

Ben-Porath Model

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As we talked about earlier, the Mincer model is not structural in the classic sense

The Ben-Porath model is a structural model of investment on the job

The model is

- Finite lived to time T
- continuous time
- interest rate r
- earnings $E(t)$

So people make human capital investment decisions to maximize the present value of income

$$\int_0^T e^{-rt} E(t) dt$$

We assume that earnings take the form

$$E(t) = H(t) [1 - I(t)] - D(t)$$

Where

- $I(t)$: time spent investing in human capital
- $H(t)$: Human capital itself
- $D(t)$: Direct costs of human capital investment

Thus the present value of earnings can be written as

$$\int_0^T e^{-rt} (H(t) [1 - I(t)] - D(t)) dt$$

The human capital production function is defined as

$$\dot{H} = A(IH)^\alpha D^\beta - \sigma H$$

where σ is the rate of depreciation in human capital

The one other thing we need to solve this model is the initial level of human capital $H(0)$

Now we can write down the Hamiltonian as

$$\mathcal{H} = e^{-rt} (H(t) [1 - I(t)] - D(t)) + \mu(t) \left[A (IH)^\alpha D^\beta - \sigma H \right]$$

This gives first order conditions:

$$\begin{aligned} I &: e^{-rt} H = \mu \alpha A I^{\alpha-1} H^\alpha D^\beta \\ D &: e^{-rt} = \mu \beta A (IH)^\alpha D^{\beta-1} \end{aligned}$$

and

$$\begin{aligned} \dot{\mu} &= \frac{-\partial \mathcal{H}}{\partial H} \\ &= -e^{-rt} (1 - I) - \mu \left[\alpha A I^\alpha H^{\alpha-1} D^\beta - \sigma \right] \end{aligned}$$

Take the ratio of the first two first order conditions:

$$\begin{aligned} H &= \frac{\mu\alpha A I^{\alpha-1} H^\alpha D^\beta}{\mu\beta A (IH)^\alpha D^{\beta-1}} \\ &= \frac{\alpha D}{\beta I} \end{aligned}$$

or

$$D = \frac{\beta}{\alpha} IH$$

Since direct costs of investment D are just a multiple of time costs IH , the distinction between the two is not interesting (of course with borrowing constraints this would no longer be true)

That is we can redefine the model so that

$$\begin{aligned}I^* &= \left(1 + \frac{\beta}{\alpha}\right) I \\ \alpha^* &= \alpha + \beta \\ A^* &= A \left(\frac{\beta}{\alpha}\right)^\beta \left(\frac{\alpha}{\alpha + \beta}\right)^{\alpha + \beta}\end{aligned}$$

With this notation, you can see that

$$\begin{aligned}A^* (I^* H)^{\alpha^*} &= A \left(\frac{\beta}{\alpha}\right)^\beta \left(\frac{\alpha}{\alpha + \beta}\right)^{\alpha + \beta} \left(\left(1 + \frac{\beta}{\alpha}\right) IH\right)^{\alpha + \beta} \\ &= A \left(\frac{\beta}{\alpha}\right)^\beta (IH)^{\alpha + \beta} \\ &= A (IH)^\alpha \left(\frac{\beta}{\alpha} IH\right)^\beta \\ &= A (IH)^\alpha (D)^\beta\end{aligned}$$

It is easy to show that everything else goes through as well.

Thus there is no need to worry about D

Lets abstract from it by using the redefined model (without the * notation)

Then we have first order conditions:

$$\begin{aligned}e^{-rt} &= \mu \alpha A I^{\alpha-1} H^{\alpha-1} \\ \dot{\mu} &= -e^{-rt} (1 - I) - \mu \left[\alpha A I^{\alpha} H^{\alpha-1} D^{\beta} - \sigma \right] \\ &= -e^{-rt} + \sigma \mu + I \left[e^{-rt} - \mu \alpha A I^{\alpha-1} H^{\alpha-1} \right] \\ &= -e^{-rt} + \sigma \mu\end{aligned}$$

Define

$$g(t) = e^{rt} \mu$$

Then

$$\begin{aligned} \frac{\partial g}{\partial t} &= r e^{rt} \mu + e^{rt} \dot{\mu} \\ &= r e^{rt} \mu - 1 + e^{rt} \sigma \mu \\ &= (r + \sigma)g - 1 \end{aligned}$$

We want to solve for this differential equation, but we don't know $g(0)$.

However, we do know that $\mu(T) = 0$ which implies that $g(T) = 0$.

This is straight forward to solve, it yields

$$g(t) = \frac{1 - e^{(\sigma+r)(t-T)}}{\sigma + r}$$

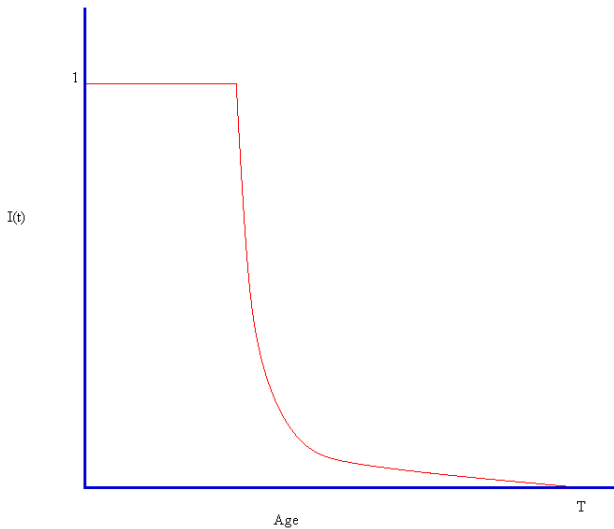
You can see that

- $g(T) = 0$
- $g(t)$ is strictly decreasing with t
- From the first order condition for investment

$$I(t)H(t) = (\alpha Ag(t))^{\frac{1}{1-\alpha}}$$
$$I(t) = \frac{(\alpha Ag(t))^{\frac{1}{1-\alpha}}}{H(t)}$$

- investment IH , is decreasing with t
- IH doesn't depend on $H(0)$ (Ben-Porath neutrality)
- Investment is decreasing with H
- What happens to $H(t)$ depends on investment versus depreciation
- It makes sense to impose that Investment time is bounded from above by 1

This gives a pattern of investment



And a pattern of earnings

