than t^p, who is weakly better than t^*; thus, the most the seller could benefit in any deviation is

$$\mu(b, s_1) - \mu(b, M^p(b)) \quad \leq \quad \mu(b, s_1) - \mu(b, s_{t^p}) \quad \leq \quad \mu(b, s_1) - \mu(b, s_{t^*}) \quad \leq \quad \mu(b_1, s_1) - \mu(b_1, s_{t^*})$$

since $v_b \leq v_1$ and the gain from better quality is increasing in v_b.

By similar arguments, we can show that any seller s who is not paired up to trade under M^p cannot be part of a jointly-profitable deviation and that any seller who is paired up to trade cannot gain more than $\nu(s_1, b_1) - \nu(s_1, b_{t^*})$ by deviating. □

To prove Theorem 4 part 2, we need to show that for any ε where M^u is ε-stable, M^p is ε-stable as well. By Claim 13, if M^u is ε-stable, then

$$\varepsilon \quad \geq \quad \min\{\mu(b_1, s_1) - \mu(b_1, s_{t^*}),\ \nu(s_1, b_1) - \nu(s_1, b_{t^*})\}$$

Now pick any pair (b, s). By Claim 14, either $\mu(b, s) - \mu(b, M^p(b)) \leq \mu(b_1, s_1) - \mu(b_1, s_{t^*})$ or $\nu(s, b) - \nu(s, M^p(s)) \leq 0$. Since $\mu(b_1, s_1) - \mu(b_1, s_{t^*}) \geq 0$, this means

$$D(b, s, M^p) \quad = \quad \min\{\mu(b, s) - \mu(b, M^p(b)),\ \nu(s, b) - \nu(s, M^p(s))\} \quad \leq \quad \mu(b_1, s_1) - \mu(b_1, s_{t^*})$$

Similarly, Claim 14 requires either $\nu(s, b) - \nu(s, M^p(s)) \leq \nu(s_1, b_1) - \nu(s_1, b_{t^*})$ or $\mu(b, s) - \mu(b, M^p(s)) \leq 0$; since $\nu(s_1, b_1) - \nu(s_1, b_{t^*}) \geq 0$, this means

$$D(b, s, M^p) \quad = \quad \min\{\nu(s, b) - \nu(s, M^p(s)),\ \mu(b, s) - \mu(b, M^p(s))\} \quad \leq \quad \nu(s_1, b_1) - \nu(s_1, b_{t^*})$$

Putting these together,

$$D(b,s,M^p) \quad \leq \quad \min\{\mu(b_1,s_1)-\mu(b_1,s_{t^*}),\ \nu(s_1,b_1)-\nu(s_1,b_{t^*})\} \quad \leq \quad \varepsilon$$

Thus, M^p is ε-stable, proving the theorem. □□

A10 Theorem 5 – heterogeneity and the value of brokerage

Suppose there are many buyers and that the distribution of buyers shifts from F_b to $\widehat{F}_b$, which is more heterogeneous. This means that for any $y > x$ and any t_s,

$$V\left(\widehat{F}_b^{-1}(y),t_s\right) - V\left(\widehat{F}_b^{-1}(x),t_s\right) \quad \geq \quad V\left(F_b^{-1}(y),t_s\right) - V\left(F_b^{-1}(x),t_s\right)$$

Rearranging, this becomes

$$V\left(\widehat{F}_b^{-1}(y),t_s\right) - V\left(F_b^{-1}(y),t_s\right) \quad \geq \quad V\left(\widehat{F}_b^{-1}(x),t_s\right) - V\left(F_b^{-1}(x),t_s\right)$$

or the requirement that

$$V\left(\widehat{F}_b^{-1}(x),t_s\right) - V\left(F_b^{-1}(x),t_s\right) \tag{1}$$

must be increasing in x. The fact that the surplus under the two distributions of v_b are not ranked by first-order stochastic dominance also requires that (1) must be strictly positive at $x=1$: if not, it would be weakly negative everywhere, and $V\left(\widehat{F}_b^{-1}(x),t_s\right) \leq V\left(F_b^{-1}(x),t_s\right)$ for all x would imply that fixing t_s, the distribution of $V(\cdot,t_s)$ would be higher via FOSD under F_b than under $\widehat{F}_b$. Note also that if $\hat{F}_b$ and F_b are different distributions and are not ranked via first-order stochastic dominance, (1) cannot be the same at every x.

So now we know that for every x,

$$V\left(\widehat{F}_b^{-1}(1),t_s\right) - V\left(F_b^{-1}(1),t_s\right) \quad \geq \quad V\left(\widehat{F}_b^{-1}(x),t_s\right) - V\left(F_b^{-1}(x),t_s\right) \tag{2}$$

with the left-hand side being strictly positive and strict inequality holding for a positive measure of x. This implies that

$$V\left(\widehat{F}_b^{-1}(1),t_s\right) - V\left(F_b^{-1}(1),t_s\right) \quad \geq \quad \max\left\{0, V\left(\widehat{F}_b^{-1}(x),t_s\right)\right\} - \max\left\{0, V\left(F_b^{-1}(x),t_s\right)\right\} \tag{3}$$

again with strict inequality on a positive measure of x. (If $V\left(\widehat{F}_b^{-1}(x),t_s\right) \leq 0$, then the right-hand side of (3) is weakly negative; therefore, (3) holds strictly; if $V\left(\widehat{F}_b^{-1}(x),t_s\right) \geq 0$, then the right-hand side of (3) is weakly lower than the right-hand side of (2); thus, (3) holds by transitivity.) This means

$$V\left(\widehat{F}_b^{-1}(1),t_s\right) - V\left(F_b^{-1}(1),t_s\right) \quad > \quad \int_0^1 \left(\max\left\{0, V\left(\widehat{F}_b^{-1}(s),t_s\right)\right\} - \max\left\{0, V\left(F_b^{-1}(s),t_s\right)\right\}\right) ds$$

Therefore, by continuity, for x sufficiently close to 1,

$$x\left[V\left(\widehat{F}_b^{-1}(x),t_s\right)-V\left(F_b^{-1}(x),t_s\right)\right] \quad > \quad \int_0^1 \left(\max\left\{0,V\left(\widehat{F}_b^{-1}(s),t_s\right)\right\}-\max\left\{0,V\left(F_b^{-1}(s),t_s\right)\right\}\right)ds \tag{4}$$

Now,

- under random matching, the change in surplus generated by a seller of type t_s due to a shift from F_b to $\widehat{F}_b$ is given by the right-hand side of (4)
- we know already that if $\frac{N_b}{N_s}$ is sufficiently large, then with probability going to 1, there are more "high-type" buyers than there are sellers, and therefore, every seller trades with a high-type buyer; in addition, under unit fees when costs dominate or percentage fees when quality dominates, the probability goes to 1 that the matching is assortative; and thus, fixing $\frac{N_b}{N_s}$, a given seller would trade with the same "percentile" buyer under either F_b or $\widehat{F}_b$
- if $\frac{N_b}{N_s} \geq 1-x$, then as the market grows, the left-hand side of (4) gives a lower bound on the increase in surplus generated by a seller of type t_s from stretching F_b to $\widehat{F}_b$ under brokerage
- thus, if $\frac{N_b}{N_s}$ is sufficiently large, the expected surplus generated by each type of seller increases more under brokerage than under random matching when F_b is stretched to $\widehat{F}_b$, which is the same as saying that brokerage is worth more in surplus (relative to random matching) under $\widehat{F}_b$ than under F_b

If instead there are many sellers (N_b/N_s small), the analysis is similar. When quality dominates, a shift from F_s to $\widehat{F}_s$ that is more heterogeneous similarly implies that

$$V\left(v_b,\widehat{F}_s^{-1}(x)\right)-V\left(v_b,F_s^{-1}(x)\right) \tag{5}$$

is increasing in x and strictly positive at $x=1$; when costs dominate, it implies (5) is decreasing in x and strictly positive at $x=0$. We can once again use this to show that when there are sufficiently many sellers per buyer, each type of buyer gains in the move from F_s to $\widehat{F}_s$ under brokerage than under random matching, which is the same as brokerage being worth more under $\widehat{F}_s$ than under F_s. □

A11 Simulation results on the social value of brokerage

As noted in the text, we used numerical simulation to compare the outcome with a broker to the outcome under random matching to give a bit more understanding to Theorem 1. The simulations were performed in NetLogo.

For all simulations, we let $U(v, q) = vq$. For our baseline specification for the costs-dominate case, buyer types v_b were drawn from the uniform distribution on $[5, 10]$, quality was $q_s = 1$ for all sellers, and seller costs c_s were drawn from the uniform distribution on $[5, 10]$. For our baseline specification for the quality-dominates case, v_b was drawn uniformly from $[5, 10]$, q_s drawn uniformly from $[5, 10]$, and costs were $c_s = 50$ for all sellers. Throughout, we assumed $\phi = \frac{1}{2}$ (the buyer and seller split the gains from trade evenly) and that the broker's fees were small enough to ignore.

For each specification, for markets with different numbers of buyers and sellers, we randomly generated $100, 000$ copies of the economy (sets of buyer and seller preferences); calculated the expected surplus realized under random matching based on 1,000 randomly-generated matchings of buyers to sellers; and calculated the surplus realized in the matchings that maximize unit fees, percentage fees, and total surplus. We then averaged over the 100,000 simulations to obtain an estimate of the expected surplus for each matching process (random, unit fees, percentage fees, efficient matching).

In the tables below, for each size market and each specification of the model, we show the percentage increase in total surplus from switching from random matching to each of the others.

To begin, we look at "balanced markets"—markets with equal numbers of buyers and sellers. The first row of Table 13 shows that in a market with 2 buyers and 2 sellers, in the baseline costs-dominate specification, switching from random matching to brokerage with unit fees would increase the total surplus by 10%; switching from random matching to brokerage with percentage fees would leave the total surplus unchanged; and switching from random matching to the most efficient possible matching would increase total surplus by 20%. Our takeaway from Table 13 is that compared to random matching, brokerage provides a benefit that can easily be quite marginal or significantly negative in balanced markets.

Table 13: Baseline specification, balanced markets

		Costs Dominate			*Quality Dominates*		
Number of Buyers	Number of Sellers	Gain from Unit Fees	Gain from Pct Fees	Gain from Efficient	Gain from Unit Fees	Gain from Pct Fees	Gain from Efficient
2	2	10%	0%	20%	-3%	5%	16%
3	3	11%	-3%	29%	-6%	4%	24%
5	5	8%	-9%	36%	-12%	1%	30%
10	10	-4%	-22%	43%	-23%	-9%	36%
20	20	-19%	-37%	46%	-34%	-21%	40%
50	50	-41%	-55%	49%	-46%	-34%	41%

Next, we consider markets where buyers outnumber sellers. Table 14 shows that in these markets, the gains from brokerage are quite large and nearly as large as moving to the efficient matching regardless of which type of fees are used, and the gains from brokerage are larger as the market becomes more unbalanced.

Next, we consider markets where sellers outnumber buyers. Table 15 shows that the gains from brokerage are again large but now depend on using the "right kind of fees"—unit fees when costs dominate and percentage fees when quality dominates. While the gains from brokerage with

Table 14: Baseline specification, buyers outnumber sellers

		Costs Dominate			*Quality Dominates*		
Number of Buyers	Number of Sellers	Gain from Unit Fees	Gain from Pct Fees	Gain from Efficient	Gain from Unit Fees	Gain from Pct Fees	Gain from Efficient
4	2	63%	60%	72%	52%	55%	63%
6	3	66%	63%	80%	53%	59%	70%
10	5	66%	63%	88%	54%	62%	77%
20	10	62%	61%	93%	53%	64%	82%
40	20	58%	57%	97%	52%	65%	85%
6	2	94%	93%	100%	79%	83%	87%
9	3	98%	97%	108%	82%	87%	93%
15	5	101%	100%	114%	84%	91%	99%
30	10	101%	101%	119%	86%	95%	103%
60	20	101%	101%	122%	86%	97%	106%
10	2	128%	127%	131%	107%	109%	111%
15	3	132%	132%	137%	110%	114%	116%
25	5	136%	135%	142%	112%	117%	120%
50	10	138%	138%	146%	114%	120%	124%
100	20	139%	139%	148%	115%	122%	125%

the appropriate fee type are large and become larger as the market becomes more unbalanced, brokerage with the "wrong kind of fees" is worse than random matching (and becomes even worse as the market becomes more unbalanced).

Table 15: Baseline specification, sellers outnumber buyers

		Costs Dominate			*Quality Dominates*		
Number of Buyers	Number of Sellers	Gain from Unit Fees	Gain from Pct Fees	Gain from Efficient	Gain from Unit Fees	Gain from Pct Fees	Gain from Efficient
2	4	63%	-15%	71%	-12%	56%	63%
3	6	66%	-26%	80%	-20%	59%	70%
5	10	66%	-42%	88%	-31%	62%	77%
10	20	62%	-62%	94%	-45%	64%	82%
20	40	58%	-77%	97%	-54%	65%	85%
2	6	94%	-30%	100%	-23%	83%	87%
3	9	98%	-44%	108%	-32%	87%	93%
5	15	101%	-60%	114%	-42%	91%	99%
10	30	101%	-76%	119%	-52%	95%	104%
20	60	101%	-87%	122%	-59%	97%	106%
2	10	128%	-52%	131%	-35%	109%	111%
3	15	132%	-64%	137%	-43%	113%	116%
5	25	136%	-76%	142%	-50%	117%	120%
10	50	138%	-87%	146%	-56%	120%	124%
20	100	139%	-93%	148%	-59%	122%	125%

Tables 16 and 17 show further simulation results, with variations to the distributions of buyer valuations, seller quality, or seller costs.

First, we consider two variations to the specification of the costs-dominate case. The baseline specification had both buyer valuations v_b and seller costs c_s distributed uniformly on the interval $[5, 10]$ so that a randomly-chosen buyer and seller would be 50% likely to be compatible. Table 16 reproduces a few representative results for this specification and compares it to two others: one where $v_b \sim U[5, 15]$ and $c_s \sim U[5, 10]$ so that 75% of randomly-chosen pairs would be compatible and one where $v_b \sim U[5, 10]$ and $c_s \sim U[5, 15]$ so that only 25% of randomly-chosen pairs would be compatible. The results are qualitatively unchanged; not surprisingly, when compatible pairs become harder to find, the gains from brokerage are larger, as random matching performs poorly.

Table 16: Different specifications of costs-dominate case ($q_s = 1$ for every seller)

Number of Buyers	Number of Sellers	Range of Valuations	Range of Costs	% Pairs Compatible	Gain from Unit Fees	Gain from Pct Fees	Gain from Efficient
3	9	5-10	5-10	50%	98%	-44%	108%
5	5	5-10	5-10	50%	8%	-9%	36%
9	3	5-10	5-10	50%	98%	97%	108%
5	15	5-10	5-10	50%	101%	-76%	119%
20	20	5-10	5-10	50%	-19%	-37%	46%
15	5	5-10	5-10	50%	101%	100%	114%
3	9	5-15	5-10	75%	40%	-36%	42%
5	5	5-15	5-10	75%	-2%	-8%	10%
9	3	5-15	5-10	75%	89%	89%	89%
5	15	5-15	5-10	75%	41%	-40%	44%
20	20	5-15	5-10	75%	-8%	-13%	13%
15	5	5-15	5-10	75%	93%	93%	93%
3	9	5-10	5-15	25%	178%	68%	208%
5	5	5-10	5-15	25%	53%	47%	69%
9	3	5-10	5-15	25%	122%	121%	126%
5	15	5-10	5-15	25%	176%	47%	228%
20	20	5-10	5-15	25%	57%	55%	91%
15	5	5-10	5-15	25%	132%	132%	138%

Second, we consider two variations on the quality-dominates case. Here, we maintain $v_b \sim U[5, 10]$ and $q_s \sim U[5, 10]$ but vary seller costs c_s from the base case of 50 (in which 61% of randomly-chosen buyer-seller pairs would be compatible) to $c_s = 40$ (where 85% of pairs would be compatible) and to $c_s = 70$ (where only 20% of pairs would be compatible). Once again, the qualitative takeaway is the same across specifications. (In addition, once again, the gains from brokerage are larger when compatible traders are hard to find, since random matching would perform poorly.)

Table 17: Different specifications of quality-dominates case ($v_b \sim U[5, 10]$, $q_s \sim U[5, 10]$)

Number of Buyers	Number of Sellers	Seller Costs	% Pairs Compatible	Gain from Unit Fees	Gain from Pct Fees	Gain from Efficient
3	9	50	61%	-32%	87%	93%
5	5	50	61%	-12%	1%	30%
9	3	50	61%	82%	87%	93%
5	15	50	61%	-42%	91%	99%
20	20	50	61%	-34%	-21%	40%
15	5	50	61%	84%	91%	99%
3	9	40	85%	-38%	64%	64%
5	5	40	85%	-11%	-2%	13%
9	3	40	85%	59%	64%	64%
5	15	40	85%	-43%	67%	67%
20	20	40	85%	-16%	0%	18%
15	5	40	85%	61%	67%	67%
3	9	70	20%	45%	187%	200%
5	5	70	20%	46%	74%	94%
9	3	70	20%	181%	187%	200%
5	15	70	20%	26%	204%	222%
20	20	70	20%	28%	60%	129%
15	5	70	20%	197%	203%	222%

A12 List of Sources Used

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