

Explaining Rising Wage Inequality:
Explorations With a Dynamic General
Equilibrium Model of Labor Earnings with
Heterogeneous Agents

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Large increase in earnings inequality in last 20-30 years due to increases in the “Returns to Skill”

In response to these changes, many policies that promote skill formation have been formulated

We develop a dynamic GE model of schooling and OJT

- Skill-biased technical change yields results that match the recent rise in wage inequality
- Prices do not equal wages - inequality growth largest for young workers
- Cross-section estimates of rates of return are misleading

Outline

“Micro” model of schooling and On-The-Job Training

Extending the model to general equilibrium

Empirical methodology and estimates of the model

The changing wage structure

Conclusions and Extensions

A model of Schooling and On the Job Training

An individual chooses human capital investment through school and on the job to maximize the present value of after-tax earnings

$$V^S(\theta) = \sum_{a=1}^{\bar{a}} \left(\frac{1}{1+r} \right)^a E_a$$

We allow individuals to choose between two schooling levels, High School and College. We follow them at the end of high school.

Once they enter the labor force they spend a fraction of their time, I_a^S , on the job investing in human capital, and the rest working.

The earnings in school group S at age a are

$$E_a^S = R_a^S H_a^S (1 - I_a^S),$$

where R_a^S is the rental rate on human capital and H_a^S is the stock of human capital.

Human capital on the job is produced according to the production function

$$H_{a+1}^S = A^S(\theta) I_a^{\alpha_S} H_a^{\beta_S} + (1 - \sigma^S) H_a^S.$$

We add heterogeneity to the model by assuming that persons can be indexed by θ . We allow the initial stock of human capital in each schooling group, $H_0^S(\theta)$ and the productivity parameter in human capital $A^S(\theta)$ to depend on θ .

For each θ we can solve for the optimal level of human capital investment over the lifecycle and form $V^S(\theta)$.

People choose schooling to maximize lifetime utility

$$\hat{S} = \operatorname{argmax}[V^S(\theta) - D^S - \varepsilon^S],$$

where D^S is the present value of direct costs of schooling, and ε^S is the nonpecuniary benefits of schooling.

The distribution of ε^S may depend on θ .

Individuals make savings and consumption choices to maximize utility

$$\sum_{a=1}^{\bar{a}} \delta^a \frac{C_a^\gamma - 1}{\gamma}$$

subject to the budget constraint,

$$\sum_{a=1}^{\bar{a}} \left(\frac{1}{1+r} \right)^a C_a = V^S(\theta) - D^S.$$

Embedding the model in a General Equilibrium Framework

- Skill is perfectly substitutable across ages, but not substitutable across schooling groups.
- There are three factors of production High School Human Capital, College Human Capital, and Physical Capital.
- The model is embedded in an Auerbach- Kotlikoff style overlapping generations model.

Each period there are \bar{a} cohorts.

We assume that the distribution of heterogeneity θ is identical within cohorts.

We obtain aggregate stocks according to

$$\bar{H}_t^S = \sum_{t_c=t-a_R}^{t-1} \int H_{t-t_c}^S(\theta, P_{t_c})(1 - I_{t-t_c}^S(\theta, P_{t_c}))N^S(\theta, t_c)dG(\theta)$$

$$\bar{K}_t = \sum_{t_c=t-a_R}^{t-1} \int \sum_{s=1}^{\bar{S}} K_{t-t_c}^S(\theta, P_{t_c})N^S(\theta, P_{t_c})dG(\theta).$$

Demand Side of Model

We assume a competitive economy with aggregate production function

$$F_t(\bar{H}_t^1, \bar{H}_t^2, \bar{K}_t).$$

The rental rates in each period thus take the form,

$$R_t^1 = \frac{\partial F_t(\bar{H}_t^1, \bar{H}_t^2, \bar{K}_t)}{\partial \bar{H}_t^1}$$

$$R_t^2 = \frac{\partial F_t(\bar{H}_t^1, \bar{H}_t^2, \bar{K}_t)}{\partial \bar{H}_t^2}$$

$$r_t = \frac{\partial F_t(\bar{H}_t^1, \bar{H}_t^2, \bar{K}_t)}{\partial \bar{K}_t}$$

Estimating the Human Capital Production Function

- We use wage and schooling data on white males from the NLSY
- We assume that there are four observable θ types which we define according to AFQT quartile.
- We assume that the interest rate is fixed at $r = 0.05$ and that rental rates are fixed and normalized to one.

For any given (a, θ, S) and any set of parameters π we can calculate the optimal wage

$$w(a, \theta, S; \pi).$$

We assume that these wages are measured with error and we estimate the parameters, π , using nonlinear least squares, minimizing

$$\sum_{i=1}^N \sum_a (w_{i,a}^* - w(a, \theta, S; \pi))^2$$

where $w_{i,a}^*$ is the observed wage.

Human Capital Production

$$H_{a+1}^S = A^S(\theta) I_a^{\alpha_S} H_a^{\beta_S} + (1 - \sigma) H_a^S$$

$$S = 1, 2$$

	High School ($S = 1$)	College ($S = 2$)
α	0.945(0.017)	0.939(0.026)
β	0.832(0.253)	0.871(0.343)
$A(1)$	0.081(0.045)	0.081(0.072)
$H_{a_R}(1)$	9.530(0.309)	13.622(0.977)
$A(2)$	0.085(0.053)	0.082(0.074)
$H_{a_R}(2)$	12.074(0.403)	14.759(0.931)
$A(3)$	0.087(0.056)	0.082(0.077)
$H_{a_R}(2)$	13.525(0.477)	15.614(0.909)
$A(4)$	0.086(0.054)	0.084(0.083)
$H_{a_R}(4)$	12.650(0.534)	18.429(1.095)

Figure 1: Predicted vs Actual Hourly Wages (in 1992 dollars)
by AFQT Quartile (High School Category)

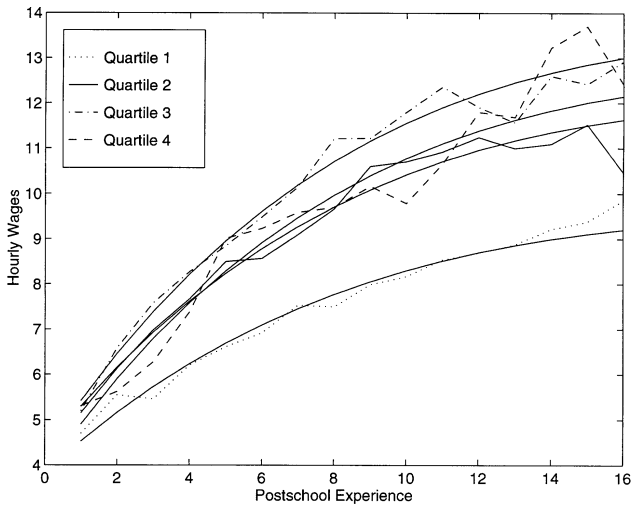


Figure 2: Predicted vs Actual Hourly Wages (in 1992 dollars)
by AFQT Quartile (College Category)

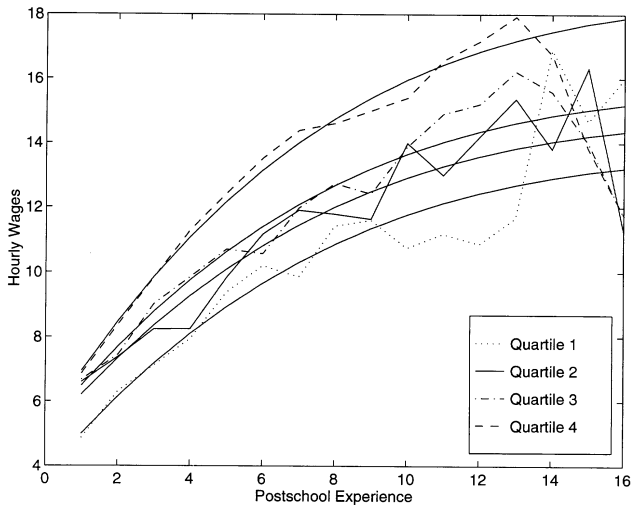


Figure 3A: Comparison of Mincer vs. Estimated Investment Profiles
High School

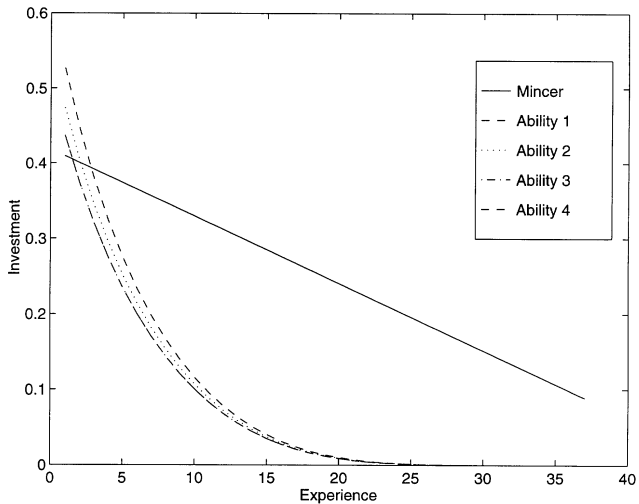
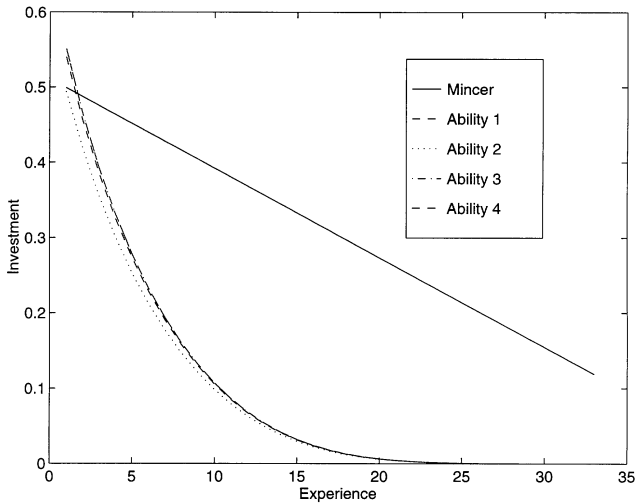


Figure 3B: Comparison of Mincer vs. Estimated Investment Profiles
College



Given these estimated parameters, we can obtain the present value of earnings for each type as college graduates or high school graduates, \widehat{V}_θ^S .

**Human Capital Production Function
and Schooling Decision**
Units are thousands of Dollars

	Human Capital Production	
	High School ($S = 1$)	College ($S = 2$)
$H^S(1)$	8.042(0.094)	11.117(0.424)
$H^S(2)$	10.0634(0.118)	12.271(0.325)
$H^S(3)$	11.1273(0.155)	12.960(0.272)
$H^S(4)$	10.361(0.234)	15.095(0.323)
Present Value Earnings 1	260.304(3.939)	289.618(12.539)
Present Value Earnings 2	325.966(5.075)	319.302(10.510)
Present Value Earnings 3	360.717(6.352)	337.260(9.510)
Present Value Earnings 4	335.977(8.453)	393.138(11.442)

College Decision: Attend College if

We assume that the nonpecuniary tastes for college are normally distributed, so

$$\Pr(\text{Coll} \mid D^S, \theta) = \Phi \left(\frac{(1 - \tau) (V_\theta^2 - V_\theta^1) - D^S + \mu_\theta}{\sigma_\varepsilon} \right)$$

Using data on state tuition we estimate this model as a probit.

College Choice Equation
 $P(\delta^2 = 1) = \Lambda(-\lambda D^2 + \alpha(\theta))$

	Probit Parameters	Average Derivatives
λ	0.166(0.062)	-0.0655(0.025)
$\alpha(1)$	-1.058(0.097)	-
$\alpha(2)$	-0.423(0.087)	0.249(0.037)
$\alpha(3)$	0.282(0.089)	0.490(0.029)
$\alpha(4)$	1.272(0.101)	0.715(0.018)
Sample Size:		
Persons	869	1069
Person Years	7996	11626

(1) D^2 is the discounted tuition cost of attending college.

(2) $\alpha(\theta)$ is the nonparametric estimate of $(1 - \tau)[V^2(\theta) - V^1(\theta)]$ the monetary

	555.011(8.105)	555.100(11.112)
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College Decision: Attend College if
 $(1 - \tau)V^2(\theta) - D^2 + \varepsilon_i \geq (1 - \tau)V^1(\theta)$
 $\varepsilon_\theta \sim N(\mu_\theta, \sigma_\varepsilon)$

σ_ε (Std. deviation of ε)	22.407(8.425)
Nonpecuniary costs by ability level	
μ_1 (Lowest Ability Quartile)	-53.0190(16.770)
μ_2 (Second Ability Quartile)	-2.8173(12.760)
μ_3 (Third Ability Quartile)	29.7712(11.540)
μ_4 (Highest Ability Quartile)	-28.6494(16.966)

(1) $V^i(\theta)$ is the monetary value of going to schooling level i for a person of AFQT quartile t

Estimating/Calibrating Utility and Aggregate Production Parameters

We take

$$\delta = 0.96$$

$$\gamma = 0.10$$

We calibrate the model to “look like” the NLSY in the original steady state:

$$r = 0.05$$

$$R^1 = 2.00$$

$$R^2 = 2.00$$

In order to match the capital-output ratio, we need a transfer from old cohorts to young. We take an exogenous transfer from

We estimate a nested CES production function allowing for a linear time trend in $\log[a_1/(1 - a_1)]$

$$a_3 \left(a_2 \left(a_1 (\bar{H}_t^1)^{\rho_1} + (1 - a_1) (\bar{H}_t^2)^{\rho_1} \right)^{\rho_2/\rho_1} + (1 - a_2) \bar{K}_t^{\rho_2} \right)^{1/\rho_2}$$

Table 3
Estimates of Aggregate Production Function
Estimated from Factor Demand Equations (III-1) and (III-2)
1965-1990
Allowing for Technical Progress Through a Linear Trend
(Standard Errors in Parentheses)

Instruments	ρ_1	Implied Elasticity of Substitution (σ_1)	Time Trend	ρ_2	Implied Elasticity of Substitution (σ_2)	Time Trend
OLS (Base Model)	0.306 (0.089)	1.441 (0.185)	0.036 (0.004)	-0.034 (0.200)	0.967 (0.187)	-0.004 (0.007)
Percent Working Pop. < 30 & Defense Percent of GNP	0.209 (0.134)	1.264 (0.215)	0.039 (0.005)	-0.036 (0.200)	0.965 (0.187)	-0.004 (0.007)
Defense Percent of GNP	0.157 (0.125)	1.186 (0.175)	0.041 (0.004)	-0.171 (0.815)	0.854 (0.594)	-0.008 (0.024)
Percent Working Pop. < 30	0.326 (0.182)	1.484 (0.400)	0.036 (0.006)	0.364 (1.150)	1.572 (2.842)	0.007 (0.034)

We use $\rho_1 = 1.441$ and $\rho_2 = 0$ based on those estimates.

We calibrate (a_1, a_2, a_3) and the transfer to yield prices (r, R^1, R^2) and a capital-output ratio of 4 in the initial steady state.

Skill-Biased Technical Change

- Economy Starts in Steady State based on NLSY
- Unexpected shock resulting in a constant decline in $\log[a_1/(1 - a_1)]$ for 30 years
- Declines by 3.6% as in our estimates
- Perfect foresight
- Transition period of 200 years

Figure 5: Estimated Trend in a_1 for 30 years

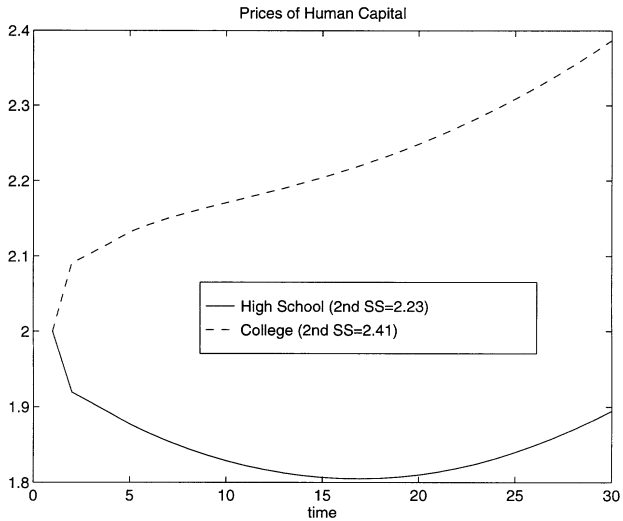
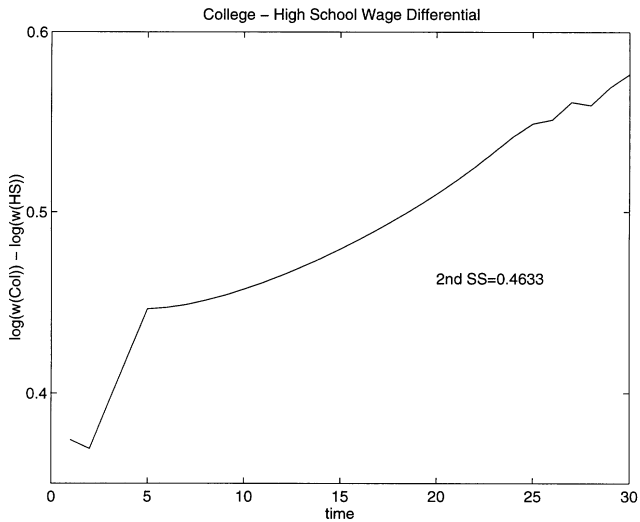


Figure 6: Estimated Trend in a_1 for 30 years



In response to change in relative demand for college graduates

- Increased enrollment in college
- Adjustment occurs only for Young, old do not return to college
- Young “overcompensate” for old
- As old die off, there is an oversupply of college graduates
- Return to College Falls
- This Yields Cycles

Figure 11A: Estimated Trend in a_1 for 30 years

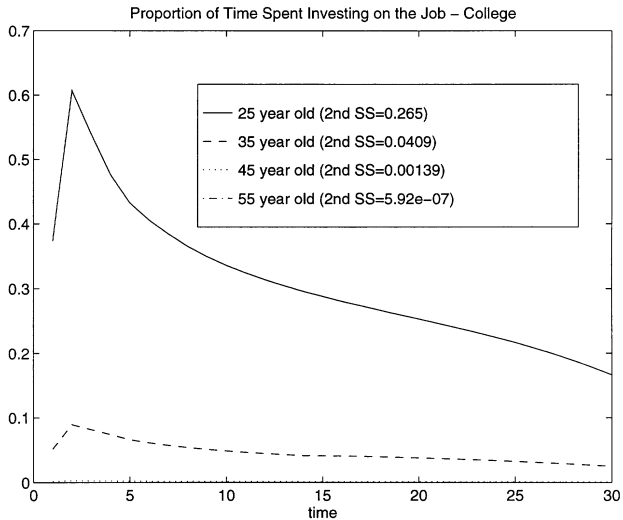


Figure 11B: Estimated Trend in a_1 for 30 years

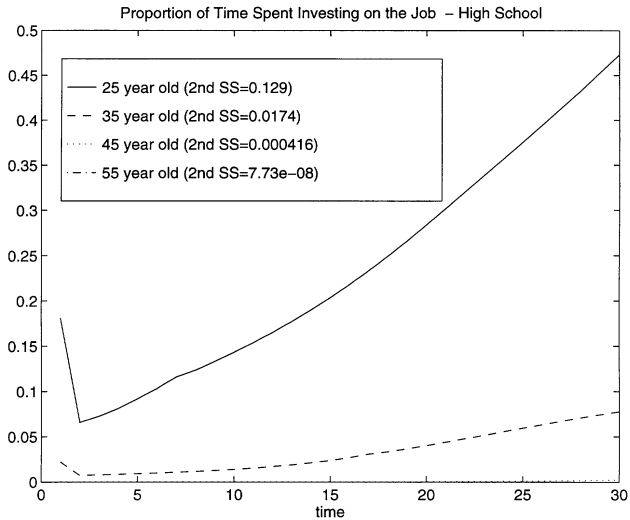


Figure 10: Estimated Trend in a_1 for 30 years

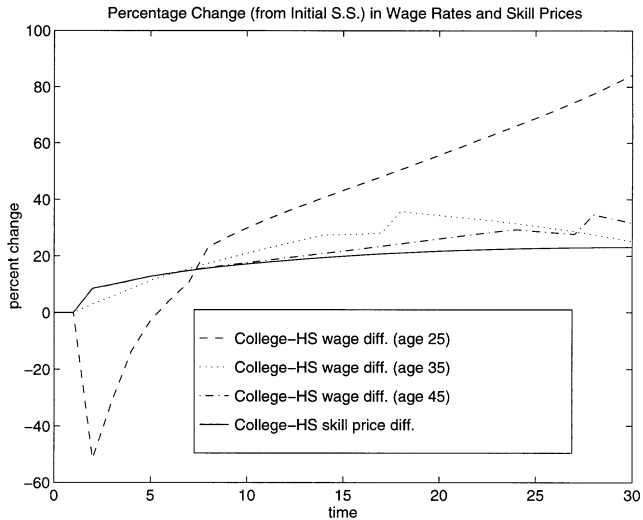


Figure 20: Estimated Trend in a_1 for 30 years
Baby boom (Expansion of Cohort Size by 32%) between years 1965-80

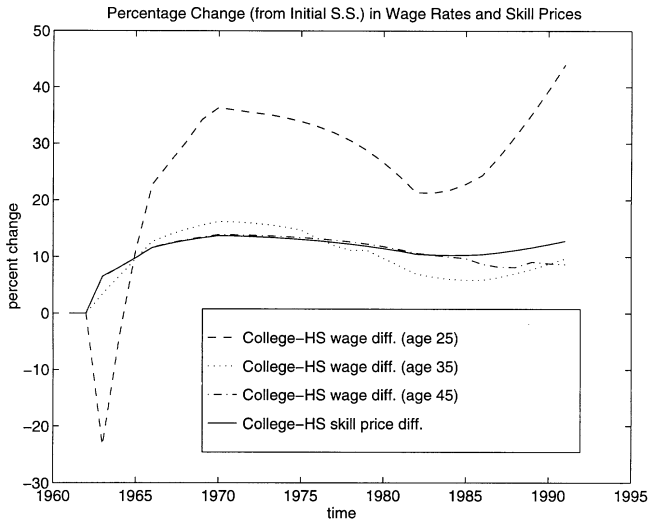


TABLE I
U. S. REAL WEEKLY WAGE CHANGES FOR FULL-TIME WORKERS, 1963-1987^a

Group	Change in log average real weekly wage (multiplied by 100)			
	1963-1971	1971-1979	1979-1987	1963-1987
All	19.2	-2.8	-0.3	16.1
Gender:				
Men	19.7	-3.4	-2.4	13.9
Women	17.6	-0.8	6.1	22.9
Education (years of schooling):				
8-11	17.1	0.3	-6.6	10.9
12	16.7	1.4	-4.0	14.1
13-15	16.4	-3.4	1.5	14.4
16+	25.5	-10.1	7.7	23.1
Experience (men):				
1-5 years	17.1	-3.5	-6.7	6.8
26-35 years	19.4	-0.6	0.0	18.8
Education and Experience (men):				
Education 8-11				
Experience 1-5	20.5	1.5	-15.8	6.2
Experience 26-35	19.3	-0.4	-1.9	17.0
Education 12				
Experience 1-5	17.4	0.8	-19.8	-1.6
Experience 26-35	14.3	3.2	-2.8	14.7
Education 16+				
Experience 1-5	18.9	-11.3	10.8	18.4
Experience 26-35	28.1	-4.0	1.8	25.9

The Effects of Skill-Biased Technology Change

- Movements in measured wages are different from movements in skill prices, especially for young workers
- Without intervention, economy converges to a new steady state with lower wage inequality than before the technology change
- In the long run, society is richer and all types are better off. In the short run, low ability/low skilled workers caught in the transition are worse off.
- In the new steady state, there are more high skilled workers, but human capital per skilled worker is lower
- During transition periods, cross-section estimates of “returns” to skill are substantially different from the actual returns faced by cohorts making educational decisions

Summary

- We develop an empirically-grounded dynamic overlapping generations general-equilibrium model of skill formation with heterogeneous human capital
- Model roughly consistent with changing wage structure
- We distinguish between effects measured in a cross-section and the effects on different cohorts