ANGLO-DUTCH PREMIUM AUCTIONS IN EIGHTEENTH-CENTURY AMSTERDAM*

Christiaan van Bochove, Utrecht University
Lars Boerner, Freie Universität Berlin
Daniel Quint, University of Wisconsin

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Abstract. This paper studies Anglo-Dutch premium auctions used in the secondary market for financial securities in eighteenth-century Amsterdam, Europe's financial capital at the time. An Anglo-Dutch premium auction consists of an English auction followed by a Dutch auction, with a cash premium paid to the winner of the first round regardless of the second-round outcome. To rationalize the introduction and continued use of this auction format, we need to determine whether bidding behavior was consistent with equilibrium play. We model this auction format theoretically, and show that the likelihood of a bid in the second round should be higher when there is greater uncertainty about the value of the security being sold. We then test this prediction on data from 16,854 securities sold at auction on 469 days over an 18-year period in the late 1700s; using several different proxies for the uncertainty of a given security's value, we find support for this theoretical prediction.

*Contact: C.J.vanBochove@uu.nl, Lars.Boerner@fu-berlin.de, dquint@ssc.wisc.edu. The authors thank Helmut Bester, Francesco Decarolis, Oscar Gelderblom, Joost Jonker, Johannes Münster, Moritz Schularick, Nikolaus Wolf, seminar participants at Universität Mannheim, Humboldt and Freie Universität Berlin, Université Paris X, and University of Wisconsin for helpful comments. In particular, the authors thank Oscar Gelderblom and Joost Jonker for access to the auction database, which has been compiled by research assistants sponsored by Vidi and Euryi grants.

1 Introduction

While the use of auctions to sell specific goods can be documented as far back as Mesopotamia and ancient Rome,[1] their use and importance increased greatly in the Early Modern period. From the seventeenth century onward, the big Dutch trading companies used auctions to sell wholesale goods from intercontinental trade (Philips 1924). Especially in the Dutch Republic, the use of auctions spread to other wholesale markets such as wine and wood, as well as (as we discuss in this paper) certain financial markets. This new use of auctions as a market clearing mechanism spread across Europe in the eighteenth and nineteenth centuries, in particular among countries with substantial intercontinental trade (Kröhne 1907).

Along with the increased use of auctions came experimentation with a variety of auction formats – among them the "standard" formats still used today (ascending or English auctions, first-price sealed-bid auctions, and descending or Dutch auctions), as well as lesser-known, more complex mechanisms. Our focus is one of the latter: a two-stage auction which we refer to as an Anglo-Dutch premium auction. This type of auction can be documented as early as 1529;[2] it was used throughout the Dutch Republic in the eighteenth and nineteenth centuries to sell real estate, goods from intercontinental trade, wine, and financial securities. It is still used today for real estate auctions in Amsterdam; and a similar, though not identical, mechanism is used for fire sales of bankrupt companies in the United States.

An Anglo-Dutch premium auction has two rounds of (possible) bidding. The first round is an ascending auction, where the price keeps rising as long as multiple bidders remain willing to raise their bids. Once nobody is willing to bid higher in the first round, the winner (the high bidder at that point) receives a pre-set cash premium. The second round is a descending auction in which all bidders are still eligible: the auctioneer begins at a sufficiently high price, and calls out lower and lower prices until someone bids. The first buyer to bid in the second round wins the object being auctioned at that price. If there are no new bids before the price falls to the level of the highest bid from the first round, the first-round winner pays that price, and receives both the object and the premium.[3]

[1]See Thielmann (1961) and Shubik (1999). Early auction use seems to have been limited to specific objects such as bankruptcy goods, estates, or rights to state revenues (Stieda 1907).

[2]Noordkerk (1748). We are not aware of any evidence of the use of this type of auction during the Middle Ages.

[3]Hence the name: as in Klemperer (2002), "Anglo-Dutch" refers to an English auction followed by a Dutch auction;

In this paper, we have two main goals. The first is to document the use of the Anglo-Dutch premium auction in financial markets in the late 1700s. The second is to understand why it was used – why this particular format was introduced, and once introduced, why it remained in use as long as it did.

While we cannot document the exact motivations of the auction organizers, we conjecture that this auction format combined the advantages of an English auction (strategic simplicity, information aggregation, and good revenue properties) with the advantages of a Dutch auction, which is more robust to collusion and more encouraging of participation. This answer, however, relies on the assumption that bidding behavior in this auction resembled equilibrium play – a strong assumption for a complicated game played over two centuries ago. We are therefore interested in understanding whether bidders actually appeared to be playing an equilibrium in this auction. This requires answering two questions: first, what would equilibrium bidding be in this type of auction? And second, does that match the actual observed behavior of bidders?

Neither our theoretical results nor the available data are complete enough to exhaustively test whether actual bidding was consistent with equilibrium play, but we are able to address this question in a more indirect way. We analyze a theoretical model of equilibrium play in Anglo-Dutch premium auctions, and show that a second-round bid is more likely to occur when, relative to the size of the premium, there is greater uncertainty about the value of the security being auctioned. We then analyze auction outcomes in a dataset covering 16,854 securities sold at auction in Amsterdam on 469 auction days, held roughly every second week over an 18-year period from 1766 to 1783. This data supports the prediction of our theory model: a number of different variables which are associated with greater uncertainty in value, also predict more second-round bidding. Bonds of Dutch provinces and other European governments, which were associated with less risk and less uncertainty in value, saw substantially less second-round bidding than Caribbean and South American plantation loans and debt of private companies. Securities in foreign (non-Dutch) currency, associated with greater uncertainty due to exchange rate risk, saw more second-round bidding. There was dramatically more second-round bidding after the onset of the Amsterdam financial crisis of 1773, which had substantial and long-term effects (Sautijn Kluit 1865, Jonker and

as in Goeree and Offerman (2004) and Hu, Offerman, and Zou (2010), "premium auction" refers to an auction where a cash bonus is paid to one or more bidders.

Sluyterman 2000). The effect of the crisis on bidding behavior was strongest for plantation loans, consistent with findings (van de Voort 1973, de Vries and van der Woude 1997) that the overseas colonies were most dramatically impacted by the crisis due to contemporaneous local crises in South America and the Caribbean. Second-round bidding was also more common in winter, when the flow of information was slower due to fewer ships arriving in Amsterdam; and less common when identical securities had already been sold, reducing the uncertainty about their value.

Thus, auction outcomes respond to across-auction differences in exactly the way the theory predicts, giving us more confidence that, either through introspection, experimentation, or myopic self-interest, bidders were perhaps playing close to equilibrium strategies. This then validates our view of the relative strengths of the English and Dutch auction formats, and therefore the possible motivation for combining the two formats. Once the decision has been made to combine the two auction formats, the payment of the premium makes sense as an effort to get bidders to bid seriously in the first round, to prevent bidders from simply waiting and bidding in the second round and having the auction degenerate to a pure Dutch auction.

The remainder of the paper is structured as follows. Section 2 surveys the most closely related literature, both historical and theoretical. Section 3 gives some general history of the Anglo-Dutch premium auction and its use. Section 4 introduces a theoretical model and gives a partial characterization of equilibrium behavior. Section 5 describes our data, explains our empirical approach, and presents our empirical findings. Section 6 concludes.

2 Related Literature

There is a small literature on the history of various auction mechanisms, including contributions by Baasch (1902), Kröhne (1907), Stieda (1907), Philips (1924), Mak van Waay (1936), and Schillemans (1947). These mostly date back to the historical school at the beginning of the twentieth century, and are primarily qualitative. More recently, Engelbrecht-Wiggans and Nonnenmacher (1999) have studied the implementation of a new auction design in early nineteenth-century New York. They show that the new auction design was seller-friendlier and thus attracted more imports to New York, which in turn supported the economic growth of the city. Another contribution by de Marchi (1995) studies Dutch (descending) art auctions during the seventeenth century, and argues that information aggregation in Dutch auctions solved an Akerlof-type lemons problem related to the quality of paintings offered on the Amsterdam art market.[4] Beyond auctions, there is a broader literature dealing with market microstructure from the seventeenth century onward – see Neal (1990), Carlos and Neal (2011), Petram (2011), and Gelderblom and Jonker (2004, 2011), among many others. These studies combine descriptive observations of market institutions with empirical work mainly related to price series, but lack formal rigorous studies of market designs.[5] Pioneering work on formal institutional analysis in economic history has been done by Greif and others (Greif 1993, 1994, 2006; Greif, Milgrom and Weingast 1994). This strand of research has focused on individual case studies during the Late Middle Ages, and has focused more on (informal) reputation mechanisms and their impact on markets rather than on formal market institutions.

On the theoretical side, the last three decades have produced a large literature on auctions in general – Klemperer (2004) offers an extensive survey. (See also Krishna (2002) for an overview of theoretical results.) Much of this literature, exemplified by the seminal paper of Milgrom and Weber (1982), focuses on understanding equilibrium behavior in existing auction formats and comparing outcomes across them. A smaller literature, such as Myerson (1981), considers the problem of designing an "optimal" format from scratch. The more recent "market design" literature (Roth 2002 and 2008, Milgrom 2011) considers the economist's role as an "engineer," designing well-functioning markets to solve matching or allocation problems and studying both theoretical properties of market-clearing mechanisms and logistical challenges in implementing them.

To our knowledge, there has been no study, either theoretical or empirical, of the Anglo-Dutch premium auction as it was practiced historically. There are, however, several papers on other, related two-stage auction formats, beginning with Klemperer (2002). Klemperer discusses the vulnerability of ascending auctions to collusion, and the fact that weak entrants are more likely to participate in first-price or descending auctions. He proposes a hybrid mechanism to combine these advantages of Dutch auctions with other advantages of English auctions. Klemperer's proposed mechanism is an ascending auction, followed by a first-price auction between only the top two bidders from the first round, which he labels an Anglo-Dutch auction.[6] Azacis and Burguet (2008)

[4]Other recent contributions on auctions are mainly descriptive: see Middelhoven (1978), Montias (2002), and Wegener Sleeswijk (2007).

[5]For recent attempts in historical market design studies with a formal institutional analysis see Boerner and Quint (2010) or Boerner and Hatfield (2010).

[6]Klemperer also observes that ascending auctions generate higher revenue when bidder information is affiliated,

compare the Anglo-Dutch auction to another mechanism, a two-stage English auction with a high reserve price in the first stage. They show that when entry is considered, this "Anglo-Anglo" auction is more efficient than a one-stage auction and generates more revenue than an Anglo-Dutch auction.

Most closely related to our work is a paper by Goeree and Offerman (2004) on a different hybrid auction format, which they label the Amsterdam auction. This is an ascending auction, followed by either a first- or second-price sealed-bid auction between the two highest bidders from the first round. Both participants in the second round receive a bonus, which is determined as a function of their second-round bids. Goeree and Offerman offer theoretical results suggesting that such a mechanism may generate greater revenue than standard auction formats when there is a single "strong" bidder and several "weak" bidders, and offer experimental evidence to support this claim. Hu, Offerman and Zou (2010) study the Amsterdam auction in the presence of risk aversion, and find that depending on risk preferences, it can either outperform or underperform a standard English auction.

McLean and Postelwaite (2004) propose a different two-stage mechanism to allocate an object efficiently when valuations are a mix of common and private values and bidders have separate information about both. They propose a first stage where bidders are rewarded for reporting their information about the common component of value, followed by a Vickrey auction to allocate the object efficiently. They define the "informational size" of a single bidder's signal, and relate that to the size of the rewards needed to elicit truthful revelation in the first stage (and therefore the "cost" of allocating the object efficiently). Their notion of informational size is analogous to the measure we introduce below to capture the overall common-value uncertainty in the value of an asset (our β) and the share of this contained in each bidder's signal (our $\frac{\beta}{N}$), though given our simpler model of valuations, our measure is defined differently.

but first-price auctions generate higher revenue when bidder information is independent and bidders are risk-averse. He posits that with both correlated values and risk-aversion, the Anglo-Dutch auction might therefore outperform either a pure ascending or pure first-price auction even without collusion or participation concerns; Levin and Ye (2008) show an example where this is indeed the case.

3 Anglo-Dutch Premium Auctions in Historical Context

Prior to the Early Modern period, auction use was limited to a few particular goods or services.[7] This changed with the start of intercontinental trade during the Early Modern period. There is evidence of extensive use of auctions, in particular in the Dutch Republic, from the seventeenth century onwards.[8] The large Dutch trading companies used auctions to sell goods, particularly spices at first, then later cotton, tobacco, coffee and tea. Auctions were used sporadically during the early seventeenth century, becoming more common from the 1640s onwards (Philips 1924). During the eighteenth and nineteenth centuries, the use of auctions to clear wholesale markets can be further documented in other countries involved in intercontinental trade (Kröhne 1907). The expanded use of auctions was supported by an institutional change: while town officials previously had the exclusive right and obligation to hold auctions, brokers and private business people increasingly gained the right to run auctions as well.[9]

Not only were auctions more common in this period, but a wide variety of different auction formats were employed.[10] In earlier periods, auctions had mostly been limited to simple ascending auctions, but now merchants started using a variety of formats: in addition to ascending auctions, in this period historians have documented the use of descending, sealed bid, and Anglo-Dutch premium auctions.

Use of the Anglo-Dutch premium auction can first be documented in housing auction regulations from 1529; the first use of this auction format outside of real estate seems to have been for ships, ship parts, and shipping gear, presumably during the seventeenth century.[11] By the early eighteenth century, this mechanism was commonly used to sell colonial goods and wine.[12] According to Philips (1924), use of this mechanism in auctions run by the VOC (Dutch East India Company) dates back to 1730.

[7] See Thielmann (1961) and Shubik (1999) for auctions in the Babylonian and Roman empires, where they were limited to bankruptcy and estate goods or the right to tax farming. Lopez (1976) documents a number of major institutional changes during the Commercial Revolution of the Late Middle Ages, but nothing to do with auctions; see Stieda (1907) and Simonsfeld (1886) for more.

[8] Timber: Schillemans 1947; Middelhoven 1978; Van Prooije 1990; Ebeling 1992. Wine: Wegener Sleeswijk 2007. Art: De Marchi 1995; Montias 2002. Tulips: Goldgar 2007. See also ACA, Willige verkopingen; ACA, Koopmanschappen; ACA, Houtwaren; ACA, Scheepsverkopingen.

[9] For an interesting case study on this change in Hamburg, see Baasch (1902).

[10] The full range of auction techniques is documented in merchant manuals such as Le Moine de l'Espine (1694-1801) and Ricard (1722).

[11] See Noordkerk (1748).

[12] Ricard (1722); Le Moine de l'Espine (1694-1801); Wegener Sleeswijk (2007).

Anglo-Dutch premium auctions were also used to sell financial securities throughout the eighteenth century. The auctions we study were held in Amsterdam between 1766 and 1783, but at least sporadic data is available from this same market as early as 1701 and as late as 1897.[13] There is also direct source evidence from Utrecht, where auctions were smaller and held less regularly; the same mechanism was likely used in other cities as well (van Bochove 2011). These auctions seem to have been used primarily to sell off estates: the Amsterdam archive contains auction booklets mentioning the estate to which the securities belonged; the notarial deeds from the town of Utrecht similarly mention in many instances that the securities belonged to an estate.[14] Auctions were not the only way to sell securities; brokers often arranged private sales, and were also hired to estimate the value of securities for division among heirs;[15] but auctions offered the advantage of usually liquidating the entire offering at once.[16]

The design of the Anglo-Dutch premium auction is documented in Ricard (1722) and Le Moine de l'Espine (from 1744 onwards), as well as in our source material. In a first round, the price increased in small increments as long as two or more bidders were still willing to bid against each other.[17] As in a standard ascending (English) auction, the first round ended when nobody was willing to outbid the current high bidder. The seller of the good then awarded the winner of the first round a cash premium, known as the *plokpenning*.[18] In the second round, the auctioneer started with an asking price much higher than the winning bid of the first round, and decreased the price like in a Dutch auction until somebody made an offer. The first bidder to bid in the second round

[13]Editions of the *Prijs-courant der Effecten* (12 December 1842), *Amsterdamsch Effectenblad* (17 December 1869), and *Nieuw Algemeen Effectenblad* (24 December 1897) all included lists of auctioned securities; the December 1901 issues of the *Prijscourant*, however, did not contain such lists.

[14]Van Bochove (2011). For example, securities belonging to the estate of the late Clara Jacoba Pellicorne were auctioned off on 3 September 1742; the estate of Maurits Walraven similarly was auctioned on 23 August 1745. ACA, Willige verkopingen, inv.nr. 296. The records kept by the town secretaries also illustrate this. See, for example, ACA, Willige verkopingen, inv.nr. 130 (deed 24 March 1766). And for Utrecht: UCA, Notarissen.

[15]Van Bochove (2011). For the latter service, fees were typically 1%. See Noordkerk (1748).

[16]Le Moine de l'Espine (1734), pp 300-301 notes that relative to private ("over the counter") sales, the greater transparency and open competation in an auction protect brokers from being suspected of dealing dishonestly, but that "one will resort to an auction only when the goods can bear the costs." A thorough characterization of the advantages and disadvantages of auctions relative to private sales is unfortunately outside of the scope of this paper; we are primarily concerned with why the Anglo-Dutch premium auction was used when an auction was called for, and how it performed.

[17]It is not clear from historical sources whether the auctioneer called out successive price levels or whether bidders called out their own bids. In our data, nearly all winning bids were in increments of 0.25% of the face value of the security, but a very small number (about 100 out of the 16,854 sales in our sample) were in increments of 0.125% or some other amount.

[18]The premium is sometimes also referred to as *trekgeld* or *strijkgeld* (Mak van Waay 1936, 24-25). According to Ricard (1722), the premium was placed in front of the auction desk such that everybody could see it during the first round of bidding.

paid his bid and received the object, and the first-round winner received the premium and paid nothing. If the price reached the first-round price with no new bids, the first-round bidder won the object at that price, and still received the premium. All bidders were allowed to participate in the second round, regardless of whether they had bid in the first.

While our focus is on the use of the Anglo-Dutch premium auction in the Early Modern period, there are at least two modern (21st-century) settings in which this auction format, or a somewhat analogous mechanism, is still employed. First, about 600 years after it was introduced for the sale of real estate in Amsterdam, the Anglo-Dutch premium auction is still used for that same purpose. Each year, the Eerste Amsterdamse Onroerend Goed Veiling B.V. (First Amsterdam Real Estate Auction) holds around 30 auctions, selling a total of about 400 properties with a total value averaging 142 million euros. Their website details the selling rules,[19] which are exactly the mechanism described above. Goeree and Offerman (2004) note that similar auction formats are used in other towns in the Netherlands and Belgium.

And second, when U.S. investment banks broker "fire sales" of bankrupt companies or divisions of companies, the mechanism used is not literally an Anglo-Dutch premium auction, but shares many features with it. The bank first solicits an initial bid for the bankrupt company; the initial bidder becomes known as the "stalking horse." A standard auction is then run, but the stalking horse receives a "breakup fee" if he loses the auction, and must therefore be outbid by at least that much to lose.[20] Relative to the premiums considered in our historical data, the breakup fees – and therefore the stalking horse's bidding advantage – are huge: LoPucki and Doherty (2007) report breakup fees averaging 2.3% of the initial bid. In addition, on average, a bidder must bid at least 3.7% more than the stalking horse's standing bid in order to bid at all. Given the magnitude of this advantage (which led to stalking horses becoming buyers in 85% of the cases LoPucki and Doherty examine), it might be reasonable to suppose that much of the true competition arises in the determination of the stalking horse, analogous to the first round of the Anglo-Dutch premium auction. While this is not determined by an actual auction, to the extent that the investment

[19] http://www.mva-makelaars.nl/veilingx/english.shtml

[20] Unlike in Anglo-Dutch premium auctions, the stalking horse does not receive the breakup fee if it wins the auction. However, since the stalking horse has a bidding advantage equal to the breakup fee in the second round, the outcome is exactly the same as if he received the breakup fee regardless of the second round outcome, and bid that much higher in the second round while competing on equal footing with the other bidders. Thus, other than the stalking horse bid not being chosen via a formal auction, the mechanism is nearly identical to the one we study.

banker solicits initial bids from multiple buyers and chooses the highest one, the mechanism is closely analogous.

Thus, in addition to playing an important role in certain historical markets, the Anglo-Dutch premium auction, or other similar selling procedures, is still economically relevant, and understanding how it behaves, and under what conditions it works well, is an important question to answer.

4 Theory

In this section, we aim to answer two questions: first, why would the Anglo-Dutch premium auction have appealed as a possible auction format; and second, what equilibrium behavior would we expect in this type of auction?

4.1 Why Anglo-Dutch, and why Premium?

To answer the first question, rather than committing to a particular theoretical model, we draw on insights from a number of different models to give a broad, impressionistic view of why this particular auction format might have appealed. We believe the use of a hybrid (two-stage) mechanism was an attempt to combine the advantages of two common auction formats, an ascending (English) auction and a descending (Dutch) auction. Thus, understanding the motivation for the Anglo-Dutch premium auction must begin with a discussion of the relative strengths of these two other auction formats. Note that descending auctions are strategically equivalent to first-price sealed-bid auctions, so insights from the literature on first-price auctions apply to Dutch auctions.

Advantages of English Auctions

Ascending auctions have several strengths. Milgrom and Weber (1982) show that in a symmetric model with affiliated information and interdependent values, ascending auctions lead to higher expected revenue than any other standard auction format.[21] Further, since bidders observe their opponents' bidding behavior and make inferences about their information about the object's value, they are better able to correct for the winner's curse, and are therefore less likely to experience ex

[21] However, this result can be reversed when bidders are risk-averse or asymmetric.

post regret. In a descending auction, a bidder has no information about his opponents when he bids, and might, upon winning the auction and then learning what the other bidders had been planning to bid, realize he is worse off than if he had not participated.[22] In addition, in a private-values setting, ascending auctions allocate an object efficiently even in the presence of asymmetries among bidders, while descending auctions may not (Maskin and Riley 2000). Finally, ascending auctions are strategically simple;[23] auction complexity is often ignored by theorists, but could potentially deter less sophisticated bidders.

Advantages of Dutch Auctions

On the other hand, ascending auctions have some weaknesses relative to other formats such as descending auctions. First, as noted in Klemperer (2002), they are more vulnerable to collusion. Imagine a coalition of bidders agreeing to collude by keeping the price low. In an ascending auction, any deviation from the agreement would be instantly detected, and could be "punished" immediately by being outbid. In a descending or a sealed-bid auction, however, a deviation from a collusive agreement would not be detected until the auction was over, making it harder to punish/deter.

Second, common-value ascending auctions have a multiplicity of equilibria, some of which yield very low revenue;[24] a first-price or Dutch auction would not have such equilibria.

Finally, ascending auctions are disadvantageous to weaker bidders, which could discourage entry by marginal competitors; and ascending auctions sometimes earn low revenue when one bidder is stronger than others. In a private-values model, Maskin and Riley (2000) show that if one bidder has ex-ante stochastically lower valuations than his opponent, he wins less often, and earns lower payoffs, in an ascending auction than in a first-price or Dutch auction. Klemperer (1998) shows that if values are "mostly common" but one bidder has a slight private-value disadvantage, he cannot earn positive profits in an ascending auction, and drops out immediately in equilibrium,

[22] In a simple model, such ex post regret does not hurt the seller. But in a setting where the winning bidder might attempt to back out of the deal, or where a dissatisfied bidder might stop participating in future auctions, this could impose a cost on the seller.

[23] In private-values models, bidders effectively have a dominant strategy: keep bidding until the price reaches your valuation for the object, then stop. Even with interdependent values, the dynamic nature of the auction allows bidders to bid by answering a series of yes-no questions ("given what I know right now, if I bid this price and win, will I be happy?") rather than seeking the optimal trade-off between a better deal and a higher likelihood of winning.

[24] Consider the following strategies: one bidder, regardless of his private information, plans to keep bidding indefinitely, and all other bidders do not bid at all. These strategies are a Nash equilibrium, and lead to zero revenue. These particular strategies fail the standard equilibrium refinements of sequential rationality, but more complex equilibrium strategies can be constructed that achieve the same outcome in a Perfect Bayesian equilibrium.

leading to low revenue, while he could profitably compete in a first-price auction. And in a pure common-values setting where one bidder is perfectly informed about the value of the object and his opponents are not, the winner's curse would force the uninformed bidders to drop out immediately in an ascending auction, since at any price where the informed bidder allowed them to win, they would regret the purchase; Engelbrecht-Wiggans, Milgrom and Weber (1983) show that in a first-price auction, uninformed bidders do bid seriously and win half the time, and revenue is substantial.[25]

Motivation for the Anglo-Dutch Premium Auction

Thus, ascending auctions will tend to outperform Dutch auctions when bidders are symmetric and in genuine competition with one another; but Dutch auctions will tend to perform better when bidders are asymmetric, especially when weaker bidders might be deterred from participating at all, and when bidders might be in collusion with one another. We believe the main motivation for a hybrid auction combining the two formats was to capture the "best of both worlds" – an ascending auction to elicit information and increase revenue, combined with a second-round descending auction to stymie collusion and encourage participation. The auctions we study had a group of regular bidders who participated and won frequently, making collusion at least a possibility. New regulations passed in 1677 for real estate auctions expliclty banned collusion (along with other disruptive practices) under penalty of corporal punishment, suggesting that collusion among bidders was at least something of a concern, although we have no evidence of whether or how strictly this rule was enforced.[26]

Of course, once a two-stage mechanism has been chosen, there is the question of whether bidders will bid seriously in the first round (at the cost of revealing their interest in winning the security), or simply wait to bid in the second round, undermining the point of using a two-stage auction.[27] The two-stage mechanisms discussed in Klemperer (2002) and Goeree and Offerman (2004) solve this problem in one way: by limiting participation in the second round to the highest

[25] In a setting where the object's value is uniformly distributed over an interval $[0, \overline{v}]$, average revenue is two-thirds the value of the object in a first-price auction, but 0 in an ascending auction.

[26] The new regulation contained the rule that "...no one, and especially not Brokers, may cause any commotion, confusion, disorder, assembling or collusion or other actions, leading to the disruption of the sale, or to depress [the prices] of Goods and Merchandise, directly or indirectly, on the penalty of corporal punishment..." See Noordkerk (1748).

[27] This appears to be the logic considered by Cassidy (1967), who writes, "The attainment of a sufficiently high first-phase bid is so crucial to a successful [Anglo-] Dutch auction that sometimes bidders in the ascending phase are encouraged by means of a small bonus, called in Holland plok or plokgelden."

two bidders in the first round. But this comes at a cost: many of the relative advantages of the Dutch auction (robustness to collusion, attractiveness of entry to weaker bidders) might be lost if participation in the Dutch auction was rationed via an English auction. Goeree and Offerman show their mechanism performs well when there is a single "strong" bidder and multiple weak bidders; but with just two strong bidders, weak bidders would have little chance of advancing to the second round, and entry by weak bidders would therefore be discouraged. The payment of a first-round premium is a different way to solve the problem: it adds an additional reason to bid in the first round (and makes very low first-round prices implausible), while still allowing all bidders access to the Dutch part of the auction, potentially disrupting collusion and encouraging entry (especially in the suspected presence of collusion by the "usual" bidders, exactly when entry is most important).

Thus, in total, the Anglo-Dutch premium auction seems an attempt to combine the positive informational and revenue properties of an ascending auction with the anticollusive and pro-entry features of a descending auction, with the premium required to elicit serious bidding in the first round.

4.2 Equilibrium Bidding Behavior

To answer the second theoretical question – what type of bidding behavior would be predicted in equilibrium – we introduce a particular theoretical model of the Anglo-Dutch premium auction.

Model

Since the objects sold in the auctions we study are financial instruments – basically, a series of (uncertain) future cash flows – we model bidder valuations as pure common values. There are N identical bidders, each of whom receives some private signal s_i about the value of a security; given the realizations of all bidders' signals $\mathbf{s} = (s_1, s_2, \ldots, s_N)$, the value of the security is $v(s_1, s_2, \ldots, s_N)$ regardless of the winner. For ease of exposition, we use a very simple model of valuations: we assume that bidder signals are independent draws from the uniform distribution on $[0, 1]$, and that the value of the security is linear in the average of the bidders' signals,

$$v(\mathbf{s}) \quad = \quad c + \beta \frac{\sum_{i=1}^{N} s_i}{N}$$

The parameter β measures the absolute variability of v (the most it could vary based on all bidders' information), and $\frac{\beta}{N}$ is the share of that variability attributed to each bidder's signal.[28]

As for the auctions themselves, we follow the theoretical literature and model prices as continuous. We model the ascending round as a continuous-time button auction, as in Milgrom and Weber (1982): the price rises continuously (as on a clock) from a low level, and each bidder keeps his hand raised as long as he wants to remain active in the auction, then lowers his hand when he chooses to drop out. Bidders all know who is still active at each point and at what prices other bidders have dropped out; the first round ends when the second-to-last bidder drops out. In the second round, the price drops continuously from a high level, and the first bidder to shout "mine!" ends the auction and wins at that price. In the unlikely event of a tie (simultaneous bids in the second round, or all remaining bidders dropping out simultaneously in the first round), a winner is selected at random.

We let ρ denote the value of the premium paid to the first-round winner.

Theoretical Results

Below, *equilibrium* refers to Perfect Bayesian Equilibrium, the standard equilibrium concept for a dynamic game of incomplete information. *Ex post equilibrium* refers to an equilibrium in which no bidder could profitably deviate from his equilibrium strategy *even if he knew all bidders' private information at the start of the auction* – thus, an equilibrium in which no bidder experiences ex post regret once all information has been revealed.

Theorem 1. *For any $N \geq 2$, $\rho > 0$, and $\beta > 0$, an equilibrium exists. Further...*

1. *If $\beta \leq N\rho$, there is a symmetric, pure-strategy equilibrium with only first-round bidding.*
    - *If $\beta \leq \frac{1}{2}N\rho$, this equilibruim is an ex post equilibrium.*
2. *If $\beta > N\rho$, then any* ***symmetric*** *equilibrium (if one exists) must have a positive probability of second-round bidding.*

[28] In the appendix, we show how some of our results would extend to a more general model of signals and valuations, where bidder signals are correlated and v is symmetric and increasing but otherwise unrestricted. Besides being less complete, these more general results are more notation-heavy and therefore harder to link directly to primitives of the environment; we therefore rely on the more restrictive model shown here to convey the intuition of the results.

3. *If $\beta > N(N+1)\rho$, then* ***any*** *equilibrium must have a positive probability of second-round bidding.*

   - *The probability of a second-round bid must be at least $\frac{1}{c+\beta}\left(\frac{\beta}{N(N+1)} - \rho\right)$ in any equilibrium; this lower bound is increasing in β and decreasing in N.*

The results can be summarized as follows, and are illustrated in Figure 1. When β is small relative to ρ, there is an equilibrium in which all bidding occurs in the first round. On the other hand, when β is large relative to ρ, all equilibria involve second-round bidding; and the larger is β, the higher must be the probability of second-round bidding in any equilibrium. Thus, in rough terms, as β increases (or N decreases), we would expect to see more second-round bidding.[29]

Comparison to Standard Mechanisms

It is worth briefly discussing how the equilibrium outcomes of the Anglo-Dutch premium auction compare to the outcomes of standard auction formats, in terms of efficiency and revenue. First, we should note that since we employ a common-values model, any auction which always awards the object to *someone* will yield the same level of efficiency. Thus, in terms of overall welfare, our mechanism is exactly as good as a pure ascending auction, or a pure descending auction, or a purely random assignment of the security to one of the bidders.

Second, we should note that in the model presented in the text – independent signals and v linear – a pure Dutch auction and a pure ascending auction would be revenue-equivalent (provided the symmetric equilibrium was played in the latter); and either one would be optimal among the class of all mechanisms that always award the security to someone. However, in a more general model of common values with symmetric, affiliated signals (such as the model presented in the appendix), the pure ascending auction would generate higher expected revenue than the pure Dutch auction (Milgrom and Weber 1982).

[29] We cannot, however, state this result as a formal comparative static. For the case $\beta \leq N\rho$, Theorem 1 establishes existence of an equilibrium with no second-round bidding, but there also exist equilibria with second-round bidding. As we show in the proof of Theorem 1 in the appendix, in addition to being symmetric, the equilibrium without second-round bidding is relatively straightforward, and matches the equilibrium of a pure ascending auction, so it may well be a focal equilibrium choice to be played, but other equilibria still exist. Similarly, for the case of β between $N\rho$ and $N(N+1)\rho$, while we rule out *symmetric* equilibria without second-round bidding, we cannot rule out the possibility of asymmetric equilibria without second-round bidding. Still, at the very least, as β increases and N decreases, the lower bound on the equilibrium probability of second-round bidding increases.

As for the Anglo-Dutch premium auction, revenue conclusions depend on the equilibrium played. When $\beta \leq N\rho$, the equilibrium considered in the first part of Theorem 1 is outcome-equivalent to the symmetric equilibrium of a pure ascending auction: each bidder bids ρ higher in the first round of the Anglo-Dutch than they would have in a regular ascending auction with no premium, and nobody bids in the second round, so the premium ρ is exactly paid for by the higher winning bid. Thus, when β is low relative to ρ, one equilibrium of the Anglo-Dutch premium auction exactly matches the revenue performance of an ascending auction, which we already know is better than any other standard auction format; and does so in a way that is likely more robust to collusion (and may not have alternative low-revenue equilibria).

However, the revenue story is not always so positive. The rather contorted equilibrium constructed in the existence proof in the appendix – in which all meaningful bidding occurs in the *second* round– matches the revenue of a pure descending auction, *minus* the premium, which is effectively given away for free. Other equilibria with second-round bidding may exist, and we do not know how they perform; in general, holding β and N fixed, we suspect equilibrium revenue will tend to be higher in equilibria with less second-round bidding.[30]

Overall, then, the revenue results are mixed, and depend on equilibrium selection: one equilibrium that sometimes exists matches the (good) outcome of a pure ascending auction, while another equilibrium that always exists does strictly worse than the (worse) outcome of a pure descending auction. This also suggests one possible policy recommendation: to ensure the "good" equilibrium exists, the auctioneer should take care to make the premium large enough relative to the uncertainty in the value of the object, although this concern is mitigated when there are sufficiently many interested bidders.

[30] In any equilibrium where a second-round bid occurs with probability r, equilibrium expected revenue is at worst $r(c + \beta) + \rho$ less than the expected value of the security, although this bound is not tight.

5 Empirics

5.1 Data

We study primary source data from the Anglo-Dutch premium auctions held in Amsterdam for securities being resold on the secondary market from January 1766 to December 1783.[31] The securities sold were a mix of sovereign bonds and loans to private plantation owners and companies. These securities were not sold exclusively via Anglo-Dutch premium auction – some were also sold bilaterally, or via brokers – but in Amsterdam as well as many other cities, these auctions appear to have been the only centralized market for these securities.

The auctions were held on Mondays at 3 p.m. in a well-known and exclusive hotel (the Oude Zijds Heeren Logement) in the centre of Amsterdam, close to the Exchange. This starting time was likely tied to the hours of the Exchange, which closed at 1 p.m. (Scheltema 1846 p. 46f). Newspaper advertisements and auction booklets, published several weeks in advance, described each lot in detail.[32] Participants were thus informed about how many securities would be sold on a given day, and the issuing party, type of security, and issue date of each security up for sale.

Auctions were public events, and as such, auction dates were set by town officials.[33] Selling was done by brokers; a private person with securities to sell had to use a broker, who could arrange a private sale (or buy the asset himself) or offer it through an auction.[34] Brokers applied to the government for permission to participate on one of these dates; more than a hundred securities could easily be offered on a single day.[35]

Bidding, on the other hand, was open to anybody. While any private buyer was free to participate, many of the bidders were themselves brokers, who were buying securities for their customers. Table 4 identifies the ten most frequent winners; these ten bidders won more than 25% of the securities over our 18 years of data. The top 50 winners won more than half of all securities. However,

[31]The data has been collected by Oscar Gelderblom and Joost Jonker, and research assistants sponsored by Vidi and Euryi grants. The data are available in ACA, Willige verkopingen, inv. nrs. 70-129, and in *De maandelykse Nederlandsche Mercurius*. On the latter, see Hoes (1986); van der Steen (1996); van Meerkerk (2006).

[32]See, for example, the advertisements placed in the *Leydse Courant*.

[33]See, for example, *Mercurius* 35 (1773), 240 (December 1773), for the auction dates of 1774. Town secretaries also attended the auctions and recorded the outcome (Le Moine de l'Espine 1801 pp 311-312). Additional regulations were also added over time – for example, rules to prevent winning bidders from backing out, and rules to punish collusion among bidders – suggesting the government had an interest in this market running smoothly.

[34]Le Moine de l'Espine (1801), 308.

[35]See ACA, Scheepsverkopingen, inv.nrs. 63-72.

there were also many "one-time only" winners. Besides serious bidders, there is also evidence of participation by "premium hunters," people who hoped to win the first-round premium but did not intend to buy the security itself.[36]

Securities were auctioned one by one. Securities sold by the same broker were offered one after another;[37] within that group, securities from the same issuer were sold consecutively.[38] Figure 2 displays the sequence of securities sold on one auction day, 21 December 1779, along with their prices and which auctions were won in the second round. The day began with 26 bonds of Holland, of three different "vintages" (15 issued in 1741, then five issued in 1711, then another six issued in 1741); only one of these auctions had a second-round bid. These were followed by four bonds of Silesia (northern Austria); followed by 17 loans to the owner of a plantation in Suriname, nearly all of which sold in the second round, then another one from a different plantation in Suriname, and so on.

Our data set covers 469 such auction days. Auctions were typically held every two weeks, but fewer auctions were held during summer and around Christmas. The number of securities sold on one day ranged from very few up to 289. The total number of securities sold during the period of our data was 16,854.[39] Most securities sold had been issued between five and twenty years earlier, but a few dated back as far as the late seventeenth century. For some securities we have information on the number of times they had changed ownership: some had changed hands as many as 27 times.[40]

Most securities had a face value of 1,000 guilders, which was about three times the yearly income of an unskilled worker (de Vries and van der Woude 1997). Coupon rates ranged from 2.5% to 6%. We can classify the issuers into five broad categories. Government bonds issued by the province of Holland account for more than 5,000 of the sales. (Exact numbers can be found in Table 2.)

[36]This can be documented in regulations meant to deal with premium hunters, who were not able to pay for the object in the event that they won. These regulations and other sources (e.g. Le Moine de l'Espine 1801, 316-317) refer to these bidders as "sheep". Referring to similar auctions used for real estate, Goeree and Offerman (2004) write, "In former times, if a hanger [a premium hunter who accidentally won the auction] could not pay for the house he won, he would be sent to prison for one or two months. If it happened twice, he would be tortured." See also Noordkerk (1748).

[37]The order of the sellers was determined by the order in which they had petitioned the government to participate, or via lottery (Le Moine de l'Espine 1801, pp 310-311).

[38]ACA, Willige verkopingen, inv.nr. 130 (deed 24 March 1766) shows how one broker ordered the securities he was selling for his clients. Securities sold together often had consecutive issuing numbers, suggesting they were purchased together at original issuance.

[39]We do not know exactly how many securities were *offered* for sale on each day. For a few days, however, we took samples of auction catalogue listings where all the securities offered that day are listed; based on those samples, it appears that nearly all the securities offered were indeed sold.

[40]Information on issue dates and previous owners is not always available, however.

Other Dutch government bonds issued by the States General, other provincial estates, cities, and the Navy Boards account for about 1,700 sales. Foreign government bonds issued by the kings of Denmark, France and Sweden, the Austrian emperor, the Russian tsar, and various German dukes and cities account for about 2,000 sales. Private company loans included, among others, debt issued by a Spanish canal building company, a Swedish mining company, a Dutch mutual fund, and the VOC (and very occasionally VOC shares as well); this group accounts for about 1,000 sales. Caribbean and South American plantation loans include loans to owners of plantations in Suriname, Demerara, Essequibo, Berbice, Trinidad and Tobago, Grenada, the Danish West Indies, and other locations;[41] each loan can be ascribed to a specific debtor. Plantation loans account for about 7,000 sales. Winning bids varied substantially: Holland bonds usually sold at a premium over face value, while plantation loans nearly always sold at a discount; the other security types routinely sold either above or below face value.

For each sale, our data includes the identity of the issuer; the face value of the security and the currency it was issued in; the identity of the winner, whether he won in the first or second round, the high bid in the first round, and the second-round bid if there was one. More detailed information about the security (when it was issued, the interest rate it paid, and how many times it has been resold since being issued) is available for a subset of sales.

The first-round premium paid was not recorded separately for each auction, but we believe it was typically a constant fraction of the face value of the security for sale. More elaborate auction records from the nearby town of Utrecht report that in most cases, the premium was 0.275% of the security's face value; exceptions (usually for securities with odd denominations) were noted.[42] In Amsterdam the premium seems to have been approximately the same. When a 5,000 guilder security from the insolvent estate of Anthonij Carstens was auctioned on 22 December 1777, a premium of 12.50 guilders (or 0.25%) was awarded.[43] Le Moine de l'Espine (1801) also stressed that the size of the premium was in proportion to the value of the auctioned asset ("near de waerdije

[41]For a description of the geographical location see map 1.

[42]See UCA, Notarissen, inv.nr. U217a012 (notary W. van Vloten), deed 121 (28 April 1770); inv.nr. U247a009 (notary D.W. van Vloten), deed 108 (19 May 1770); inv.nr. U247a021 (notary D.W. van Vloten), deed 55 (1 October 1785); inv.nr. U227a013 (notary J.T. Blekman), deed 20 (10 September 1785).

[43]See ACA, Desolate Boedelkamer, inv.nr. 2896. The security was expressed in bank money rather than current guilders and the agio (the premium on bank money) was 5%, so one could argue in this instance the premium was 0.238%.

der geveilde Goederen").[44]

5.2 Empirical Strategy

We will use this data to test the prediction of our model. Theory predicts that auctions for securities with more uncertain values – and auctions with fewer bidders – should have a greater likelihood of bidding in the second round. Of course, we cannot directly observe either β (the degree of uncertainty in the value of a particular security) or N (the number of bidders). However, there are a number of variables in our data which we expect to be related to either β or N; we will therefore test whether these variables are associated with more or less second-round bidding as predicted.

Definitions for all explanatory variables are given in Table 1, and descriptive statistics of these variables are given in Tables 2 and 3. The explanatory variables fall into four groups: characteristics of an individual security or its issuer; the historical context in which the auction was held; price discovery or learning effects over the course of an auction day; and other characteristics of a particular auction day, which we refer to as "auction room effects." We also consider some additional control variables, where we do not necessarily expect a particular causal link between the variable and value uncertainty, but simply want to ensure that the other relationships we measure are robust to their inclusion.

Characteristics of the Security or the Issuer

The first characteristic of a security we consider is the identity of the issuer. As noted above, we differentiate between five groups of issuers: Holland, other Dutch governments, foreign governments, Caribbean and South American plantation owners, and private companies. The available coupon rates suggest that these issuers were associated with different levels of risk. The most reputable domestic borrower, the States of Holland, paid a coupon of only 2.5% (Liesker and Fritschy 2004). The Danish case is illustrative for the rates charged to foreign borrowers (Van Bochove 2011b): the Danish kings initially paid a coupon of 5%, but this later decreased to 4%. At the same time, however, large Danish corporations continued to pay a coupon of 5%. Plantation owners in the Danish West Indies paid a coupon of 6%. Thus, private company loans and planation loans typically

[44] We have no reason to believe the premium varied systematically across sales other than with the size of the security; our empirical approach implicitly assumes that any variation in the premium (as a fraction of the security's face value) is independent of our explanatory variables.

paid higher coupons, to compensate for greater risk. We expect greater risk to also be associated with greater uncertainty in value (or more room for private information about valuations), and therefore more second-round bidding. Simple descriptive statistics suggest that this pattern does indeed hold in the bidding data: second-round bidding was more than twice as common in auctions for private company loans (32% of auctions) and plantation bonds (30%), as in auctions for Holland bonds (14%) and foreign government bonds (13%), with other Dutch bonds falling somewhere in between (20%).

While most securities in this market were denominated in Dutch guilders, a small number were denominated in foreign currencies. Due to exchange rate risk, we expect these to be associated with higher risk as well. In the sales of securities denominated in a foreign currency, 23% had a second-round bid. While this is close to the overall average in our data (22%), nearly all the securities in foreign currencies were foreign government bonds, which typically (see above) had much less second-round bidding.

Historical Events

A second set of variables is related to the historical context of the period. At the end of December 1772 and the beginning of January 1773, Amsterdam experienced a financial crisis that hurried its long-run decline as a financial centre in favour of London (Braudel 1992; Jonker and Sluyterman 2000). The crisis was triggered by liquidity problems among major Amsterdam banking houses (Sautijn Kluit 1865; Jonker and Sluyterman 2000). This led to chain reactions and spillovers on financial centres all over Europe and to further business failures. Consequently, we expect to see more risk, and more uncertainty in value, in securities sold after the onset of the crisis. Indeed, before the financial crisis, only 9% of auctions had a second-round bid, compared to 26% following the start of the crisis.

More specifically, we would expect foreign bonds to be affected more strongly. Information about domestic securities could be collected locally, but the crisis made this more difficult for foreign securities.[45] The financial crisis also coincided with several additional shocks affecting the plantations, such as tornadoes and slave rebellions in South America and the Caribbean (Postma

[45] Here we are thinking of information about the financial health of the issuer, not market information about the value of the security, as most of these securities were only traded in Amsterdam. Koudijs (2010) studies the effect of information flows on prices of assets traded in more than one market.

1990; Postma and Enthoven 2003; de Vries and van der Woude 1997; van de Voort 1973). We would therefore expect a particularly strong impact on these assets.

We also consider the effect of the Fourth Anglo-Dutch war (1780-1784), which mainly affected overseas economic activities; and the effect of winter, since during the months of January, February and March, relatively few ships came into the harbour of Amsterdam.[46] War and the commercial off-season likely slowed the arrival of information, increasing uncertainty about the values of the foreign assets. During the period of the Fourth Anglo-Dutch War, second-round bidding was even more common than usual in auctions for plantation loans (39%), but not for other types of securities.

Intra-day Learning

A third set of variables is related to intra-day learning among the auction participants. Identical securities were often auctioned consecutively; we would expect the value of the security to be less uncertain in the later sales than in the first few. To account for this sort of "price discovery" process, we control for the number of identical securities that have just been sold prior to a particular sale.

"Auction Room" Effects

A fourth set of variables considers effects specific to each auction day. While the auctions were usually held every other week, there were occasions when they were less frequent. When there was a longer gap between auctions, there would have been more time for private information and news to accummulate, leading to greater uncertainty about the value of a security; we therefore include the number of days since the previous auction day, and expect it to have a positive effect on second-round bidding.

We also consider the number of securities sold on an auction day, as we expect days with more securities for sale would likely have attracted more bidders. A larger number of bidders would be expected to decrease the likelihood of second-round bidding.

Finally, we control for the mix of bidders we know to have been present on a particular day. There is significant heterogeneity in bidding behavior across individual bidders, as shown in Table

[46] In the years 1771-1787 the share of annual entries of ships from European points of origin into Amsterdam's port (as measured through the *paalgeld* tax) was 1.5 % in January, 1.8% in February and 5.8% in March (Welling 1998). In the years 1766-1769 the share of annual passages through the Sound – northern Europe's most important shipping route – was 0.1% in January, 0.1% in February and 2.6% in March (Soundtoll 2011).

4. For instance, among the securities won by bidder Van Blomberg, 70% were won with a second-round bid; among the securities won by bidder Heimbach, on the other hand, only 14% were won with a second-round bid. For each auction day, we construct two variables meant to capture the propensity of the bidders present that day to bid, or not bid, in the second round. Obviously, the relationship between these variables and second-round bidding is mechanical (by construction); we are more interested in seeing whether the other effects we measure are robust to controlling for this type of "auction-room heterogeneity".[47]

Other Controls

Since there was often an overlap between the brokers organizing a particular day's auction and those buying securities for their clients, we include a dummy for whether the winning bidder was among that day's organizers. We also control for a linear time trend, and for four prices taken from the Amsterdam stock exchange: two financial commodity prices (Polish grain and St. Domingo sugar), the Amsterdam exchange rate on Hamburg, and the price of Amsterdam bank money (the agio).[48]

Empirical Approach

The descriptive statistics above suggest that there are indeed positive relationships between second-round bidding and several of the variables we associate with greater value uncertainty. To test these relationships more formally, we use a binary probit model. Each security sold is treated as a separate observation; the dependent variable is 1 if there was a second-round bid in the auction, 0 if not. The independent variables are those described above.[49] Tables 5 and 6 present the marginal fixed effects for a probit regression. We add variables or sets of variables one by one, both to emphasize each effect separately, and to show that each effect is robust to what else is included in the specification.

[47] The two variables are constructed as follows. We take the 82 most frequent auction winners in our data, each of whom won at least 50 auctions and who together account for 75% of winning bids. For each auction day, we look at those bidders from this group who won at least one security, and therefore who we know were present that day; and we calculate the fraction who were "high-propensity second-round bidders" (won at least 30% of their securities in the second round over the entire sample) and the fraction who were "low-propensity second-round bidders" (won less than 10% of their securities in the second round).

[48] Data from Malinowski; these four prices were selected on the basis of availability of weekly price data.

[49] All non-binary explanatory variables aside from the time trend are converted to a log scale – by adding one and then taking the natural log – prior to inclusion in the regression. While many of our explanatory variables are binary dummies, each represents a broad category, not the identity of an individual issuer or bidder, so we are not concerned about the incidental parameters problem noted by Neyman and Scott (1948).

Since several of our variables take a single value for all auctions held on the same day, we cluster standard errors by auction day.[50] Table 7 compares the results of our final specification under normal standard errors, robust standard errors, clustering of standard errors by the identity of the winner, and clustering by the auction day; the results do not change at all.

5.3 Empirical Results

Tables 5 and 6 display the main results of the binary probit estimation with different combinations of explanatory variables. Columns (1) and (2) consider only the characteristics of the security. Column (1) differentiates between other (non-Holland) Dutch bonds, foreign government bonds, plantation loans, and private company loans. Holland bonds are the omitted variable, so coefficients should be interpreted relative to the benchmark of a Holland government bond.[51] Column (2) adds a binary variable for whether the security is in a foreign currency. As expected, we find significant positive coefficients for the plantation and private company loans. The positive coefficient on foreign currency is similarly in line with the predictions.

Columns (3) through (7) incorporate different combinations of historical and time-dependent effects. Column (3) distinguishes only between auctions held before and after the onset of the financial crisis in Amsterdam, using a dummy variable for securities sold before the end of 1772. Column (4) controls for the financial crisis separately for each of the five categories of issuer. All five of these coefficients are negative and significant, as predicted. The strongest effect is in plantation loans, in line with findings by economic historians that plantation owners were especially affected by the crisis.[52] In column (5), we also control for the Fourth Anglo-Dutch war, separately depending on whether the auction was for a plantation loan or not; the effects are fairly small and not statistically significant. In column (6), we add a linear time trend, which has basically no effect. In column (7), we add a dummy for the winter season (January through March), which is characterized by less shipping activity and therefore less arrival of information from abroad, leading to greater uncertainty; as expected, this coefficient is positive, but only marginally significant.

In column (8), we add controls for price discovery or learning, adding a variable for how many of the identical security have already been sold, included separately for each type of issuer. As

[50] See Wooldridge (2002), Chapter 15.

[51] Holland bonds are an ideal benchmark since they were known as the safest investment available.

[52] See van de Voort (1973) and de Vries and van der Woude (1997).

expected, these variables all have significant, negative coefficients. Column (9) adds the number of days between auctions, which as expected has a positive (but not statistically significant) coefficient. The small magnitude of the coefficient is due to the variable being measured in days, rather than weeks. Column (10) adds the number of securities sold on the same auction day, which we expect to be positively correlated with the number of bidders (and therefore to have a negative coefficient); the coefficient is indeed negative, but not significant. Column (11) also controls for the set of bidders known to be present on that day, a significant source of otherwise unobserved heterogeneity across days. (As noted above, these coefficients reflect a purely mechanical relationship; our point here is simply that the other relationships are robust to the inclusion of these controls.) Column (12) adds a dummy variable for whether the winning bidder was one of the brokers organizing that day's auction; the coefficient is small and not significant, and its inclusion has no effect on the other coefficients. Finally, column (13) adds controls for outside market effects, using price data for two commodities, an exchange rate, and an interest rate. These controls have no effect on the other coefficients. We also tried specifications involving changes in these prices rather than levels, as well as recent volatility in each price level; again, these did not effect the other results.

In Table 7 we run several robustness checks on correlated error terms. Column (4) is the final regression taken from Table 6. Column (1) reports the same regression with regular standard errors, column (2) with robust standard errors, and column (3) with standard errors clustered by the identity of the winning bidder. As noted above, standard errors are clustered by auction day in Tables 5 and 6. The different treatment of standard errors hardly changes the results at all.

The main takeaways from the empirical analysis can be summarized as follows:

- Other (non-Holland) Dutch bonds, plantation loans, and loans to private companies had significantly more second-round bidding than bonds of Holland and foreign governments
- Securities denominated in foreign currencies had more second-round bidding than securities denominated in Dutch guilders
- Auctions before the onset of the financial crisis had less second-round bidding than auctions after; the effect was strongest for plantation loans, and weakest for Dutch bonds (both Holland and other)

- Auctions held in winter had more second-round bidding
- Across all security types, second-round bidding was less likely when more identical securities had just been sold

These effects are statistically significant, and generally very stable across different empirical specifications, which controls are included, and how standard errors are clustered: the signs of the coefficients and their statistical significance hardly ever change, and the magnitude of most effects is fairly stable. And all of these effects are consistent with the prediction that second-round bidding is more likely when there is greater uncertainty about the value of the security.[53]

6 Conclusion

This paper studies Anglo-Dutch premium auctions used for the resale of sovereign and other debt in eighteenth-century Amsterdam, Europe's financial capital at the time. We analyze a theoretical model of these auctions, and predict a positive relationship between the uncertainty in the value of a security (relative to the size of the premium) and bidding in the second round of the auction. Empirically, we examine bidding bidding behavior in 16,854 sales, held on 469 auction days over an 18 years period, and confirm that such a relationship does seem to hold. A variety of different security and auction characteristics which are associated with greater uncertainty in value, also predict a higher likelihood of second-round bidding. That the auction outcomes in the data conform cleanly to our theoretical prediction gives us more comfort that even at that time, bidders might have been playing close to equilibrium bidding strategies. This then validates our view of the relative advantages of English and Dutch auctions, suggesting that a hybrid of the two might be advantageous, and therefore rationalizing both the initial and the continued use of the Anglo-Dutch premium auction.

There are still many questions we could not address fully in this paper. Since the formal model of

[53]We also examined the effect of these same variables on the increase in price between the first and second round, looking only at those sales with a second-round bid. Intuitively, the more likely other bidders are to bid in the second round, the higher each bidder must plan to bid in the second round if he wants to win; thus, we would expect the variables that predict more second-round bidding, to also predict a larger increase in price. We find exactly that: among sales with a second-round bid, the increase in price was greater on average for other Dutch bonds, plantation loans, and private company loans; loans in foreign countries; sales after the onset of the financial crisis (but only for plantation loans and private company loans); auctions held in winter; and the increase was smaller when more identical securities had just been sold.

the auction allows for multiple equilibria, a full characterization of the performance of the auction is impossible, and our view on why the auction was selected in the first place is somewhat speculative. On the larger scale, the use of auctions to clear markets for securities was part of a larger trend toward centralization of many markets started during the seventeenth century; it would be of major interest to understand the causes and welfare effects of this process more comprehensively. In this context, it would be rewarding to compare the performance of these securities auctions with other auctions and other clearing mechanisms used during the 17th and 18th centuries. Finally, learning more about price formation and learning behavior of participants in these markets – both dynamics within a given auction day, and dynamics over time as the mechanism (and the participants) became better understood – would help to illuminate the interplay between market design and price formation in greater detail.

Appendix – Proof of Theorem 1

We postpone the proof of equilibrium existence to the end.

A.1 Benchmark Equilibrium

Consider the following strategy for first-round bidding, which we will refer to as "simple bidding":

- At any history where nobody has dropped out yet, given signal s, keep bidding up to the price $v(s, s, s, \ldots, s) + \rho$
- At a history where the first bidder dropped out at price p^N, define s^N implicitly by $p^N = v(s^N, s^N, s^N, \ldots, s^N) + \rho$; at such a history, keep bidding up to price $v(s, s, s, \ldots, s, s^N) + \rho$
- At a history where the second bidder dropped out at price p^{N-1}, define s^N as above and s^{N-1} by $p^{N-1} = v(s^{N-1}, s^{N-1}, s^{N-1}, \ldots, s^{N-1}, s^N) + \rho$; at a history where one bidder dropped out at p^N and another at p^{N-1}, keep bidding up to price $v(s, s, s, \ldots, s, s^{N-1}, s^N) + \rho$
- And so on – at each history, bid as if all dropped-out bidders had revealed their types and all remaining bidders' signals match your own

This is well known to be an *ex post* equilibrium of a (one-stage) ascending auction for a good worth $v(\mathbf{s}) + \rho$.

A.2 Theorem 1 Part 1

We claim that if $\beta \leq N\rho$, all bidders bidding simply in the first round, and not bidding in the second round, is an equilibrium of the two-stage auction; and if $\beta \leq \frac{1}{2}N\rho$, it's an *ex post* equilibrium.

If $\frac{\beta}{N} \leq \frac{1}{2}\rho$, this is an ex post equilibrium.

We show the latter first. Let s_1 be bidder 1's signal, and $y^1 \geq y^2 \geq \ldots \geq y^{N-1}$ the re-ordered signals of his opponents. Following the equilibrium strategy, bidder 1 will win both the security and the premium whenever $s_1 > y^1$, and pay the price at which his last opponent drops out, for a payoff of

$$c + \frac{\beta}{N}\left(s_1 + y^1 + \textstyle\sum_{i>1} y^i\right) + \rho - \left[c + \frac{\beta}{N}\left(y^1 + y^1 + \textstyle\sum_{i>1} y^i\right) + \rho\right] \quad = \quad \frac{\beta}{N}(s_1 - y^1)$$

when $s_1 > y^1$, and a payoff of 0 when $s_1 \leq y^1$. Any other first-round strategy will give either the same payoff or a payoff of 0, so the only potential deviation would involve second-round bidding.

If bidder 1 is the second-to-last bidder to drop out, and drops out at a price revealing s_1 to be x, the first round ends at price $c + \frac{\beta}{N}\left(x + x + \sum_{i>1} y^i\right) + \rho > c + \frac{\beta}{N}\left(x + y^2 + \sum_{i>1} y^i\right) + \rho$. If bidder 1 drops out with at least two opposing bidders still active and reveals s_1 to be x, the first round ends at price $c + \frac{\beta}{N}\left(x + y^2 + \sum_{i>1} y^i\right) + \rho$. So either way, the price reaches at least $c + \frac{\beta}{N}\left(x + y^2 + \sum_{i>1} y^i\right) + \rho \geq c + \frac{\beta}{N}\left(y^2 + \sum_{i>1} y^i\right) + \rho$. So by bidding in the second round and winning the security, bidder 1's payoff is at most

$$c + \tfrac{\beta}{N}\left(s_1 + y^1 + \textstyle\sum_{i>1} y^i\right) - \left[c + \tfrac{\beta}{N}\left(y^2 + \textstyle\sum_{i>1} y^i\right) + \rho\right] \quad = \quad \tfrac{\beta}{N}\left(s_1 + y^1 - y^2\right) - \rho$$

Since $s_1 + y^1 \leq 2$, if $\frac{\beta}{N} \leq \frac{1}{2}\rho$, this is nonpositive. So simple bidding, followed by no bidding in the second round, is an ex post equilibrium when $\beta \leq \frac{1}{2}N\rho$.

If $\frac{\beta}{N} \leq \rho$, this is an equilibrium.

Consider the bidding in the first round up to a particular point where bidder 1 is still active. The history up to that point can be described by the number of bidders (including bidder 1) who are still active, K; the signals revealed by those bidders who have already dropped out, $y^K \geq y^{K+1} \geq \ldots \geq y^{N-1}$; and the lower bound on the signals of the remaining bidders (assuming simple bidding), $\underline{s}$, which is the solution to

$$P \quad = \quad c + \tfrac{\beta}{N}\left(K\underline{s} + y^K + y^{K+1} + \ldots + y^{N-1}\right) + \rho$$

where P is the current price. Conditional on equilibrium play up to that point, the signals of the remaining active bidders are independent uniform draws on the interval $[\underline{s}, 1]$, which will allow us to calculate bidder 1's expected payoff from various strategies. We will show that at any history $h = (K, (y^K, y^{K+1}, \ldots, y^{N-1}), \underline{s})$, if $s_1 > \underline{s}$, straightforward bidding offers a higher expected continuation payoff than dropping out immediately with the option to bid in the second round if that would be profitable; and if $s_1 < \underline{s}$, no strategy offers a strictly positive payoff, so bidding higher in the first round than simple bidding would suggest is never profitable.

If $s_1 > \underline{s}$ and bidder 1 plans to follow the simple bidding strategy, he will win whenever $s_1 > y^1$, earning a security worth $c + \frac{\beta}{N}\left(s_1 + y^1 + \sum_{i>1} y^i\right) + \rho$ at a price $c + \frac{\beta}{N}\left(y^1 + y^1 + \sum_{i>1} y^i\right) + \rho$ for profit of $\frac{\beta}{N}(s_1 - y^1)$. Starting at history h, then, his expected continuation payoff from following this strategy is

$$E_{y^1|h} \max\left\{0, \frac{\beta}{N}\left(s_1 - y^1\right)\right\} \quad = \quad \frac{\beta}{N}\int_{\underline{s}}^{s_1}(s_1 - x)dF_1(x|h)$$

where $F_1(\cdot|h)$ is the probability distribution of y^1, conditional on history h. Conveniently, since the signals of 1's remaining active opponents are independent uniform draws from the interval $[\underline{s}, 1]$, $F_1(x|h) = (\frac{x-\underline{s}}{1-\underline{s}})^{K-1}$, so we can evaluate this integral, which turns out to be $\frac{\beta}{NK}\frac{(s_1-\underline{s})^K}{(1-\underline{s})^{K-1}}$.

If $K = 2$ (only one opponent besides bidder 1 is still active at history h), then by dropping out immediately, bidder 1 would end the first round immediately, at price $c + \frac{\beta}{N}\left(\underline{s} + \underline{s} + \sum_{i>1} y^i\right) + \rho$. The remaining opponent's signal would then be a uniform draw from the interval $[\underline{s}, 1]$, so by bidding in the second round, bidder 1 could in expectation earn at most

$$c + \frac{\beta}{N}\left(s_1 + \frac{1+\underline{s}}{2} + \sum_{i>1} y^i\right) - \left[c + \frac{\beta}{N}\left(\underline{s} + \underline{s} + \sum_{i>1} y^i\right) + \rho\right] \quad = \quad \frac{\beta}{N}\left(s_1 + \frac{1+\underline{s}}{2} - 2\underline{s}\right) - \rho$$

If $\beta \leq N\rho$, then, the gain from dropping out now instead of sticking to the equilibrium strategy is at most

$$\begin{aligned}
&\frac{\beta}{2N}\left(2s_1 + 1 + \underline{s} - 4\underline{s}\right) - \rho - \frac{\beta}{2N}\frac{(s_1-\underline{s})^2}{1-\underline{s}} \\
&\quad\leq \quad \frac{\beta}{2N}\left(2(s_1 - \underline{s}) - 2\underline{s} - (1 - \underline{s}) - \frac{(s_1-\underline{s})^2}{1-\underline{s}}\right) \\
&\quad= \quad \frac{\beta}{2N}\frac{1}{1-\underline{s}}\left(-(1-\underline{s})^2 + 2(s_1 - \underline{s})(1-\underline{s}) - (s_1 - \underline{s})^2 - 2\underline{s}(1-\underline{s})\right) \\
&\quad= \quad \frac{\beta}{2N}\frac{1}{1-\underline{s}}\left(-(1-s_1)^2 - 2\underline{s}(1-\underline{s})\right) \\
&\quad\leq \quad 0
\end{aligned}$$

If $K > 2$, then by dropping out immediately, bidder 1 would "reveal" his signal to be $s_1 = \underline{s}$, and he would have the option of bidding in the second round after learning the realizations of $(y^2, y^3, \ldots, y^{K-1})$ (but not y^1). The first round would end at price $c + \frac{\beta}{N}\left(\underline{s} + y^2 + \sum_{i>1} y^i\right) + \rho$. Since, conditional on y^2, $y^1 \sim U[y^2, 1]$, the expected value of the security at that point would be $c + \frac{\beta}{N}\left(s_1 + \frac{1+y^2}{2} + \sum_{i>1} y^i\right)$. By bidding a tiny bit above the reserve whenever this expected value is greater than the first-round price, bidder 1 could at best achieve a payoff of

$$\begin{aligned}
&\max\left\{0, c + \frac{\beta}{N}\left(s_1 + \frac{1+y^2}{2} + \sum_{i>1} y^i\right) - \left[c + \frac{\beta}{N}\left(\underline{s} + y^2 + \sum_{i>1} y^i\right) + \rho\right]\right\} \\
&\qquad = \max\left\{0, \frac{\beta}{N}\left(s_1 - \underline{s} + \frac{1-y^2}{2}\right) - \rho\right\}
\end{aligned}$$

and so bidder 1's maximal expected payoff from dropping out at history h is

$$E_{y^2|h} \max \left\{0, \tfrac{\beta}{N}\left(s_1 - \underline{s} + \tfrac{1-y^2}{2}\right) - \rho\right\}$$

which (if it is ever strictly positive) can be written as

$$\frac{\beta}{2N}\int_{\underline{s}}^{\min\{1, 2s_1 - 2\underline{s} - 2\frac{\rho N}{\beta}+1\}}\left(2s_1 - 2\underline{s} - \frac{2N\rho}{\beta} + 1 - x\right) dF_2(x|h)$$

where $F_2(\cdot|h)$ is the distribution of y^2 conditional on h. Since the remaining opponents have signals which are independently $U[\underline{s}, 1]$, when $K \geq 3$, $dF_2(x|h) = (K-1)(K-2)\frac{1}{1-\underline{s}}\frac{1-x}{1-\underline{s}}(\frac{x-\underline{s}}{1-\underline{s}})^{K-3}$. If $\beta \leq N\rho$, the upper limit of the integral is less than 1, so the expected payoff from dropping out is no more than

$$\frac{\beta}{2N}\frac{(K-1)(K-2)}{(1-\underline{s})^{K-1}}\int_{\underline{s}}^{2s_1 - 2\underline{s} - 2\frac{\rho N}{\beta}+1}\left(2s_1 - 2\underline{s} - \frac{2N\rho}{\beta} + 1 - x\right)(1-x)\,(x-\underline{s})^{K-3}\,dx$$

After some algebra, this integrates to

$$\frac{\beta}{NK}\frac{Z^K}{(1-\underline{s})^{K-1}} - \frac{\beta}{2NK}\frac{Z^{K-1}}{(1-\underline{s})^{K-1}}K(1-\underline{s}+Z)$$

where $Z = 2s_1 - 2\underline{s} - 2\frac{\rho N}{\beta} + 1 - \underline{s}$. Now, if $\beta \leq N\rho$, then $2\frac{N\rho}{\beta} \geq 2 \geq (s_1 - \underline{s}) + (1-\underline{s})$, or $0 \geq (s_1 - \underline{s}) - 2\frac{N\rho}{\beta} + (1-\underline{s})$, which in turn implies $s_1 - \underline{s} \geq Z$, so

$$\tfrac{\beta}{NK}\tfrac{(s_1-\underline{s})^K}{(1-\underline{s})^{K-1}} \quad\geq\quad \tfrac{\beta}{NK}\tfrac{Z^K}{(1-\underline{s})^{K-1}} \quad\geq\quad \tfrac{\beta}{NK}\tfrac{Z^K}{(1-\underline{s})^{K-1}} - \tfrac{\beta}{2NK}\tfrac{Z^{K-1}}{(1-\underline{s})^{K-1}}K(1-\underline{s}+Z)$$

and the payoff to dropping out now (followed by optimal play) cannot be better than sticking to simple bidding.

Finally, at a history h with $s_1 \leq \underline{s}$, the first round is guaranteed to end at a price no lower than $c + \frac{\beta}{N}\left(\underline{s} + y^2 + \sum_{i>1} y^1\right) + \rho$, so bidder 1's payoff from bidding in the second round can be no more than

$$\begin{aligned} c + \tfrac{\beta}{N}\left(s_1 + y^1 + \textstyle\sum_{i>1} y^i\right) - \left[c + \tfrac{\beta}{N}\left(\underline{s} + y^2 + \textstyle\sum_{i>1} y^1\right) + \rho\right] &= \tfrac{\beta}{N}\left(s_1 - \underline{s} + y^1 - y^2\right) - \rho \\ &\leq \tfrac{\beta}{N} - \rho \end{aligned}$$

which is nonpositive if $\beta \leq N\rho$. So neither dropping out of the first round earlier than simple bidding would suggest, nor remaining active later, would be a profitable deviation; so simple bidding, followed by no bidding in the second round, is an equilibrium.

A.3 Theorem 1 Part 2

We want to show that if $\beta > N\rho$, there cannot be a symmetric equilibrium with no second-round bidding on the equilibrium path.

No equilibrium that's outcome-equivalent to simple bidding

First, note that any strategies which are outcome-equivalent to the simple-bidding equilibrium proven above cannot be an equilibrium when $\beta > N\rho$. This is because:

- In the event that $y^1 = \epsilon \approx 0$, such strategies would need to end the first round at a price close to $c + \rho$; so in the event that $y^2 \approx 0$, all bidders but bidder 1 and one opponent must drop out at prices close to $c + \rho$
- Once that happens, in the event that $s_1 = 1 - \epsilon \approx 1$, bidder 1 knows he will win with probability close to 1, at a price close to $c + \frac{\beta}{N} 2y^1 + \rho$, which has expected value $c + \frac{\beta}{N} + \rho$; so by following the supposed equilibrium strategy, his expected payoff would be $c + \frac{\beta}{N}(1 + \frac{1}{2}) + \rho - \left[c + \frac{\beta}{N} + \rho\right] = \frac{\beta}{2N}$
- However, by dropping out immediately and then bidding $\epsilon \approx 0$ above the current price in the second round, bidder 1 could win the security (without the premium) at a price close to $c+\rho$, for expected payoff $c + \frac{\beta}{N}(1 + \frac{1}{2}) - [c + \rho] = \frac{3\beta}{2N} - \rho$
- If $\beta > N\rho$, $\frac{3\beta}{2N} - \rho > \frac{\beta}{2N}$, so this would be a profitable deviation.

No symmetric, separating equilibrium

Next, we show that any symmetric, *separating* equilibrium (i.e., any symmetric equilibrium with strictly monotone strategies in each part of the auction, as used in Bikchandani, Haile and Riley (2002)) in which there are no second-round bids on the equilibrium path must lead to the same price as simple bidding, and therefore such an equilibrium cannot exist when $\beta > N\rho$. Bikchandani et. al. show that this would be true if there was no second round: that is, that any symmetric, separating equilibrium of a pure ascending auction would lead to the same price as simple bidding. So if there were a symmetric, separating equilibrium of the two-stage mechanism with no second-round bidding which led to a different outcome, those same strategies would not be an equilibrium in a pure ascending auction. Suppose such an equilibrium existed, and call those strategies B.

Since B is an equilibrium of the two-stage mechanism but B restricted to the first round of bidding is not an equilibrium of an ascending auction, that means some bidder must have a deviation from B which would be profitable in a pure ascending auction but not in the two-stage mechanism. This is only possible if that deviation would be "punished" by second-round bidding in the two-stage mechanism. Since by assumption, B does not have second-round bids on the equilibrium path, it must be that this deviation would therefore be recognized as an off-equilibrium-path play. However, in a symmetric, separating equilibrium, a bidder with the highest possible signal never drops out in equilibrium; so a unilateral deviation by any one bidder could not be detected until he dropped out. But at the point where the deviation calls for him to drop out, he is guaranteed a payoff no worse than 0, the same as in a pure ascending auction; so if the deviation is profitable in the pure ascending auction, the same first-round deviation (followed by no second-round bid) would be profitable in the two-stage auction. So there is no equilibrium of the two-stage auction with no second-round bidding on the equilibrium path which does not conclude at the same price as simple bidding; so when $\beta > N\rho$, no symmetric, separating equilibrium can exist with no second-round bidding on the equilibrium path.

No symmetric, non-separating equilibrium

All that is left, then, is potential symmetric equilibria which are not separating, i.e., potential equilibria where either some range of types playing mixed strategies with overlapping supports, or some range of types pooled at the same pure strategy.

Suppose first that at some history, there is some range of signals at which bidders play a mixed strategy whose supports have common overlap. Since there is some probability that all remaining bidders have types within that range, strategies at the top of that overlap must have strictly higher probability of winning then strategies at the bottom of that overlap. But bidder payoffs have strictly increasing differences in the signal and the probability of winning; so multiple signals can't make a bidder indifferent between two of those strategies.

Strict single-crossing also means that any equilibrium strategies must be weakly monotonic. All that is left is the possibility of an equilibrium where at some history, some range of types (s_*, s^*) pool and drop out at a common price P. Note that since there is a chance all remaining bidders have types within that range, pooling gives a positive probability less than 1 of winning at that price.

- If $s^* < 1$, then any pooling types who strictly prefer to win at that price could remain in slightly longer and win with discontinuously higher probability; such a deviation would not appear to be "out of equilibrium" since higher types stay in longer, so it could not be "punished" by being outbid in the second round. If no pooling types strictly prefer to win, then some pooling types strictly prefer to lose, in which case they would drop out before the pooling price. So the pooling range must be $(s_*, 1]$.
- If bidders with signals $s_i = 1$ strictly prefer to win at the pooling price, then the only way to rule out a deviation of staying in past that price is if another bidder would "punish" such a deviation by outbidding it in the second round with a high probability. But in that case, the bidders at the bottom of the pooling range could benefit by exceeding the pooling price, winning the premium, and then being outbid in the second round. (The lowest pooling type must be indifferent about winning the security *and the premium* at the pooling price, otherwise types below the pooling range would also pool; but this means winning the premium but likely losing the security is even better!)
- The only way to rule this out is for the pooling range to be all types. Then no information is revealed in the first round, and bidders play the symmetric equilibrium in the second round.
- To rule out a bidder with a high signal grabbing the object in the second round, the pooling price must be at least $c + \frac{\beta}{N}\left(1 + \frac{N-1}{2}\right)$. But since $\frac{\beta}{N} > \rho$ by assumption, this is strictly more than $c + \frac{\beta}{N}\left(0 + \frac{N-1}{2}\right) + \rho$; so a bidder with a low type gets negative expected payoff by pooling with the rest.

A.4 Extending Parts 1 and 2 to a More General Model

Parts 1 and 2 have analogs in a much more general common values model, but at the cost of more notation and less easily-interpreted results. Suppose bidder signals s_i are potentially correlated, but their distribution is symmetric and has support $[0,1]^N$; and $v(\mathbf{s})$ is any symmetric, strictly-increasing function. (This nests the standard affiliated-signals model where $(V, S_1, S_2, \ldots, S_N)$ are affiliated, since we can let $v(\mathbf{s}) = E(V|(S_1, \ldots, S_N) = \mathbf{s})$.

Let $\mathbf{y} = (y^2, y^3, \ldots, y^{N-1})$ denote all but the highest of 1's opponents. We define two measures

Δ_1 and Δ_2 relating to the variability of $v(\mathbf{s})$: let

$$\Delta_2 = \max_{\mathbf{y}} \left\{ v(1,1,\mathbf{y}) - v(0,y^2,\mathbf{y}) \right\}$$

and Δ_1 the value of ρ that solves

$$0 \;=\; \max_{h} \;\max_{s_1 \geq \underline{s}} \left\{ \begin{array}{r} \underset{\mathbf{y}|h,s_1}{E} \max\left\{0,\; \underset{y^1|\mathbf{y},s_1}{E} \{v(s_1,y^1,\mathbf{y}) - v(\underline{s},\max\{\underline{s},y^2\},\mathbf{y}) - \rho\}\right\} \\ \\ -\underset{\mathbf{y}|h,s_1}{E} \;\underset{y^1|\mathbf{y},s_1}{E} \max\left\{0,\; v(s_1,y^1,\mathbf{y}) - v(y^1,y^1,\mathbf{y})\right\} \end{array} \right\}$$

Note that in the independent-linear model presented in the text, $\Delta_2 = \frac{2\beta}{N}$ and $\Delta_1 = \frac{\beta}{N}$.[54] Parts 1 and 2 of Theorem 1 extend to the more general model in the following way:

- If $\rho \geq \Delta_2$, then simple bidding in the first round, followed by no bidding in the second round, is an ex post equilibrium of the two-stage mechanism. If $\rho < \Delta_2$, then no ex post equilibrium exists for the two-stage mechanism with no second-round bidding.
- If $\rho \geq \Delta_1$, then simple bidding in the first round, followed by no bidding in the second round, is an equilibrium of the two-stage mechanism. If $\rho < \Delta_1$, then no symmetric, separating equilibrium exists for the two-stage mechanism with no second-round bidding.

The last part of Theorem 1, however, has no analog when bidder signals are not independent.

A.5 Theorem 1 Part 3

We need to show that in the independent, linear model, if $\frac{\beta}{N(N+1)} > \rho$, then in any equilibrium, the probability of a second-round bid is bounded below by $\frac{1}{c+\beta}\left(\frac{\beta}{N(N+1)} - \rho\right)$.

To prove this, we first show that in any equilibrium, ex-ante combined bidder payoffs must be at least $\frac{\beta}{N(N+1)}$. Let $U_i(s_i)$ denote the equilibrium payoff of bidder i with signal s_i, and $Q_i(s_i)$ the probability he wins the security in equilibrium. For any $s_i' > s_i$,

$$U_i(s_i') - U_i(s_i) \quad \geq \quad Q_i(s_i)\frac{\beta}{N}(s_i' - s_i)$$

since a bidder with type s_i' could always imitate the equilibrium strategy of s_i but the security

[54] The former is easy to see, as the maximum is achieved at $\mathbf{y} = (0,0,\ldots,0)$. The latter maximum is achieved at $s_1 = 1$, at the history where all but one of bidder 1's opponents have signals 0 and have just dropped out, so that $K = 2$ and $\underline{s} = 0$, although showing that no other history exceeds this value of the maximand is nontrivial.

would be worth $\frac{\beta}{N}(s_i' - s_i)$ more whenever he wins it. So for any ϵ,

$$U_i(s_i) \geq u_i(s_i - \epsilon) + \tfrac{\beta}{N}\epsilon Q_i(s_i - \epsilon) \geq u_i(s_i - 2\epsilon) + \tfrac{\beta}{N}\epsilon Q_i(s_i - \epsilon) + \tfrac{\beta}{N}\epsilon Q_i(s_i - 2\epsilon) \geq \cdots$$

and taking $\epsilon \to 0$, $U_i(s_i) \geq \frac{\beta}{N}\int_0^{s_i} Q_i(t)dt$. Let U_i denote bidder i's ex-ante equilibrium expected payoff; then

$$U_i = E_{s_i} U_i(s_i) \geq E_{s_i} \frac{\beta}{N} \int_0^{s_i} Q_i(t)dt = \frac{\beta}{N} \int_0^1 \int_0^{s_i} Q_i(t)dtds_i$$

Reversing the order of integration,

$$U_i \geq \frac{\beta}{N} \int_0^1 \int_t^1 Q_i(t)ds_i dt = \frac{\beta}{N} \int_0^1 (1-t)Q_i(t)dt$$

Let $q_i(s_1, s_2, \ldots, s_N)$ denote bidder i's probability of winning the security at a particular signal profile (given equilibrium play by all bidders), and s_{-i} the signals of bidder i's opponents; then

$$U_i \geq \frac{\beta}{N} \int_0^1 (1-s_i)E_{s_{-i}} q_i(s_i, s_{-i})ds_i = \frac{\beta}{N} \int_{[0,1]^N} (1-s_i)q_i(\mathbf{s})d\mathbf{s}$$

Summing over i,

$$\sum_i U_i \geq \frac{\beta}{N} \int_{[0,1]^N} \left(\sum_i (1-s_i)q_i(\mathbf{s}) \right) d\mathbf{s} \geq \frac{\beta}{N} \int_{[0,1]^N} \left(\min_i \{1 - s_i\} \right) \left(\sum_i q_i(\mathbf{s}) \right) d\mathbf{s}$$

Since the auction always awards the security to somebody, $\sum_i q_i(\mathbf{s})d\mathbf{s} = 1$, so

$$\sum_i U_i \geq \frac{\beta}{N} \int_{[0,1]^N} \left(\min_i (1-s_i) \right) d\mathbf{s} = \frac{\beta}{N(N+1)}$$

Next, we use this to rule out equilibrium with no second-round bidding. Let h be the history of first-round bids. If there is no second-round bidding, then ex post payoffs given h are the expected value of the security, $E(v|h)$; plus the premium, ρ; minus the price at which the first round ended, which we label $p(h)$. By iterated expectations,

$$E_h \{E(v|h) + \rho - p(h)\} = \textstyle\sum_i U_i \geq \frac{\beta}{N(N+1)}$$

If $\frac{\beta}{N(N+1)} > \rho$, then $E_h\{E(v|h) - p(h)\} > 0$; so at some histories, the security (in expectation over all the signals that produce those first-round bids) is worth more than the first-round price. Then in a supposed equilibrium where nobody was planning to bid in the second round (and there was therefore no additional winner's curse), any losing bidder (provided his own signal was good relative to his range given h) could gain by bidding in the second round.

Next, we put a lower bound on the probability of a second-round bid. Let (h, t) denote the first-round bidding history h, followed by nobody bidding in the second round before or at price $p(h) + t$. Thus, $(h, 0)$ denotes a first-round outcome followed by nobody bidding in the second round. Let $E(v|(h, 0), s_i)$ denote the expected value of the security conditional on both $(h, 0)$ and on the actual value of bidder i's signal.

Suppose we are in any equilibrium, and consider a particular first-round history h. Pure strategies for the second-round game consist of a price $t(s_i)$ such that bidder i, given signal s_i, plans to bid in the second round at price $p(h) + t(s_i)$ if nobody has bid by then. $t(s_i) = 0$ means a plan not to bid. By strict dominance, we can assume $t(s_i) \in [0, \overline{v}]$; mixed strategies are allowed.

Claim 1. *Second-round equilibrium strategies satisfy strict single crossing: if both are reached with positive probability, if bidder i weakly prefers t' to $t < t'$ at signal s_i, he must strictly prefer t' to t at any signal $s'_i > s_i$.*

This is straightforward, since the probability of winning is monotonic in t, and $t' > t$ can only be as good if the win probability is strictly higher. This leads to equilibrium strategies being weakly monotonic; which means that the conditional expected value of the security can only go down as the second round progresses, except at the moment when someone bids.

Claim 2. *In any equilibrium, for any h, if the history $(h, 0)$ is reached on the equilibrium path, $E(v|(h, 0)) \leq p(h)$.*

Suppose this were false – that is, suppose for some history h, $E(v|(h, 0)) - p(h) > 0$. Consider second-round strategies following that history. Let s_i^* denote the highest signal consistent with h at which losing bidder i chooses $t(s_i) = 0$ with positive probability. We claim $E(v|(h, 0, s_i^*)) \leq p(h)$. If it was not, let $\epsilon = E(v|(h, 0, s_i^*)) - p(h) > 0$. By bidding at price $p(h) + \frac{\epsilon}{2}$, bidder i would get a payoff of $E(v|(h, \frac{\epsilon}{2}), s_i^*) - (p(h) + \frac{\epsilon}{2}) \geq E(v|(h, 0), s_i^*) - p(h) - \frac{\epsilon}{2} = \frac{\epsilon}{2}$, so not bidding would be dominated. So for all s_i such that $E(v|(h, 0, s_i)) > p(h)$, bidder i must plan to bid at some point (although he could mix over the time). So if he lets time run out, we know $E(v|(h, 0, s_i)) \leq p(h)$. So then taking the expectation of s_i over the signals at which he doesn't bid, we're done.

Claim 3. *At any history $(h, 0)$, expected ex post winner surplus is no more than ρ.*

It's $E(v|(h,0)) + \rho - p(h) \leq \rho$. If we now take expectations over h such that no bid happens in the second round, the expected ex-post payoff to a first-round winner is no more than ρ.

Claim 4. *Let r be the probability of a second-round bid; then $r \geq \frac{1}{c+\beta}\left(\frac{\beta}{N(N+1)} - \rho\right)$.*

Let $E(no)$ denote expected ex-post payoffs conditional on no second-round bid being placed, and $E(yes)$ expected ex-post payoffs conditional on a second-round bid. By iterated expectations, $\sum_i U_i = (1-r)E(no) + rE(yes)$. We just showed that $E(no) \leq \rho$; since bids are non-negative, the highest payoffs ever achievable are $c + \beta + \rho$, so

$$\frac{\beta}{N(N+1)} \leq \sum_i U_i = (1-r)E(no) + rE(yes) \leq (1-r)\rho + r(c+\beta+\rho) = \rho + r(c+\beta)$$

and so $\frac{\beta}{N(N+1)} - \rho \leq r(c+\beta)$. □

A.6 Proof of Equilibrium Existence

To prove existence, we construct a particular asymmetric equilibrium that exists for all values of β and ρ. We begin with the case of $N \geq 4$, then show what needs to be modified when $N = 3$ or $N = 2$. The equilibrium we construct will involve all bidders planning to bid in the second round on the equilibrium path.

Equilibrium Path

On the equilibrium path, bidders $2, 3, \ldots, N$ all drop out of the first round at price 0, letting bidder 1 win the premium. No information is revealed, so in the second round, beliefs still match the priors, and bidding functions are

$$b(s_i) = c + \frac{\beta}{N}\left(\frac{N-1}{N}s_i + (N-1)\frac{s_i}{2}\right)$$

To see that second-round strategies are an equilibrium, note that a bidder with signal s_i who plans to bid $b(x)$ will win whenever $s_j < x$ for every $j \neq i$, which occurs with probability x^{N-1}; in that event, the security is worth, in expectation, $c + \frac{\beta}{N}\left(s_i + (N-1)\frac{x}{2}\right)$, so the expected payoff is

$$x^{N-1}\left[c + \frac{\beta}{N}\left(s_i + (N-1)\frac{x}{2}\right) - \left(c + \frac{\beta}{N}\left(\frac{N-1}{N}x + (N-1)\frac{x}{2}\right)\right)\right] = \frac{\beta}{N}x^{N-1}\left[s_i - \frac{N-1}{N}x\right]$$

Differentiating with respect to x gives $\frac{\beta}{N}\left[(N-1)s_i x^{N-2} - (N-1)x^{N-1}\right] = \frac{\beta}{N}(N-1)x^{N-2}(s_i - x)$, which is positive when $x < s_i$ and negative when $x > s_i$. Setting $x = s_i$ gives positive payoff, and bids above $b(1)$ are dominated, so $b(s_i)$ is a best-response.

To make first-round play (in particular, dropping out immediately) a best-response, we construct a continuation game where a bidder who deviates and bids in the first round is "punished" by a combination of other bidders' beliefs, continued first-round bidding by bidder 1, and bidding in the second round. To make the punishment "credible", we construct another continuation game where bidder 1 is "punished" for failing to properly punish bidder 2. Second-round bidding in these two continuation games is defined next; off-equilibrium first-round strategies are then defined at the end.

For ease of exposition, if a single player $i \geq 2$ deviates in the first round and fails to drop out immediately, we will assume it is bidder 2. The equilibrium is symmetric in the identities of bidders $2, 3, \ldots, N$.

Continuation Game 3

Continuation Game 3 is used following consecutive deviations from equilibrium play by both players 2 and player 1. Beliefs of players $3, \ldots, N$ are that $s_1 = s_2 = 1$. Either bidder 1 or 2 is the first-round winner, at a price P. Bidding strategies are:

- Bidders 1 and 2 (regardless of their actual types) do not bid
- If $P \geq c + \frac{\beta}{N}\left(1 + \frac{N-3}{2} + 2\right)$, then nobody bids
- If $P \in \left(c + \frac{2\beta}{N},\, c + \frac{\beta}{N}\left(1 + \frac{N-3}{2} + 2\right)\right)$, then let s^* solve

$$P \;=\; c + \frac{\beta}{N}\left(s^* + (N-3)\frac{s^*}{2} + 2\right)$$

  Bidder $i \geq 3$ does not bid when $s_i \leq s^*$; when $s_i > s^*$, he bids

$$b(s_i) \;=\; c + \frac{\beta}{N}\left(\frac{N-3}{N-2}s_i + \frac{s_i}{N-2}\left(\frac{s^*}{s_i}\right)^{N-2} + (N-3)\frac{s_i}{2} + 2\right)$$

- If $P \leq c + \frac{2\beta}{N}$, then bidders $i \geq 3$ each bid

$$b(s_i) \;=\; c + \frac{\beta}{N}\left(\frac{N-3}{N-2}s_i + (N-3)\frac{s_i}{2} + 2\right)$$

Let's check this is an equilibrium. First, if $P \geq c + \frac{\beta}{N}(1 + \frac{N-3}{2} + 2)$, then a bidder $i \geq 3$ with signal s_i expects no one else to bid, and believes that the security is worth in expectation

$c+\frac{\beta}{N}(s_i+\frac{N-3}{2}+1+1)\leq P$, and therefore does not bid. Bidder $i\in\{1,2\}$ believes the security to be worth in expectation $c+\frac{\beta}{N}(s_i+\frac{N-2}{2}+1)<P$ and therefore does not bid either.

For the second case, a bid above $c+\frac{\beta}{N}\left(1+\frac{N-3}{2}+2\right)$ is dominated; a bid by bidder $i\geq 3$ equal to $b(x)$ $(x>s^*)$ wins whenever $s_j<x$ for all $j\in\{3,4,\ldots,N\}-\{i\}$, which is with probability x^{N-3}, and therefore delivers expected payoff of

$$\begin{aligned} &x^{N-3}\left[c+\tfrac{\beta}{N}\left(s_i+(N-3)\tfrac{x}{2}+2\right)-\left(c+\tfrac{\beta}{N}\left(\tfrac{N-3}{N-2}x+\tfrac{x}{N-2}\left(\tfrac{s^*}{x}\right)^{N-2}+(N-3)\tfrac{x}{2}+2\right)\right)\right]\\ &\quad= x^{N-3}\tfrac{\beta}{N}\left[s_i-\tfrac{N-3}{N-2}x-\tfrac{(s^*)^{N-2}}{N-2}x^{-(N-3)}\right] \;=\; \tfrac{\beta}{N}\left[s_ix^{N-3}-\tfrac{N-3}{N-2}x^{N-2}-\tfrac{(s^*)^{N-2}}{N-2}\right]\end{aligned}$$

Differentiating with respect to x gives

$$\tfrac{\beta}{N}\left[(N-3)s_ix^{N-2}-(N-3)x^{N-3}\right]$$

which is positive for $x<s_i$ and negative for $x>s_i$, so $b(s_i)$ is a best-response among bids above P. Bidding P (or slightly above P) would give expected payoff

$$\tfrac{\beta}{N}\left[s_i(s^*)^{N-3}-\tfrac{N-3}{N-2}(s^*)^{N-2}-\tfrac{(s^*)^{N-2}}{N-2}\right] \;=\; \tfrac{\beta}{N}\left[s_i(s^*)^{N-3}-(s^*)^{N-2}\right]$$

which is positive for $s_i>s^*$ and negative for $s_i<s^*$. So not bidding is a best-response for $s_i\leq s^*$, and $b(s_i)$ a best-response for $s_i>s^*$.

Bidder $i\in\{1,2\}$ who is *not* the first-round bidder would, by bidding $b(x)$, earn expected payoff

$$\begin{aligned} &x^{N-2}\left[c+\tfrac{\beta}{N}\left(s_i+(N-2)\tfrac{x}{2}+1\right)-\left(c+\tfrac{\beta}{N}\left(\tfrac{N-3}{N-2}x+\tfrac{x}{N-2}\left(\tfrac{s^*}{x}\right)^{N-2}+(N-3)\tfrac{x}{2}+2\right)\right)\right]\\ &\quad= \tfrac{\beta}{N}x^{N-2}\left[s_i-1+\tfrac{x}{2}-\tfrac{N-3}{N-2}x-\tfrac{x}{N-2}\left(\tfrac{s^*}{x}\right)^{N-2}\right] \;<\; 0\end{aligned}$$

and so prefers not to bid. Bidder $i\in\{1,2\}$ who *is* the first-round bidder is effectively bidding $b(s^*)$ by not making a new bid; but since

$$\begin{aligned} &\tfrac{d}{dx}\left\{\tfrac{\beta}{N}\left[(s_i-1)x^{N-2}+x^{N-1}\left(\tfrac{1}{2}-\tfrac{N-3}{N-2}\right)-\tfrac{x}{N-2}(s^*)^{N-2}\right]\right\}\\ &= \tfrac{\beta}{N}\left[(N-2)x^{N-3}(s_i-1)+(N-1)x^{N-2}\left(\tfrac{1}{2}-\tfrac{N-3}{N-2}\right)-\tfrac{1}{N-2}(s^*)^{N-2}\right] \;<\; 0\end{aligned}$$

he prefers $b(s^*)$ to a higher bid, and does not bid again.

For the third case, a bid of $b(x)$ by bidder $i\geq 3$ gives expected payoff

$$x^{N-3}\left[c+\tfrac{\beta}{N}\left(s_i+(N-3)\tfrac{x}{2}+2\right)-c-\tfrac{\beta}{N}\left(\tfrac{N-3}{N-2}x+(N-3)\tfrac{x}{2}+2\right)\right] \;=\; \tfrac{\beta}{N}x^{N-3}\left[s_i-\tfrac{N-3}{N-2}x\right]$$

which has derivative $\frac{\beta}{N}\left((N-3)x^{N-2}s_i-(N-3)x^{N-3}\right)$, which once again is positive for $x<s_i$ and negative for $x>s_i$; setting $x=s_i$ gives strictly positive payoff, so $b(s_i)$ is a best-response. For bidder $i\in\{1,2\}$ (whether or not the first-round winner, counting the premium as a "sunk benefit"), bidding $b(x)$ gives payoff

$$\begin{aligned} x^{N-2}\left[c+\tfrac{\beta}{N}\left(s_i+(N-2)\tfrac{x}{2}+1\right)-c-\tfrac{\beta}{N}\left(\tfrac{N-3}{N-2}x+(N-3)\tfrac{x}{2}+2\right)\right] \\ = \tfrac{\beta}{N}x^{N-2}\left[s_i-1+\tfrac{x}{2}-\tfrac{N-3}{N-2}x\right] \leq 0 \end{aligned}$$

so not bidding is a best-response.

Finally, define

$$P^{**}=\begin{cases} c+\frac{\beta}{N}\frac{N+2}{2}+\rho & if \quad \rho\geq\frac{\beta}{2N} \\ c+\frac{\beta}{N}\left(2+\frac{N-1}{2}\left(\frac{2N\rho}{\beta}\right)^{\frac{1}{N-1}}\right) & if \quad \rho<\frac{\beta}{2N} \end{cases}$$

If $\rho\geq\frac{\beta}{2N}$, then $P^{**}\geq c+\frac{\beta}{N}\frac{N+3}{3}$, so if $P=P^{**}$, nobody will bid in the second round; in that case, the first-round winner wins the security for sure (regardless of others' signals), so if $s_1=s_2=1$, his expected payoff is

$$c+\tfrac{\beta}{N}\left(2+\tfrac{N-2}{2}\right)-\left[c+\tfrac{\beta}{N}\tfrac{N+2}{2}+\rho\right] = -\rho$$

which, combined with winning the premium ρ in the first round, gives a net expected payoff of 0.

If $\rho\geq\frac{\beta}{2N}$, then when $P=P^{**}$, s^* solves

$$c+\tfrac{\beta}{N}\left(2+\tfrac{N-1}{2}\left(\tfrac{2N\rho}{\beta}\right)^{\frac{1}{N-1}}\right) = c+\tfrac{\beta}{N}\left(s^*+\tfrac{N-3}{2}s^*+2\right)$$

and therefore $s^*=\left(\frac{2N\rho}{\beta}\right)^{\frac{1}{N-1}}$. So with probability $(s^*)^{N-2}$, nobody bids in the second round, and the security is worth, in expectation (assuming $s_1=s_2=1$), $c+\frac{\beta}{N}\left((N-2)\frac{s^*}{2}+2\right)$. So the first-round winner $i\in\{1,2\}$ at price P^{**}, assuming $s_1=s_2=1$, gets expected payoff (gross of winning the premium)

$$\begin{aligned} (s^*)^{N-2}\left[c+\tfrac{\beta}{N}\left((N-2)\tfrac{s^*}{2}+2\right)-c-\tfrac{\beta}{N}\left((N-1)\tfrac{s^*}{2}+2\right)\right] \\ = \tfrac{\beta}{N}(s^*)^{N-2}\left[-\tfrac{s^*}{2}\right] = -\tfrac{\beta}{2N}(s^*)^{N-1} = -\tfrac{\beta}{2N}\tfrac{2N\rho}{\beta} = -\rho \end{aligned}$$

and so including winning the premium, the first-round winner would get ex-ante expected payoff 0.

Continuation Game 2

Continuation Game 2 is used following a deviation by bidder 2 only. All bidders commonly believe that $s_2 = 1$. Strategies are:

- Bidder 2 does not bid (regardless of his actual type)
- If $P \geq c + \frac{\beta}{N}\left(1 + \frac{N}{2}\right)$, then nobody bids
- If $P \in \left(c + \frac{\beta}{N}, c + \frac{\beta}{N}\left(1 + \frac{N}{2}\right)\right)$, then let s^* solve $P = c + \frac{\beta}{N}\left(1 + \frac{N}{2}s^*\right)$. Bidder $i \neq 2$ does not bid when $s_i \leq s^*$; when $s_i > s^*$, he bids

$$b(s_i) \quad = \quad c + \frac{\beta}{N}\left(\frac{N-2}{N-1}s_i + \frac{s_i}{N-1}\left(\frac{s^*}{s_i}\right)^{N-1} + (N-2)\frac{s_i}{2} + 1\right)$$

- If $P \leq c + \frac{\beta}{N}$, then bidders $i \neq 2$ each bid

$$b(s_i) \quad = \quad c + \frac{\beta}{N}\left(\frac{N-2}{N-1}s_i + (N-2)\frac{s_i}{2} + 1\right)$$

Proof that these are equilibrium strategies are exactly analogous to those in Continuation Game 3. Define

$$P^* = \begin{cases} c + \frac{\beta}{N}\frac{N+1}{2} + \rho & if \quad \rho \geq \frac{\beta}{2N} \\ c + \frac{\beta}{N}\left(1 + \frac{N}{2}\left(\frac{2N\rho}{\beta}\right)^{\frac{1}{N}}\right) & if \quad \rho < \frac{\beta}{2N} \end{cases}$$

We will show that if $s_2 = 1$ and bidder 2 wins the first round at price P^*, he earns ex-ante expected payoff 0.

If $\rho \geq \frac{\beta}{2N}$, then $P^* = c + \frac{\beta}{N}\frac{N+1}{2} + \rho \geq c + \frac{\beta}{N}\frac{N+2}{2}$, and so if the first round reaches price P^*, nobody bids in the first round. In that case, by winning the first round at price P^* (and not updating beliefs about s_1), bidder 2 earns second-round payoff

$$c + \frac{\beta}{N}\left(s_2 + \frac{N-1}{2}\right) - \left[c + \frac{\beta}{N}\frac{N+1}{2} + \rho\right] \quad = \quad \frac{\beta}{N}(s_2 - 1) - \rho$$

Combined with winning the premium, then, bidder 2's expected payoff would be nonpositive.

If $\rho < \frac{\beta}{2N}$, then if $P = P^* = c + \frac{\beta}{N}\left(1 + \frac{N}{2}\left(\frac{2N\rho}{\beta}\right)^{\frac{1}{N}}\right)$, s^* solves

$$c + \frac{\beta}{N}\left(1 + \frac{N}{2}\left(\frac{2N\rho}{\beta}\right)^{\frac{1}{N}}\right) \quad = \quad c + \frac{\beta}{N}\left(1 + \frac{N}{2}s^*\right)$$

and so $s^* = \left(\frac{2N\rho}{\beta}\right)^{\frac{1}{N}}$. After winning the first round at price P^*, bidder 2 expects to be "stuck with" the security with probability $(s^*)^{N-1}$, for second-round expected payoff

$$\begin{aligned} & (s^*)^{N-1}\left[c + \tfrac{\beta}{N}\left(s_2 + (N-1)\tfrac{s^*}{2}\right) - \left(c + \tfrac{\beta}{N}\left(1 + \tfrac{N}{2}s^*\right)\right)\right] \\ = \; & \tfrac{\beta}{N}(s^*)^{N-1}\left[s_2 - 1 - \tfrac{1}{2}s^*\right] \;=\; -\tfrac{\beta}{N}(s^*)^{N-1}(1-s_2) - \tfrac{\beta}{2N}(s^*)^N \\ = \; & -\tfrac{\beta}{N}(s^*)^{N-1}(1-s_2) - \tfrac{\beta}{2N}\tfrac{2N\rho}{\beta} \;=\; -\tfrac{\beta}{N}(s^*)^{N-1}(1-s_2) - \rho \end{aligned}$$

So including winning the premium in the first round, winning the first round at price P^* gives expected payoff 0 to bidder 2 if $s_2 = 1$, and negative expected payoff otherwise.

First Round Strategies

First-round strategies, then, are:

- Bidder 1 does not drop out at $P = 0$
- At any history where $P > 0$ and there is one other active bidder, bidder 1 believes him to have signal $s_i = 1$, and
  - If $P < P^*$, bidder 1 remains active
  - If $P \geq P^*$, bidder 1 drops out
- Each bidder $i \geq 2$ drops out at $P = 0$
- At any history where $P > 0$ and bidder i deviated and is still active along with bidder 1,
  - If $P \leq P^*$, bidder i believes $s_1 \sim U[0,1]$, and remains active if $s_i = 1$ (and drops out if $s_i < 1$)
  - If $P^* < P < P^{**}$, bidder i believes $s_1 = 1$ and remains active if $s_i = 1$
  - If $P^* \geq P^{**}$, bidder i drops out immediately

Note that "punishing" bidder i is credible for bidder 1, since he believes $s_i = 1$ and expects bidder i to stay in past P^*; dropping out at P^* (following his equilibrium strategy) still gives bidder 1 nonnegative expected payoff in the second round, while dropping out before P^* would trigger continuation game 3 and earn bidder 1 zero expected payoff. Staying in past P^* is not profitable, since bidder 1 could only win the first round at price P^{**}, and would therefore get zero expected payoff if $s_1 = s_i = 1$ and negative otherwise.

As for bidder i, once he's deviated, assuming he has signal $s_i = 1$, bidder i gets zero expected payoff from staying in and winning the first round at P^*, and zero payoff (via continuation game

3) from dropping out earlier, so it's credible to remain in until P^*. Past P^*, bidder i believes $s_1 = 1$, and is therefore happy to stay in until P^{**} as long as $s_i = 1$, since (again) dropping out earlier would lead to zero expected payoff via CG3. Since dropping out at price 0 earns nonnegative expected payoffs in round 2 (strictly positive if $s_i > 0$), bidder i drops out at price 0.

If bidder 1 drops out at price 0 (and so the premium is awarded at random or not at all), beliefs are not updated and second-round play is the same as on the equilibrium path; so bidder 1 gains nothing by doing so.

Any history not addressed above would require simultaneous deviations by multiple bidders, and we therefore do not specify what happens at those histories.

What if $N = 3$?

If $N = 3$, define

$$P^{**} \quad = \quad \max\left\{P^*,\ c + \tfrac{\beta}{3}\left(2 + \sqrt{\tfrac{6\rho}{\beta}}\right)\right\}$$

If $P^{**} = P^*$, modify both bidder 1's strategy following a deviation by bidder 2 to be, "stay in till P^*, drop out at P^*, and drop out immediately at any $P > P^*$, and bidder 2's strategy following a deviation by himself to be, "stay in till P^*, do not drop out at P^*, drop out immediately at any $P > P^*$." Play at all histories other than Continuation Game 3 are otherwise unchanged.

In Continuation Game 3 when $N = 3$, bidders play the "drainage tract auction" equilibrium considered in Engelbrecht-Wiggans, Milgrom and Weber (1983), Milgrom and Weber (1982b), and Hendricks, Porter and Wilson (1994), where the one bidder with private information plays a pure strategy and the uninformed bidders play mixed strategies and earn zero expected profit. While there are many such equilibria, we will select the one where only one uninformed bidder bids – the round-one winner.

Let i denote the identity of the first-round winner, and j whichever of 1 and 2 is not i. Recall that this continuation game includes the common beliefs that $s_1 = s_2 = 1$. Let $\underline{s}_3$ solve $P = c + \frac{\beta}{3}(2 + \underline{s}_3)$, or $\underline{s}_3 = 0$ if $P < c + \frac{2\beta}{3}$. Define a function $\Gamma : [\underline{s}_3, 1] \to \Re^+$ by

$$\Gamma(t) \quad = \quad c + \tfrac{\beta}{3}\left(2 + \tfrac{1}{2}t + \tfrac{1}{2}\tfrac{\underline{s}_3^2}{t}\right)$$

and note that $\Gamma'(t) = \frac{\beta}{3}\left(\frac{1}{2} - \frac{1}{2}\frac{\underline{s}_3^2}{t^2}\right) > 0$ on $(t, 1]$, so Γ is strictly increasing. Also note that $\Gamma(\underline{s}_3) = P$.

Define a probability distribution G with support $[P, \Gamma(1)]$ by

$$G(x) = \begin{cases} 1 & if \quad x \geq \Gamma(1) \\ \exp\left(-\int_x^{\Gamma(1)} \left(\frac{1}{c+\frac{\beta}{3}(2+\Gamma^{-1}(t))-t}\right) dt\right) & if \quad x \in [P, \Gamma(1)) \\ 0 & if \quad x < P \end{cases}$$

(Note that this distribution has a point mass at P.)

Bidding strategies are as follows:

- Bidder 3 bids $\Gamma(s_3)$ if $s_3 > \underline{s}_3$, and does not bid if $s_3 \leq \underline{s}_3$
- If $s_i = 1$, bidder i mixes such that the CDF of his bids is G (with the point mass at P indicating no new bid); if $s_i < 1$, bidder i does not bid
- Bidder j does not bid

To see this is an equilibrium, we first show that if bidder 3 plays this strategy, bidder i is indifferent between not bidding and between any bid in the support of b_3. Treat the first-round bonus won by bidder i as a "sunk benefit", and note that (since bidder 3 never bids exactly P) not bidding is identical to bidding $P = \Gamma(\underline{s}_3)$ for bidder i. By bidding $\Gamma(x)$ for $x \in [\underline{s}_3, 1]$, bidder i wins when $s_3 < x$; if bidder i has signal s_i and believes $s_j = 1$, this gives expected payoff

$$\begin{aligned} x\left[c + \frac{\beta}{3}\left(1 + s_i + \frac{x}{2}\right) - \Gamma(x)\right] &= x\left[c + \frac{\beta}{3}\left(1 + s_i + \frac{x}{2}\right) - c - \frac{\beta}{3}\left(2 + \frac{1}{2}x + \frac{1}{2}\frac{\underline{s}_3^2}{x}\right)\right] \\ &= \frac{\beta}{3}x\left[s_i - 1 - \frac{1}{2}\frac{\underline{s}_3^2}{x}\right] = -\frac{\beta}{3}(1-s_i)x - \frac{\beta}{6}\underline{s}_3^2 \end{aligned}$$

If $s_i = 1$, then this is the same for all x, so bidder i is indifferent among all bids in the range of Γ and not bidding (effectively bidding P), and is therefore best-responding by playing a mixed strategy.

As for bidder j, by those same calculations, if $s_i = 1$, bidding $\Gamma(x)$ would give an expected payoff of $\left(-\frac{\beta}{3}(1-s_j)x - \frac{\beta}{6}\underline{s}_3^2\right) G(x)$, which is nonpositive for any x and any s_j, so j best-responds by not bidding.

Finally, knowing the value of the security is $c + \frac{\beta}{3}(2 + s_3)$, bidder 3 solves the maximization problem

$$\max_{b \in (P, \Gamma(1)] \cup \{0\}} \ln\left[G(b)\left(c + \frac{\beta}{3}(2 + s_3) - b\right)\right]$$

Taking the derivative with respect to b gives

$$\frac{g(b)}{G(b)} - \frac{1}{c+\frac{\beta}{3}(2+s_3)-b} \quad = \quad \frac{1}{c+\frac{\beta}{3}(2+\Gamma^{-1}(b))-b} - \frac{1}{c+\frac{\beta}{3}(2+s_3)-b}$$

This is positive when $\Gamma^{-1}(b) < s_3$, or $b < \Gamma(s_3)$ and negative when $b > \Gamma(s_3)$; so bidder 3's log-profit problem is strictly quasiconcave and solved at $b = \Gamma(s_3)$. For $s_3 > \underline{s}_3$,

$$c+\tfrac{\beta}{3}(2+s_3)-\Gamma(s_3) \quad = \quad c+\tfrac{\beta}{3}(2+s_3)-c-\tfrac{\beta}{3}\left(2+\tfrac{1}{2}s_3+\tfrac{1}{2}\tfrac{\underline{s}_3^2}{s_3}\right) \quad = \quad \tfrac{\beta}{3}\left(\tfrac{1}{2}s_3-\tfrac{1}{2}\tfrac{\underline{s}_3^2}{s_3}\right) \quad > \quad 0$$

so bidder 3 prefers bidding to not bidding; and for any $s_3 \leq \underline{s}_3$, $c+\frac{\beta}{3}(2+s_3) \leq P$, so bidder 3 prefers not bidding to bidding.

What we need from CG3 is for bidder j (the one who is not the first-round winner) to earn 0 payoffs (which is true since he doesn't bid), and if $P \geq P^{**}$, for player i to earn non-postiive ex-ante payoffs. To see the latter, note that

$$c+\tfrac{\beta}{3}(2+\underline{s}_3) \quad = \quad P \quad \geq \quad P^{**} \quad = \quad c+\tfrac{\beta}{3}\left(2+\sqrt{\tfrac{6\rho}{\beta}}\right)$$

means $\underline{s}_3 \geq \sqrt{\frac{6\rho}{\beta}}$; bidder i's second-round profit is therefore

$$-\tfrac{\beta}{3}(1-s_i)x-\tfrac{\beta}{6}\underline{s}_3^2 \quad \leq \quad -\rho$$

and therefore even counting the first-round bonus, bidder i's profits are not positive.

What if $N = 2$?

In the case of $N = 2$, the equilibrium is essentially the same, but with the drainage tract equilibrium (with $s_1 \sim U[0,1]$ and $s_2 = 1$) replacing CG2, both bidders bidding $c+\beta$ in the second round (basically Bertrand-competing with the beliefs $s_1 = s_2 = 1$) replacing CG3, and P^* appropriately redefined so that give bidder 2 earns zero ex-ante profit if he wins the first round at price P^*.

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Table 1: Variable Names and Description

Variable	Description
	issuer/ security characteristics
Holland bonds	equals 1 if the security is a bond from the province of Holland
other Dutch bonds	equals 1 if a bond from any other Dutch state, province, or city government
foreign government bonds	equals 1 if a bond from a foreign (non-Dutch) government
plantation loans	equals 1 if a plantation loan
private company loans	equals 1 if a loan from a private company
securities in foreign currency	equals 1 if the security was not issued in Dutch guilders
	historical events
pre-financial crisis	equals 1 if the auction took place between 1666-1672
pre-financial crisis Holland bonds	equals 1 if the auction took place between 1666-1672 and the security is a Holland bond
pre-financial crisis other Dutch bonds	equals 1 if the auction took place between 1666-1672 and the security is an other Dutch bond
pre-financial crisis foreign government bonds	equals 1 if the auction took place between 1666-1672 and the security is a foreign government bond
pre-financial crisis plantation loans	equals 1 if the auction took place between 1666-1672 and the security is a plantation loan
pre-financial crisis private company loans	equals 1 if the auction took place between 1666-1672 and the security is a private company loan
Anglo-Dutch war plantation	equals 1 if the auction took place between 1780-1783 and the security is a plantation loan
Anglo-Dutch war others	equals 1 if the auction took place between 1780-1783 and the security is not a plantation loan
time_trend	linear time trend, measured in months
season	equals 1 if the auction took place in January, February, or March
	learning
same Holland bonds before	number of Holland bonds of the same debtor sold immediately before this one
same other Dutch bonds before	number of other Dutch bonds of the same debtor sold immediately before this one
same foreign government bonds before	number of foreign government bonds of the same debtor sold immediately before this one
same plantation loans before	number of plantation loans of the same debtor sold immediately before this one
same private company loans before	number of private company loans of the same debtor sold immediately before this one

auction room effects	
days between auction	number of days between this auction and the previous auction
goods sold	number of securities sold during this auction day
top82	number of the top 82 winners (those who won at least 50 securities in our data) who won at least one security on this auction day
over30	number of the top 82 winners who won at least one security on tjos auction day and, over our data, won at least 30% of their securities with a second-round bid
under10	number of the top 82 winners who won at least one security on the auction day in question and, over our data, won less than 10% of their securities with a second-round bid
second stage winning bidders	over30 divided by top82
first stage winning bidders	under10 divided by top82
other auction effects	
own auction	equals 1 if winner is also one of this day's auction organizers
exchange markets	
grain price	price of Polish grain on the Amsterdam exchange the day of the auction
sugar price	price of sugar from St. Domingo on the Amsterdam exchange the day of the auction
agio price	price of bank guilders expressed in current guilders
Hamburg price	exchange rate on Hamburg (Stuivers/Thaler)

Table 2: Descriptive Statistics, Binary Variables

Type of Security	*Number of Observations*	*First Year Present*	*Last Year Present*	*% Won in 2nd Round*
all securities	16,854	1766	1783	22%
Holland bonds	5,316	1766	1783	14%
other Dutch bonds	1,669	1766	1783	20%
foreign government bonds	1,988	1766	1783	13%
plantation loans	6,959	1766	1783	30%
private company loans	922	1766	1783	32%
securities in foreign currency	394	1766	1783	23%
pre-financial crisis	3,617	1766	1772	9%
pre-crisis, Holland	1,704	1766	1772	10%
pre-crisis, other Dutch	611	1766	1772	11%
pre-crisis, foreign government	415	1766	1772	2%
pre-crisis, plantation	717	1766	1772	4%
pre-crisis, private company	169	1766	1772	18%
Anglo-Dutch war, plantation	652	1781	1783	39%
Anglo-Dutch war, others	2,343	1781	1783	17%
season (winter)	4,187	1766	1783	23%
own security	3,414	1766	1783	23%

Table 3: Descriptive Statistics, Other Variables

Variable	*# Obs*	*Mean*	*Std Dev*	*Min*	*Max*
same Holland bonds before	5,316	18.66	20.65	0	146
same other Dutch bonds before	1,669	11.42	16.79	0	104
same foreign government before	1,988	0.36	1.77	0	33
same plantation loans before	6,959	3.54	10.07	0	128
same private company loans before	922	0.16	1.27	0	30
days between auction	16,849	12.48	9.40	1	63
goods sold	16,854	72.35	55.79	1	289
over30/top82	16,549	0.18	0.12	0	1
under10/top82	16,549	0.27	0.16	0	1
grain price	16,846	201.35	24.93	145.6	268.8
sugar price	16,853	0.27	0.09	0.10	0.45
agio price	16,532	4.67	0.32	1.5	5.13
Hamburg price	16,758	33.46	0.72	30.69	36

Table 4: Most Frequent Auction Winners

Name of Bidder	*Securities Won*	*First Year*	*Last Year*	*% Won in 2nd Round*
W. Zeelt	604	1769	1783	17%
W.H. Stoopendaal	577	1766	1783	35%
G. Jarman	519	1766	1783	25%
A. van Ketwich	401	1766	1783	15%
D. Leuveling	379	1770	1783	30%
J.P. Heimbach	364	1766	1782	14%
E. Croese	288	1767	1783	37%
H. van Blomberg	280	1772	1783	70%
A. van Vloten	277	1768	1783	25%
Tideman en Scholten	249	1777	1781	22%
top 10 total	3,938	1766	1783	27%
top 20 total	5,865	1766	1783	26%
top 50 total	9,261	1766	1783	24%
top 82 total	11,249	1766	1783	23%
total	16,854	1766	1783	22%

Table 5: Determinants of Second Round Bidding: Probit Regressions, error terms clustered by auction day (part 1)

Dependent variable:	*second stage winning (1) vs. first stage winning (0)*						
second stage winning	(1)	(2)	(3)	(4)	(5)	(6)	(7)
			issuer/ security characteristics				
other Dutch bonds	0.08** (0.03)	0.08*** (0.03)	0.09*** (0.03)	0.11*** (0.04)	0.11*** (0.04)	0.11*** (0.04)	0.11*** (0.04)
foreign government bonds	-0.00 (0.03)	-0.03 (0.03)	-0.05* (0.03)	-0.02 (0.03)	-0.02 (0.03)	-0.02 (0.03)	-0.01 (0.03)
plantation loans	0.17*** (0.03)	0.17*** (0.04)	0.13*** (0.02)	0.16*** (0.02)	0.15*** (0.02)	0.15*** (0.02)	0.15*** (0.02)
private company loans	0.25*** (0.03)	0.24*** (0.04)	0.21*** (0.04)	0.24*** (0.04)	0.23*** (0.03)	0.23*** (0.03)	0.25*** (0.04)
securities in foreign currency		0.15** (0.06)	0.13** (0.06)	0.13** (0.06)	0.13** (0.06)	0.12** (0.06)	0.13** (0.06)
			historical events				
pre-financial crisis			-0.15*** (0.02)	-	-	-	-
pre-financial crisis Holland bonds				-0.08* (0.05)	-0.09** (0.04)	-0.02 (0.08)	-0.09** (0.04)
pre-financial crisis other Dutch bonds				-0.12*** (0.03)	-0.13*** (0.03)	-0.07 (0.06)	-0.13*** (0.03)
pre-financial crisis foreign government bonds				-0.17*** (0.02)	-0.17*** (0.02)	-0.15*** (0.03)	-0.17*** (0.02)
pre-financial crisis plantation loans				-0.20*** (0.01)	-0.20*** (0.01)	-0.18*** (0.02)	-0.20*** (0.01)
pre-financial crisis private company loans				-0.13*** (0.03)	-0.13*** (0.03)	-0.09* (0.05)	-0.14*** (0.03)
Anglo-Dutch war plantation					0.06 (0.04)	-0.01 (0.04)	0.06* (0.04)
AngloDutch war others					-0.03 (0.02)	-0.08*** (0.03)	-0.03 (0.02)
time trend						0.00* (0.00)	-
season							0.06* (0.03)
log pseudo likelihood	-8660.80	-8558.61	-8360.69	-8293.72	-8293.73	-8264.26	-8265.63
number of observations	16,854	16,739	16,739	16,739	16,739	16,734	16,739
LR Chi2	101.21	105.16	100.77	235.49	235.49	250.16	243.52
prob>Chi2	0	0	0	0	0	0	0
Pseudo R2	0.036	0.037	0.059	0.066	0.066	0.069	0.070

Table 6: Determinants of Second Round Bidding: Probit Regressions (part 2)

Dependent variable:	*second stage winning (1) vs first stage winning (0)*					
second stage winning	(8)	(9)	(10)	(11)	(12)	(13)
			issuer/ security characteristics			
other Dutch bonds	0.11** (0.05)	0.12** (0.05)	0.12** (0.05)	0.14** (0.05)	0.14** (0.05)	0.13** (0.05)
foreign government bonds	-0.04 (0.04)	-0.04 (0.03)	-0.04 (0.04)	-0.03 (0.04)	-0.03 (0.04)	-0.03 (0.04)
plantation loans	0.12*** (0.04)	0.11*** (0.04)	0.11** (0.04)	0.12*** (0.04)	0.12*** (0.04)	0.12*** (0.04)
private company loans	0.22*** (0.06)	0.22*** (0.06)	0.22*** (0.06)	0.23*** (0.06)	0.23*** (0.06)	0.22*** (0.06)
securities in foreign currency	0.13** (0.06)	0.13** (0.06)	0.13** (0.06)	0.12** (0.06)	0.12** (0.06)	0.12* (0.06)
			historical events			
pre-financial crisis	-	-	-	-	-	-
pre-financial crisis Holland bonds	-0.09** (0.04)	-0.09** (0.04)	-0.10** (0.04)	-0.08 (0.06)	-0.08 (0.06)	-0.03 (0.09)
pre-financial other Dutch bonds	-0.12*** (0.03)	-0.11*** (0.03)	-0.12*** (0.03)	-0.11*** (0.03)	-0.11*** (0.03)	-0.06 (0.06)
pre-financial crisis foreign government bonds	-0.17*** (0.02)	-0.17*** (0.02)	-0.16*** (0.02)	-0.17*** (0.02)	-0.17*** (0.02)	-0.14*** (0.03)
pre-financial crisis plantation loans	-0.20*** (0.01)	-0.20*** (0.01)	-0.20*** (0.01)	-0.19*** (0.01)	-0.19*** (0.01)	-0.18*** (0.02)
pre-financial crisis private company loans	-0.15*** (0.03)	-0.15*** (0.03)	-0.15*** (0.03)	-0.14*** (0.03)	-0.14*** (0.03)	-0.11** (0.04)
Anglo-Dutch war plantation	0.05 (0.03)	0.04 (0.03)	0.04 (0.03)	0.03 (0.03)	0.03 (0.03)	-0.02 (0.04)
AngloDutch war others	-0.03 (0.02)	-0.03 (0.02)	-0.03 (0.02)	-0.03 (0.02)	-0.03 (0.02)	-0.08** (0.03)
time trend	-	-	-	-	-	-
season	0.06* (0.03)	0.06* (0.03)	0.06* (0.03)	0.06** (0.03)	0.06** (0.03)	0.06** (0.03)
			learning			
same Holland bonds before	-0.04*** (0.01)	-0.04*** (0.01)	-0.04*** (0.01)	-0.03** (0.01)	-0.03** (0.01)	-0.03** (0.01)
same other Dutch bonds before	-0.06*** (0.02)	-0.06*** (0.02)	-0.06*** (0.02)	-0.06*** (0.02)	-0.06*** (0.02)	-0.06*** (0.02)
same foreign government bonds	-0.03** (0.02)	-0.04** (0.02)	-0.03** (0.02)	-0.03** (0.02)	-0.03** (0.02)	-0.03** (0.02)

before						
same plantation loans before	-0.01** (0.01)	-0.02** (0.01)	-0.02 (0.01)	-0.02* (0.01)	-0.02* (0.01)	-0.02* (0.01)
same private company loans before	-0.05*** (0.02)	-0.05*** (0.02)	-0.05** (0.02)	-0.05*** (0.02)	-0.05*** (0.02)	-0.04*** (0.02)
auction room effects						
days between auction		0.02 (0.02)	0.02 (0.02)	0.02 (0.02)	0.02 (0.02)	0.02 (0.02)
goods sold			-0.01 (0.01)	-0.00 (0.02)	-0.00 (0.02)	-0.00 (0.02)
first stage winning bidders				-0.12** (0.05)	-0.12** (0.05)	-0.11** (0.05)
second stage winning bidders				0.10** (0.04)	0.10** (0.04)	0.10** (0.04)
other auction effects						
own auction					-0.02 (0.02)	-0.02 (0.02)
exchange market						
grain price						0.00 (0.00)
agio price						-0.03 (0.02)
sugar price						0.53** (0.22)
Hamburg price						-0.00 (0.02)
log pseudo likelihood	-8187.23	-8012.20	-8011.06	-7806.83	-7804.61	-7611.10
number of observations	16,739	16,293	16,292	16,006	16,006	15,670
Wald Chi2	319.76	302.32	308.06	323.33	323.39	334.34
prob>Chi2	0	0	0	0	0	0
Pseudo R2	0.078	0.076	0.076	0.083	0.084	0.088

Table 7: Determinants of Second Round Bidding: Robustness Checks

Dependent variable:	*second stage winning (1) vs first stage winning (0)*			
second stage winning	(1)	(2)	(3)	(4)
Estimator	Probit	Probit robust	Probit cluster winner	Probit cluster auction
		issuer/ security characteristics		
other Dutch bonds	0.13*** (0.03)	0.13*** (0.03)	0.13*** (0.04)	0.13** (0.05)
foreign government bonds	-0.03 (0.02)	-0.03 (0.02)	-0.03 (0.03)	-0.03 (0.04)
plantation loans	0.12*** (0.02)	0.12*** (0.02)	0.12*** (0.03)	0.12*** (0.04)
private company loans	0.22*** (0.04)	0.22*** (0.04)	0.22*** (0.05)	0.22*** (0.06)
securities in foreign currency	0.12*** (0.03)	0.12*** (0.03)	0.12*** (0.04)	0.12* (0.06)
		historical events		
pre-financial crisis Holland bonds	-0.03* (0.01)	-0.03* (0.02)	-0.03 (0.04)	-0.03 (0.09)
pre-financial other Dutch bonds	-0.06*** (0.02)	-0.06*** (0.02)	-0.06* (0.03)	-0.06 (0.06)
pre-financial crisis foreign government bonds	-0.14*** (0.02)	-0.14*** (0.02)	-0.14*** (0.03)	-0.14*** (0.03)
pre-financial crisis plantation loans	-0.18*** (0.01)	-0.18*** (0.01)	-0.18*** (0.01)	-0.18*** (0.02)
pre-financial crisis private company loans	-0.11*** (0.02)	-0.11*** (0.02)	-0.11*** (0.03)	-0.11** (0.04)
Anglo-Dutch war plantation	-0.02 (0.02)	-0.02 (0.02)	-0.02 (0.02)	-0.02 (0.04)
Anglo-Dutch war others	-0.08*** (0.01)	-0.08*** (0.01)	-0.08*** (0.02)	-0.08** (0.03)
season	0.06*** (0.01)	0.06*** (0.01)	0.06*** (0.01)	0.06** (0.03)
		learning		
same Holland bonds before	-0.03*** (0.01)	-0.03*** (0.01)	-0.03*** (0.01)	-0.03** (0.01)
same other Dutch bonds before	-0.06*** (0.01)	-0.06*** (0.01)	-0.06*** (0.02)	-0.06*** (0.02)
same foreign	-0.03***	-0.03***	-0.03**	-0.03**

government bonds before	(0.01)	(0.01)	(0.01)	(0.02)
same plantation loans before	-0.02*** (0.00)	-0.02*** (0.00)	-0.02** (0.01)	-0.02* (0.01)
same private company loans before	-0.04*** (0.01)	-0.04*** (0.01)	-0.04** (0.02)	-0.04*** (0.02)
auction room effects				
days between auction	0.02*** (0.01)	0.02*** (0.01)	0.02** (0.01)	0.02 (0.02)
goods sold	-0.01 (0.01)	-0.00 (0.01)	-0.01 (0.01)	-0.00 (0.02)
first stage winning bidders	-0.11*** (0.02)	-0.11*** (0.02)	-0.11*** (0.04)	-0.11** (0.05)
second stage winning bidders	0.10*** (0.02)	0.10*** (0.02)	0.10*** (0.04)	0.10** (0.04)
other auction effects				
own auction	-0.02* (0.01)	-0.02* (0.01)	-0.02 (0.02)	-0.02 (0.02)
exchange market				
grain price	0.00*** (0.00)	0.00*** (0.00)	0.00** (0.00)	0.00 (0.00)
agio_price	-0.03*** (0.01)	-0.03*** (0.01)	-0.03** (0.02)	-0.03 (0.02)
sugar price	0.53*** (0.07)	0.53*** (0.07)	0.53*** (0.12)	0.53** (0.22)
Hamburg price	-0.00 (0.01)	-0.00 (0.01)	-0.00 (0.01)	-0.00 (0.02)
log pseudo likelihood	-7611.10	-7611.10	-7611.10	-7611.10
number of observations	15,670	15,670	15,670	15,670
LR Chi2	1485.29	1305.87	533.82	334.34
prob>Chi2	0	0	0	0
Pseudo R2	0.089	0.089	0.089	0.088

Map 1: Geographical Location of Plantations

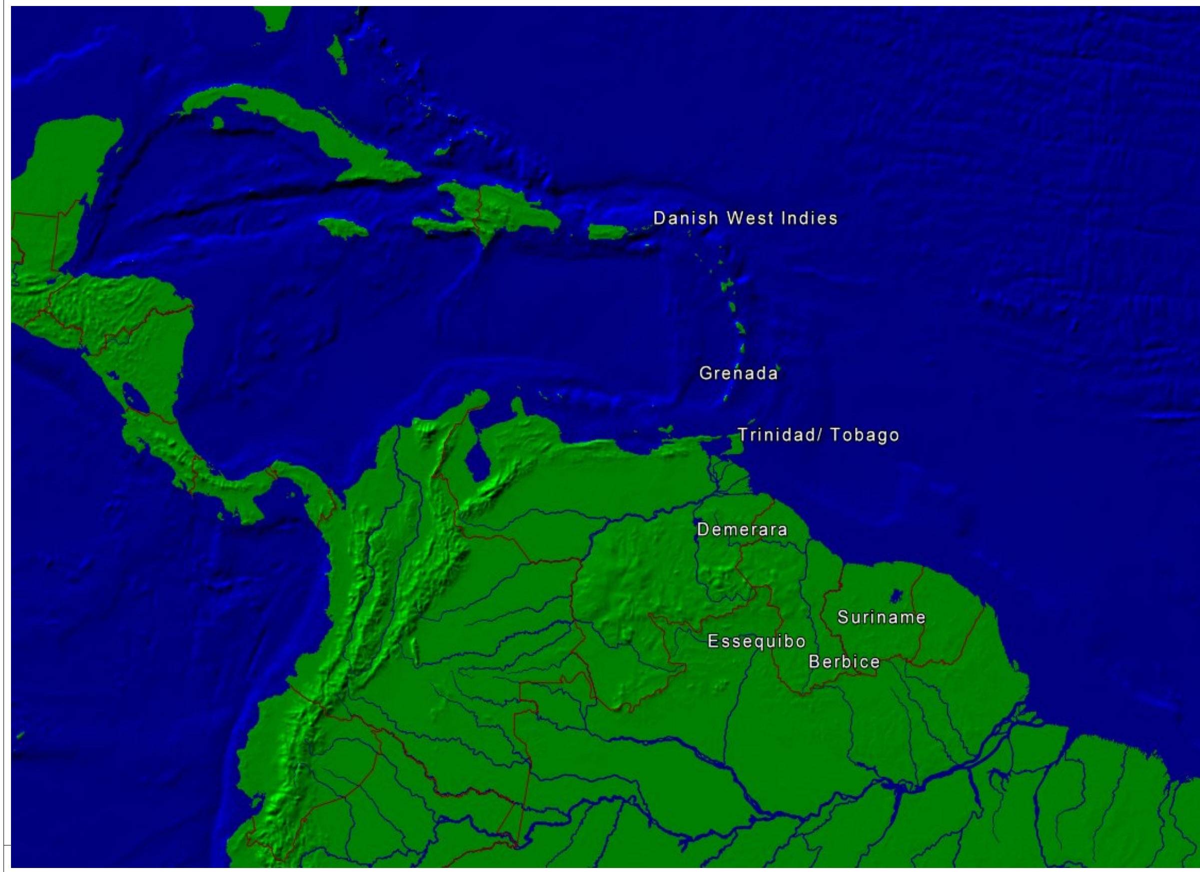

Figure 1: Summary of Results in Theorem 1

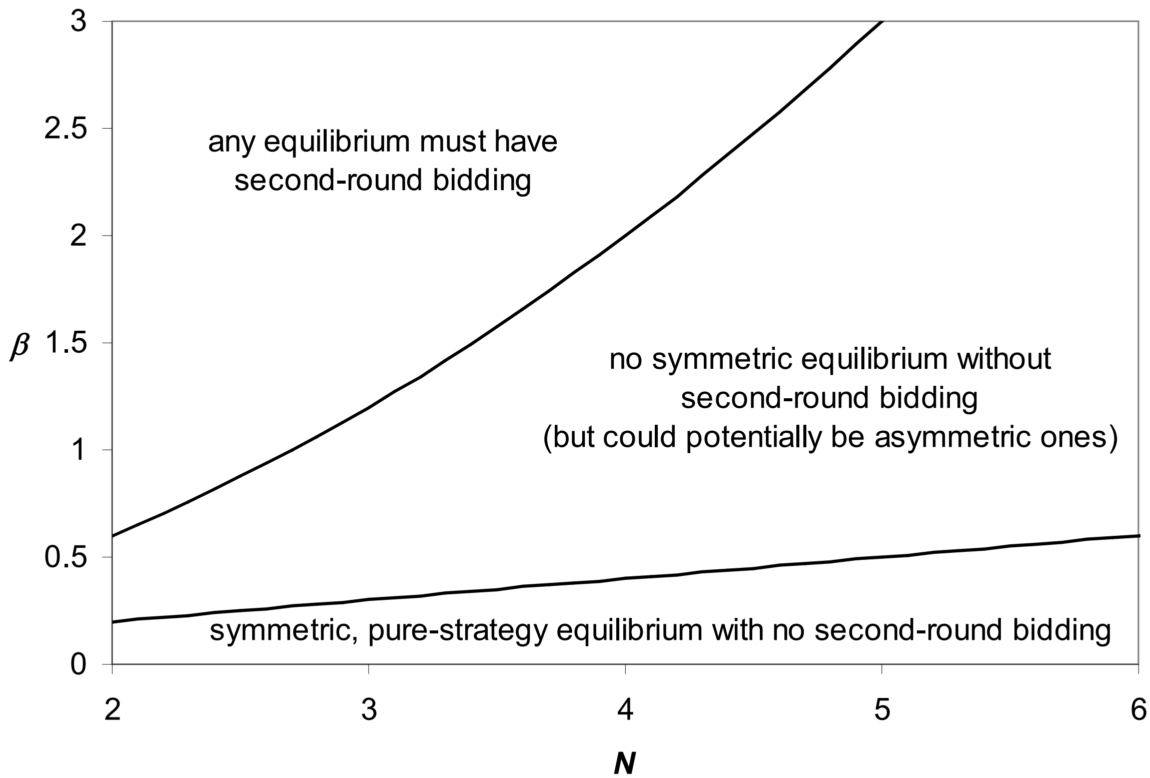

Figure 2: An Example of One Auction Day