The Real Exchange Rate, Real Interest Rates, and the Risk Premium

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Abstract

The well-known uncovered interest parity puzzle arises from the empirical regularity that, among developed country pairs, the high interest rate country tends to have high expected returns on its short term assets. At the same time, another strand of the literature has documented that high real interest rate countries tend to have currencies that are strong in real terms – indeed, stronger than can be accounted for by the path of expected real interest differentials under uncovered interest parity. These two strands – one concerning short-run expected changes and the other concerning the level of the real exchange rate – have apparently contradictory implications for the relationship of the foreign exchange risk premium and interest-rate differentials. This paper documents the puzzle, and shows that existing models appear unable to account for both empirical findings. The features of a model that might reconcile the findings are discussed.

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This study concerns two prominent empirical findings in international finance that have achieved almost folkloric status. The well-known interest parity puzzle in foreign exchange markets finds that over short time horizons (from a week to a quarter) when the interest rate (one country relative to another) is higher than average, the securities of the high-interest rate currency tend to earn an excess return. That is, the high interest rate country tends to have the higher expected return in the short run. A risk-based explanation of this anomaly requires that the securities in the high-interest rate country are relatively riskier, and therefore incorporate an excess return as a reward for risk-bearing.

The second stylized fact concerns evidence that when a country's relative real interest rate rises above its average, its currency tends to be stronger than average in real terms. Moreover, the strength of the currency tends to be greater than is warranted by rational expectations of future short-term real interest differentials. One way to rationalize this finding is to appeal to the influence of prospective future risk premiums on the level of the exchange rate. That is, the country with the relatively high real interest rate has the lower risk premium and hence the stronger currency. When a country's real interest rate rises, its currency appreciates not only because its assets pay a higher interest rate but also because they are less risky.

This paper produces evidence that confirms these empirical regularities for the exchange rates of the G7 countries (Canada, France, Germany, Italy, Japan and the U.K.) relative to the U.S. However, these findings, taken together, constitute a previously unrecognized puzzle regarding how cumulative excess returns or foreign exchange risk premiums affect the level of the real exchange rate. Theoretically, a currency whose assets are perceived to be risky prospectively – looking forward from the near to the distant future – should be weaker, ceteris paribus. The evidence cited implies that when a country's relative real interest rate is high, the country's securities are expected to yield an excess return over foreign securities in the short run; but, because the high-interest rate currency tends to be stronger, over longer horizons the foreign asset is expected to yield an excess return. This behavior of excess returns in the foreign exchange market poses a challenge for conventional theories of the foreign exchange risk premium.

In brief, when one country's interest rate is high, its currency tends to be stronger than average in real terms, it tends to keep appreciating for awhile, and then depreciates back toward its long-run value. But leading models of the forward-premium anomaly do not account for the behavior of the level of the real exchange rate: they predict that the high-interest rate currency will be weaker than average in real terms and appreciate over both the short- and long-run. A risk-based explanation for the empirical regularities requires a story of some sort of reversal of the risk premium – the securities of the high-interest rate country must be relatively riskier in the short-run, but expected to be less risky than the other country's securities in the more distant future. It may be difficult to rationalize this pattern by focusing on the risk premium required by a single agent in each economy, as many theoretical models do. Instead, a full explanation may require interaction of more than one type of agent and perhaps also requires introducing some sort of "stickiness" in the financial markets – delayed reaction to news, slow adjustment of expectations, liquidity constraints, momentum trading, or other sorts of imperfections.

Figure 1, which will be explained in detail later, illustrates the point dramatically. The chart plots estimates based on a vector autoregression for the U.S. relative to data constructed as a weighted average of the other G7 countries. The line labeled Q(t+j) shows the estimates of the slope coefficient of a regression of the real exchange rate in period t+j on the U.S.-Foreign real interest differential in period t. When the U.S. real interest rate is relatively high, the dollar tends to be strong in real terms (the real exchange rate is below its long run mean.) Over the ensuing months, in the short run, the dollar on average appreciates even more when the U.S. real interest rate is high, before depreciating back toward its long-run mean. The other two lines in the chart represent hypothetical behavior of the real exchange rate implied under two different models. The line labeled –R(t+j) shows how the real exchange rate would behave if uncovered interest parity held (based on the VAR forecasts of future real interest differentials.) Relative to the interest parity norm, the actual real exchange rate behavior is notably different: when the U.S. real interest rate is high, (1) the real value of the dollar is stronger than implied by interest parity (the real exchange rate is lower); (2) the dollar continues to appreciate in the short run (while interest parity implies a depreciation.) The line labeled Model illustrates the implied behavior of the real exchange rate in a class of models based on risk averse behavior of single agents in each economy. The models have been developed to account for the uncovered interest parity puzzle – the depreciation of the currency that tends to accompany a relatively high Home interest rate. Referring to the line labeled Q(t+j), the models are built to explain the initial negative slope of the line. However, the models miss the overall picture badly, because they predict the effect of interest rates on the level of real exchange rates with the wrong sign and therefore get the subsequent dynamics wrong as well.[1]

The literature on the forward premium anomaly is vast. Classic early references include Bilson (1981) and Fama (1984). Engel (1996) surveys the early work that establishes this puzzle, and discusses the problems faced by the literature that tries to account for the regularity. There have been many recent important contributions, including prominent papers by Backus, Foresi, and Telmer (2002), Lustig and Verdelhan (2007), Burnside et. al. (2010a, 2010b), Verdelhan (2010), Bansal and Shaliastovich (2010), Backus et. al. (2010). Below, we survey the implications of the recent theoretical work for real exchange rate behavior.

Dornbusch (1976) and Frankel (1979) are the original papers to draw the link between real interest rates and real exchange rates in the modern, asset-market approach to exchange rates. The connection has not gone unchallenged, principally because the persistence of real exchange rates and real interest differentials makes it difficult to establish their comovement with a high degree of uncertainty. For example, Meese and Rogoff (1988) and Edison and Pauls (1993) treat both series as non-stationary and conclude that evidence in favor of cointegration is weak. However, more recent work that examines the link between real interest rates and the real exchange rate, such as Engel and West (2006), Alquist and Chinn (2008), and Mark (2009), has tended to reestablish evidence of the empirical link. Another approach connects surprise changes in real interest rates to unexpected changes in the real exchange rate. There appears to be a strong link of the real exchange rate to news that alters the expected real interest differential – see, for example, Faust et. al. (2007), Andersen et. al. (2007) and Clarida and Waldman (2008).

The behavior of exchange rates and interest rates described here is closely associated with the notion of “delayed overshooting”. The term was coined by Eichenbaum and Evans (1995), but is used to describe a hypothesis first put forward by Froot and Thaler (1990). Froot and Thaler’s explanation of the forward premium anomaly was that when, for example, the Home interest rate rises, the currency appreciates as it would in a model of interest parity such as Dornbusch’s (1976) classic paper. But, they hypothesize that the full reaction of the market is delayed, perhaps because some investors are slow to react to changes in interest rates, so that the currency keeps on appreciating in the months immediately following the interest rate increase.

[1] The models referred to here tend to treat the real exchange rate as nonstationary, in contrast to the evidence we present in section 2. As explained below, the line in Figure 2 refers to the model’s prediction for the stationary component of the real exchange rate.

Bacchetta and van Wincoop (2010) build a model based on this intuition. Much of the empirical literature that has documented the phenomenon of delayed overshooting has focused on the response of exchange rates to identified monetary policy shocks.[2] But in the original context, the story was meant to apply to any shock that leads to an increase in relative interest rates. Risk-based explanations of the interest parity puzzle have not confronted this literature's finding that high interest rate currencies are strong currencies. Our empirical findings are consistent with Froot and Thaler's hypothesis of delayed overshooting, but with one important modification. The empirical methods here allow us to uncover what the level of the real exchange rate would be if uncovered interest parity held, and to compare the actual real exchange rate with this notional level. We find the level of the real exchange rate is excessively sensitive to real interest differentials. That is, when a country's real interest rate increases, its currency appreciates more than it would under uncovered interest parity. Then it continues to appreciate for a number of months, before slowly depreciating back to its long run level.

Section 1 develops the approach of this paper. Section 2 presents empirical results. Section 3 develops some general conditions that have to be satisfied in order to account for our empirical findings. We discuss the difficulties that "representative agent" models face, and illustrate the problem by showing that some recent models based on non-standard preferences are unable to match the key facts we develop.[3] In section 4, we consider various caveats to our findings. Finally, in the concluding section, we discuss features of models that may be able to account for these empirical regularities.

1. Excess Returns and Real Exchange Rates

We develop here a framework for examining behavior of excess returns and the level of the real exchange rate. The approach developed here is essentially mechanical. We relate the concepts here to economic theories of risk and return in section 3.

Our set-up will consider a Home and Foreign country. In the empirical work of section 2, we always take the US as the Home country (as does the vast majority of the literature), and

[2] See, for example, Eichenbaum and Evans (1995), Kim and Roubini (2000), Faust and Rogers (2003), Scholl and Uhlig (2008), and Bjornland (2009).

[3] Here, "representative agent" models is a somewhat inadequate label for models of the risk premium that are developed off of the Euler equation of agents with the minimum variance stochastic discount factor, generally taking the consumption stream as exogenous.

consider other major economies as the Foreign country. Let i_t be the one period nominal interest in Home. We denote Foreign variables throughout with a superscript *, so i_t^* is the Foreign interest rate. s_t denotes the log of the foreign exchange rate, expressed as the Home currency price of Foreign currency. $E_t s_{t+1}$ refers to the expectation, conditional on time *t* information, of the log of the spot exchange rate at time $t+1$. We define the "excess return", λ_t, as:

(1) $\lambda_t \equiv i_t^* + E_t s_{t+1} - s_t - i_t$.

This definition of excess returns corresponds with the definition in the literature. We can interpret $i_t^* + E_t s_{t+1} - s_t$ as a first-order log approximation of the expected return in Home currency terms for a Foreign security. As Engel (1996) notes, the first-order log approximation may not really be adequate for appreciating the implications of economic theories of the excess return. For example, if the exchange rate is conditionally log normally distributed, then $\ln\left(E_t(S_{t+1}/S_t)\right) = E_t s_{t+1} - s_t + \frac{1}{2}\text{var}_t(s_{t+1})$, where $\text{var}_t(s_{t+1})$ refers to the conditional variance of the log of the exchange rate. Engel (1996) points out that this second-order term is approximately the same order of magnitude as the risk premiums implied by some economic models. However, we proceed with analysis of λ_t defined according to equation (1) both because it is the object of almost all of the empirical analysis of excess returns in foreign exchange markets, and because the theoretical literature that we consider in section 3 seeks to explain λ_t as defined above including possible movements in $\text{var}_t(s_{t+1})$.

The well-known uncovered interest parity puzzle comes from the empirical finding that the change in the log of the exchange rate is negatively correlated with the Home less Foreign interest differential, $i_t - i_t^*$. That is, estimates of $\text{cov}(s_{t+1} - s_t, i_t - i_t^*) = \text{cov}(E_t s_{t+1} - s_t, i_t - i_t^*)$ tend to be negative. As Engel (1996) surveys, and subsequent empirical work confirms, this finding is consistent over time among pairs of high-income, low-inflation countries.[4] From equation (1), we note that the relationship $\text{cov}(E_t s_{t+1} - s_t, i_t - i_t^*) < 0$ is equivalent to $\text{cov}(\lambda_t, i_t - i_t^*) < -\text{var}(i_t - i_t^*) < 0$. That is, when the Home interest rate is relatively high, so $i_t - i_t^*$ is above average, the excess return on Home assets also tends to be above average: λ_t is below

[4] Bansal and Dahlquist (2000) find that the relationship is not as consistent among emerging market countries, especially those with high inflation.

average. This is considered a puzzle because it has been very difficult to find plausible economic models that can account for this relationship.

Let p_t denote the log of the consumer price index at Home, and $\pi_{t+1} = p_{t+1} - p_t$ is the inflation rate. The log of the real exchange rate is defined as $q_t = s_t + p_t^* - p_t$. The ex ante real one-period interest rates, Home and Foreign, are given by $r_t = i_t - E_t \pi_{t+1}$ and $r_t^* = i_t^* - E_t \pi_{t+1}^*$. Note also $E_t q_{t+1} - q_t = E_t s_{t+1} - s_t + E_t \pi_{t+1}^* - E_t \pi_{t+1}$. We can rewrite (1) as:

(2) $$\lambda_t = r_t^* - r_t + E_t q_{t+1} - q_t .$$

We take as uncontroversial the proposition that the real interest differential, $r_t - r_t^*$, and excess returns, λ_t, are stationary random variables without time trends, and denote their means as $\bar{r}$ and $\bar{\lambda}$, respectively. We will also stipulate that there is no deterministic time trend or drift in the log of real exchange rates, so that the unconditional mean of $E_t q_{t+1} - q_t$ is zero. Rewriting (2):

(3) $$q_t - E_t q_{t+1} = -(r_t - r_t^* - \bar{r}) - (\lambda_t - \bar{\lambda}) .$$

Previewing one of the empirical findings of section 2, the uncovered interest parity puzzle still holds when returns are expressed in real terms as in equation (2). That is, we find $\operatorname{cov}(E_t q_{t+1} - q_t, r_t - r_t^*) < 0$, where we construct measures of expected real exchange rates and expected inflation from VARs. This finding implies $\operatorname{cov}(\lambda_t, r_t - r_t^*) < 0$, analogous to the findings when returns are expressed in nominal terms.

Next, we iterate equation (3) forward, applying the law of iterated expectations, to get:

(4) $$q_t - \lim_{j\to\infty}\left(E_t q_{t+j}\right) = -R_t - \Lambda_t ,$$

where

(5) $$R_t \equiv \sum_{j=0}^{\infty} E_t (r_{t+j} - r_{t+j}^* - \bar{r}) \text{, and}$$

(6) $$\Lambda_t \equiv \sum_{j=0}^{\infty} E_t (\lambda_{t+j} - \bar{\lambda}) .$$

We label R_t as the "prospective real interest differential". It is the expected sum of the current and all future values of the Home less Foreign real interest differential (relative to its unconditional mean). It is important to note that R_t is not the real interest differential on long-term bonds, even hypothetical infinite-horizon bonds. R_t is the difference between the real return from holding an infinite sequence of short-term Home bonds and the real return from the infinite sequence of short-term Foreign bonds. An investment that involves rolling over short term assets has different risk characteristics than holding a long-term asset. Hence we coin the phrase "prospective" real interest differential to avoid the trap of calling R_t the long-term real interest differential.

Similarly, Λ_t is the expected infinite sum of excess returns on the Foreign security. We label this the "prospective excess return".

The left-hand side of (4), $q_t - \lim_{j\to\infty}\left(E_t q_{t+j}\right)$, can be interpreted as the transitory component of the real exchange rate. In fact, according to our empirical findings reported in section 2, we can treat the real exchange rate as a stationary variable, so $\lim_{j\to\infty}\left(E_t q_{t+j}\right) = \bar{q}$. As is well known, even if the real exchange rate is stationary, it is very persistent. Engel (2000), in fact, argues that it may be practically impossible to distinguish between the stationary case and the unit root case under plausible economic conditions. We proceed in examining $q_t - \bar{q}$, assuming stationarity, but note that our methods could be applied to the transitory component of the real exchange rate, taken as the difference between q_t and a measure of the permanent component, $\lim_{j\to\infty}\left(E_t q_{t+j}\right)$. In section 4, we note how Engel's (2000) interpretation implies that in practice it may not be possible to distinguish a permanent and transitory component, but make the case that the economic analysis of that paper argues for treating the real exchange rate as stationary.

In section 3, we discuss the common assumption in theoretical models of excess returns that the real exchange rate is equal to the difference between the marginal utility of a (particular) Home consumer and (particular) Foreign consumer. We note here that stationarity of the real exchange rate is completely compatible with a unit root in the log of consumption, or in the marginal utility of consumption. It requires simply that Home and Foreign marginal utilities of consumption be cointegrated, which is a natural condition among well-integrated economies such as the highly developed countries used in this study. It is analogous to the assumption made in almost all closed-economy models that we can treat the marginal utilities of different consumers within a country as cointegrated.

Under the stationarity assumption, we can write (4) as:

(7) $q_t - \bar{q} = -R_t - \Lambda_t$.

From this formulation, we see that the prospective excess return, Λ_t, captures the potential effect of risk premiums on the *level* of the real exchange rate, holding the prospective real interest differential constant.

In the next section, we present evidence that $\text{cov}(r_t - r_t^*, R_t) > 0$ and $\text{cov}(r_t - r_t^*, \Lambda_t) > 0$. Taken together, these two findings imply from (7) that $\text{cov}(r_t - r_t^*, q_t) > 0$, which jibes with the concept familiar from Dornbusch (1976) and Frankel (1979) that when a country's real interest rate is high (relative to the foreign real interest rate, relative to average), its currency tends to be strong in real terms (relative to average.) But if $\text{cov}(r_t - r_t^*, \Lambda_t) > 0$, the strength of the currency cannot be attributed entirely to the prospective real interest differential, as it would be in Dornbusch and Frankel (who both assume uncovered interest parity, or that $\lambda_t \equiv 0$.) The relationship between excess returns and real interest differential plays a role in determining the relation between the real exchange rate and real interest rates.

It is entirely unsurprising that we find $\text{cov}(r_t - r_t^*, R_t) > 0$. This simply implies that there is not a great deal of non-monotonicity in the adjustment of real interest rates toward the long run mean.

The central puzzle raised by this paper concerns the two findings, $\text{cov}(\lambda_t, r_t - r_t^*) < 0$ and $\text{cov}(r_t - r_t^*, \Lambda_t) > 0$. The short-run excess return on the Foreign security, λ_t, is negatively correlated with the real interest differential, consistent with the many empirical papers on the uncovered interest parity puzzle. But the prospective excess return, Λ_t, is positively correlated. Given the definition of Λ_t in equation (6), we must have that for at least $j = 0$ and possibly for some $j > 0$, $\text{cov}(E_t\lambda_{t+j}, r_t - r_t^*) < 0$, but for other $j > 0$, $\text{cov}(E_t\lambda_{t+j}, r_t - r_t^*) > 0$. The sum of the latter covariances must exceed the sum of the former to generate $\text{cov}(r_t - r_t^*, \Lambda_t) > 0$. As we discuss in section 3, our risk premium models of excess return are not up to the task of explaining this finding. In fact, while they are constructed to account for $\text{cov}(\lambda_t, r_t - r_t^*) < 0$, they have the counterfactual implication that $\text{cov}(r_t - r_t^*, \Lambda_t) < 0$.

The empirical approach of this paper can be described simply. We estimate VARs in the variables q_t, $i_t - i_t^*$, and $i_{t-1} - i_{t-1}^* - (\pi_t - \pi_t^*)$. From the VAR estimates, we construct measures of $E_t\left(i_t - i_t^* - (\pi_{t+1} - \pi_{t+1}^*)\right) = r_t - r_t^*$. Using standard projection formulas, we can also construct estimates of R_t. To measure Λ_t, we take the difference of $q_t - \bar{q}$ and R_t. From these VAR estimates, we calculate our estimates of the covariances just discussed. As an alternative approach, we estimate VARs in q_t, $i_t - i_t^*$, and $\pi_t - \pi_t^*$, and then construct the needed estimates of $r_t - r_t^*$, R_t, and Λ_t. The estimated covariances under this alternative approach are very similar to those from the original VAR. Our approach of estimating undiscounted expected present values of interest rates from VARs is presaged in Mark (2009) and Brunnermeier et. al. (2009).

2. Empirical Results

We investigate the behavior of real exchange rates and interest rates for the U.S. relative to the other six countries of the G7: Canada, France, Germany, Italy, Japan, and the U.K. WE also consider the behavior of U.S. variables relative to an aggregate weighted average of the variables from these six countries.[5] Our study uses monthly data. Foreign exchange rates are noon buying rates in New York, on the last trading day of each month, culled from the daily data reported in the Federal Reserve historical database. The price levels are consumer price indexes from the Main Economic Indicators on the OECD database. Nominal interest rates are taken from the last trading day of the month, and are the midpoint of bid and offer rates for one-month Eurorates, as reported on Intercapital from Datastream. The interest rate data begin in June 1979. Most of our empirical work uses the time period June 1979 to October 2009. In some of the tests for a unit root in real exchange rates, reported below, we use a longer time span from June 1973 to October 2009. It is important for our purposes to include these data well into 1979 because it has been noted in some recent papers that there was a crash in the "carry trade" in 2008, so it would perhaps bias our findings if our sample ended prior to this crash.[6]

[5] The weights are determined by the value of each country's exports and imports as a fraction of the average value of trade over the six countries.
[6] See, for example, Brunnermeier, et. al. (2009) and Jordà and Taylor (2009).

2.1 Tests for unit root in real exchange rates

It is well known that real exchange rates among advanced countries are very persistent.[7] There is no consensus on whether these real exchange rates are stationary or have a unit root. Two recent studies of uncovered interest parity, Mark (2010) and Brunnermeier, et. al. (2009) estimate statistical models that assume the real exchange rate is stationary, but do not test for stationarity. Jordà and Taylor (2010) demonstrate that there is a profitable carry-trade strategy that exploits the uncovered interest parity puzzle when the trading rule is enhanced by including a forecast that the real exchange rate will return to its long-run level when its deviations from the mean are large. That paper assumes a stationary real exchange rate and includes statistical tests that cannot reject cointegration of s_t with $p_t - p_t^*$. However, that study does not indicate whether the cointegrating vector is insignificantly different than [1,-1], so the tests are not equivalent to a test for a unit root in q_t.

We present evidence here that favors the hypothesis of no unit root in the real exchange rate. Clearly the real exchange rate is very persistent, and the evidence in favor of stationarity is not incontrovertible.

Table 1 presents standard ADF tests for a unit root. The null is not rejected for any currency except the U.K. pound at the 10 percent level. The table also includes tests for a unit root based on the GLS test proposed by Elliott et. al. (1996). These tests show stronger evidence against a unit root – the null is rejected at the 5% level for three currencies, at the 10% level for two others, and not rejected for the Canadian dollar or Japanese yen. However, the test statistic is based on the assumption that there may be a trend in the real exchange rate under the alternative, which is not a realistic assumption for these real exchange rates.

We next follow much of the recent literature on testing for a unit root in real exchange rates by exploiting the power from panel estimation. The lower panel of Table 1 reports estimates from a panel model. The null model in this test is:

(8) $$q_{it} - q_{it-1} = \mu_i + \sum_{j=1}^{k_i} c_i (q_{it-j} - q_{it-j-1}) + \varepsilon_{it}.$$

[7] See Rogoff (1996) for example.

Under the null, the change in the real exchange rate for country *i* follows an autoregressive process of order k_i. Note that the parameters and the lag lengths can be different across the currencies. Under the alternative:

(9) $$q_{it} - q_{it-1} = \mu_i + \alpha q_{it-1} + \sum_{j=1}^{k_i} c_i (q_{it-j} - q_{it-j-1}) + \varepsilon_{it},$$

with a common α for the currencies.

We estimate α for the six currencies from (9).[8] We find the lag length for each currency by first estimating a univariate version of (9), and using the BIC criterion. The estimated value of α is reported in the lower panel of Table 1, in the row labeled "no covariates".

This table also reports the bootstrapped distribution of α. The bootstrap is constructed by estimating (8), then saving the residuals for the six real exchange rates for each time period. We then construct 5000 artificial time series (each of length 440, corresponding to our sample of 440 months) for the real exchange rate by resampling the residuals and using the estimates from (8) to parameterize the model.

The lower panel of Table 1, in the row labeled "no covariates" reports certain points of the distribution of α from the bootstrap. We see that we can reject the null of a unit root at the 5 percent level.

We also consider a version of the panel test in which we include covariates. Specifically, we investigate the possibility that the inflation differential (with the U.S.) helps account for the dynamics of the real exchange rate. We follow the same procedure as above, but add lagged own relative inflation terms to equation (9). To generate the distribution of the estimate of α, we estimate a VAR in the change in the real exchange rate (as in (8)) and the inflation rate. For each country, the real exchange rate and inflation rates depend only on own-country lags under the null. The bootstrap proceeds as in the model with no covariates.

The bottom panel of Table 1 reports the estimated α and its distribution for the model with covariates in the row labeled "with covariates". Adding covariates does not alter the conclusion that we can reject a unit root at the 5 percent level.

Based on these tests, we will proceed to treat the real exchange rate as stationary, though we note that the evidence favoring stationarity is thin for the Canadian dollar and Japanese yen real exchange rates.

[8] We do not include the average G6 real exchange rate as a separate real exchange rate in this test.

2.2 Fama regressions

Table 2 reports results from the standard "Fama regression" that is the basis for the forward premium puzzle. The change in the log of the exchange rate between time t+1 and t is regressed on the time t interest differential:

(10) $s_{t+1} - s_t = \beta_0 + \beta_1(i_t - i_t^*) + u_{t+1}$.

Under uncovered interest parity, $\beta_0 = 0$ and $\beta_1 = 1$.

We can rewrite this regression as:

$$i_t - (i_t^* + s_{t+1} - s_t) = -\beta_0 + (1 - \beta_1)(i_t - i_t^*) + u_{t+1}.$$

The left-hand side of the regression is the ex post excess return on the home security. If $\beta_0 = 0$ but $\beta_1 < 1$, then the high-interest rate currency tends to have a higher excess return. There is a positive correlation between the excess return on the Home currency and the Home-Foreign interest differential.

The Table reports the 90% confidence interval for the regression coefficients, based on Newey-West standard errors. For five of the six currencies, the 90% confidence interval for β_1 lies below one (Italy being the exception.) For four of the six, zero is inside the 90% confidence interval for β_0. (In the case of the U.K., the confidence interval barely excludes zero, while for Japan we find strong evidence that β_0 is greater than zero.)

The G6 exchange rate (the weighted average exchange rate, defined in the data section) appears to be less noisy than the individual exchange rates. In all of our tests, the standard errors of the coefficient estimates are smaller for the G6 exchange rate than for the individual country exchange rates, suggesting that some idiosyncratic movements in country exchange rates gets smoothed out when we look at averages. Table 2 reports that the 90% confidence interval for this exchange rate lies well below one – in fact, it lies below zero.[9] This is the standard finding in empirical work on interest parity (see the survey by Engel (1996)) – we can not only reject that the interest differential is unbiased predictor of the change in the exchange rate ($\beta_0 = 0$ and $\beta_1 = 1$), but we find that the interest differential tends to predict the exchange rate in the opposite direction from interest parity ($\beta_1 < 0$).

[9] The intercept coefficient, on the other hand is very near zero, and the 90% confidence interval easily contains zero.

2.3 Fama regressions in real terms

The Fama regression in real terms can be written as:

(11) $q_{t+1} - q_t = \beta_0 + \beta_1(\hat{r}_t - \hat{r}_t^*) + u_{t+1}$.

In this regression, $\hat{r}_t - \hat{r}_t^*$ refers to estimates of the ex ante real interest rate differential, $r_t - r_t^* \equiv i_t - E_t\pi_{t+1} - (i_t^* - E_t\pi_{t+1}^*)$ and. We estimate the real interest rate from VARs. As noted above, we consider two different VAR models. Model 1 is a VAR with 3 lags in the variables q_t, $i_t - i_t^*$, and $i_{t-1} - i_{t-1}^* - (\pi_t - \pi_t^*)$. From the VAR estimates, we construct measures of $E_t\left(i_t - i_t^* - (\pi_{t+1} - \pi_{t+1}^*)\right) = r_t - r_t^*$. Model 2 is a 3-lag VAR in q_t, $i_t - i_t^*$, and $\pi_t - \pi_t^*$.

There are two senses in which our measures of $\hat{r}_t - \hat{r}_t^*$ are estimates. The first is that the parameters of the VAR are estimated. But even if the parameters were known with certainty, we would still only have estimates of $r_t - r_t^*$ because we are basing our measures of $r_t - r_t^*$ on linear projections. Agents certainly have more sophisticated methods of calculating expectations, and use more information than is contained in our VAR.

The findings for the Fama regression in real terms are similar to those when the regression is estimated on nominal variables. For four of the six currencies, the estimates of β_1, reported in Table 3A, are negative, and all are less than one. In addition, the estimated coefficient for the G6 aggregate is close to -1. This summary is true for both VAR models.

Table 3A reports three sets of confidence intervals. All of the subsequent tables also report three sets of confidence intervals for each parameter estimate.

The first is based on Newey-West standard errors, ignoring the fact that $\hat{r}_t - \hat{r}_t^*$ is a generated regressor. The second two are based on bootstraps.

For both bootstraps, we construct pseudo-samples using the VAR estimates.[10] For each pseudo-sample, we estimate the VAR. We estimate all of the regression coefficients reported in Tables 3A, 3B, 4 and 5, and calculate the Newey-West standard errors for each of those regressions.
We repeat this exercise 1000 times.

[10] Initial values are set at the sample means. We generate samples of 865 observations, then use the last 365 observations, corresponding to the length of the time series we use in estimation.

The first confidence interval based on the bootstraps (the second confidence interval reported for each coefficient estimate) uses the coefficient estimates reported in the tables. Let $\hat{\beta}$ refer to any of the coefficient estimates reported in Tables 3A, 3B, 4 and 5. From the regressions on the pseudo-samples, we order the coefficient estimates from these 1000 replications from smallest to largest - $\hat{\beta}_1$ is the smallest and $\hat{\beta}_{1000}$ be the largest. The confidence interval reported in the tables is based on $([\hat{\beta}-(\hat{\beta}_{950}-\hat{\beta}),\hat{\beta}+(\hat{\beta}-\hat{\beta}_{50})]$. That is, the reported confidence interval corrects for the asymmetry in the distribution of $\hat{\beta}_i$ from the regressions on the pseudo-samples.

Hansen (2010) argues that the first bootstrap method performs poorly when the $\hat{\beta}_i$ do not have a symmetric distribution. Instead, he recommends the following procedure. As above, let $\hat{\beta}$ refer to the estimated coefficient in the data, and $\hat{\sigma}$ to be the Newey-West standard error in the data. For each pseudo-sample *i*, we will record analogous estimates: $\hat{\beta}_i$ and $\hat{\sigma}_i$. θ_i is defined by: $\theta_i=\frac{\hat{\beta}_i-\hat{\beta}}{\hat{\sigma}_i}$. We arrange these θ_i from smallest to largest, so that θ_1 is the smallest and θ_{1000} is the largest. The third confidence interval reported for each coefficient estimate is given by $[\hat{\beta}-\hat{\sigma}\theta_{950},\hat{\beta}-\hat{\sigma}\theta_{50}]$. It turns out that our two bootstraps generally produce very similar confidence intervals.

Indeed, from Table 3A, all three sets of confidence intervals are similar. For the individual currencies, for both Model 1 and Model 2, the confidence interval for β_1 lies below one for Germany, Japan, and the U.K. It contains one for Canada and Italy, and contains one for France except using the second confidence interval.

The findings are clear using the G6 average exchange rate: the coefficient estimate is -0.93 when the real interest estimate comes from Model 1, and -0.91. All of the confidence intervals lie below one, though they all contain zero. For both models, the estimate of β_0 is very close to zero, and all confidence intervals contain zero.

In summary, the evidence on the interest parity puzzle is similar in real terms as in nominal terms. The estimate of the coefficient β_1, tends to be negative. There is less evidence in the real regressions that the coefficient is significantly less than zero, but strong evidence that it is less than one. Even in real terms, the country with the higher interest rate tends to have short-run excess returns (i.e., excess returns and the interest rate differential are positively correlated.)

The Fama regression finds a strong negative correlation between $s_{t+1} - s_t$ and $i_t - i_t^*$. It is well known that for the currencies of low-inflation, high-income countries, $s_{t+1} - s_t$ is highly correlated with $q_{t+1} - q_t$, which suggests $q_{t+1} - q_t$ is negatively correlated with $i_t - i_t^*$. Since $i_t - i_t^* = r - r_t^* + E_t(\pi_{t+1} - \pi_{t+1}^*)$, for exploratory reasons we consider a regression of $q_{t+1} - q_t$ on $\hat{r}_t - \hat{r}_t^*$ and $\hat{E}_t(\pi_{t+1} - \pi_{t+1}^*)$, where the latter is our measure of the expected inflation differential generated from the VARs. These regressions are reported in Table 3B. Specifically, Table 3B reports the estimation of:

(12) $$q_{t+1} - q_t = \beta_0 + \beta_1(\hat{r}_t - \hat{r}_t^*) + \beta_2 \hat{E}_t(\pi_{t+1} - \pi_{t+1}^*) + u_{t+1}.$$

The estimates of β_1 tend to be negative, and generally more negative than those reported in Table 3A for equation (11). The real surprise from Table 3b is that the estimates of β_2 are negative for all currencies in both models. Though they are not always significantly negative individually, and we do not calculate a test of their joint significance, it is nonetheless telling that all of the coefficient estimates are negative. This implies that the currency of the country that is expected to have relatively high inflation is expected to appreciate in real terms. We return to this finding in section 5.

2.4 The real exchange rate, real interest rates, and the level risk premium

Table 4 reports estimates from

(13) $$q_t = \beta_0 + \beta_1(\hat{r}_t - \hat{r}_t^*) + u_{t+1}.$$

In all cases (all currencies, for both Model 1 and Model 2), the coefficient estimate is negative. In virtually all cases, although the confidence intervals are wide, the coefficient is significantly negative.[11]

Recall from equation (7) that $q_t - \bar{q} = -R_t - \Lambda_t$, where $R_t \equiv \sum_{j=0}^{\infty} E_t(r_{t+j} - r_{t+j}^* - \bar{r})$ and $\Lambda_t \equiv \sum_{j=0}^{\infty} E_t(\lambda_{t+j} - \bar{\lambda})$. If there were no excess returns, so that $q_t - \bar{q} = -R_t$, and $\hat{r}_t - \hat{r}_t^*$ were strongly positively correlated with R_t, then we expect a negative correlation between q_t and

[11] The exceptions are that the third confidence interval contains zero for Model 1 for France, and Models 1 and 2 for the U.K.

$\hat{r}_t - \hat{r}_t^*$. That is, under uncovered interest parity, the high real interest rate currency tends to be stronger. For example, this is the implication of the Dornbusch-Frankel theory in which real interest differentials are determined in a sticky-price monetary model.

But we can make a stronger statement – there is a relationship between the real interest differential, $\hat{r}_t - \hat{r}_t^*$, and our measure of the level risk premium, $\hat{\Lambda}_t$ (where $\hat{\Lambda}_t$ is our estimate of Λ_t based on the VAR models.) Our central empirical finding is reported in Table 5. This table reports the regression:

(14) $\hat{\Lambda}_t = \beta_0 + \beta_1(\hat{r}_t - \hat{r}_t^*) + u_{t+1}$.[12]

In all cases, the estimated slope coefficient is positive. The 90 percent confidence intervals are wide, but with a few exceptions, lie above zero. The confidence for the G6 average strongly excludes zero.

This finding is surprising in light of the well-known uncovered interest parity puzzle. In the previous two subsections, we have documented that when $r_t - r_t^*$ is above average, the Home currency tends to have excess returns. That seems to imply that the high interest rate currency is the riskier currency. But the estimates from equation (14) deliver the opposite message – the high interest rate currency has the lower level risk premium. Λ_t is the level risk premium for the Foreign currency – it is positively correlated with $r_t - r_t^*$, so it tends to be high when $r_t^* - r_t$ is low.

Recall that the level risk premium is defined in equation (6) as $\Lambda_t \equiv \sum_{j=0}^{\infty} E_t(\lambda_{t+j} - \bar{\lambda})$. We have then that

(15) $$\text{cov}(\Lambda_t, r_t - r_t^*) \equiv \sum_{j=0}^{\infty} \text{cov}[E_t(\lambda_{t+j}), r_t - r_t^*].$$

The short-run interest parity puzzle establishes that $\text{cov}(\lambda_t, r_t - r_t^*) < 0$. Clearly if $\text{cov}(\Lambda_t, r_t - r_t^*) > 0$, then we must have $\text{cov}[E_t(\lambda_{t+j}), r_t - r_t^*] > 0$ for at least some $j > 0$. That is,

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[12] To be precise, $\hat{\Lambda}_t$ is calculated as the difference between q_t and our VAR estimate of R_t. To calculate our estimate of R_t, given by the infinite sum of equation (5), we demean $r_{t+j} - r_{t+j}^*$ by its sample mean. We use the sample mean rather than maximum likelihood estimate of the mean because it tends to be a more robust estimate.

in order for $\text{cov}(\Lambda_t, r_t - r_t^*) > 0$, we must have a reversal in the correlation of the short-run risk premiums with $r_t - r_t^*$ as the horizon extends.

This is illustrated in Figure 2, which plots estimates of the slope coefficient in a regression of $\hat{E}_t(\lambda_{t+j-1})$ on $\hat{r}_t - \hat{r}_t^*$ for $j = 1, \ldots, 100$. For the first few observations, this coefficient is negative, but it eventually turns positive at longer horizons.

The Figure also plots the slope of regressions of $\hat{E}_t(r_{t+j-1} - r_{t+j-1}^*)$ on $\hat{r}_t - \hat{r}_t^*$ for $j = 1, \ldots, 100$. These tend to be positive at all horizons.

The Figure also includes a plot of the slope coefficients from regressing $\hat{E}_t(q_{t+j} - q_{t+j-1})$ for $j = 1, \ldots, 100$. Since $\hat{E}_t(q_{t+j} - q_{t+j-1}) = \hat{E}_t(r_{t+j-1} - r_{t+j-1}^*) + \hat{E}_t(\lambda_{t+j-1})$, these regression coefficients are simply the sum of the other two regression coefficients that are plotted. In this case, the regression coefficients start out negative for the first few months, but then turn positive for longer horizons.

To summarize, when the Home real interest rate relative to the Foreign real interest rate is higher than average, the Home currency is stronger in real terms than average. Crucially, it is even stronger than would be predicted by a model of uncovered interest parity. Excess returns or the foreign exchange risk premium contribute to this strength. If Home's real interest rate is high – in the sense that the Home relative to Foreign real interest rate is higher than average – the level risk premium on the Foreign security is higher than average.

We can project the future path of the real exchange rate when Home real interest rates are high using the facts that the currency tends to be stronger than average (the finding of regression (14)), that it continues to appreciate in the short run (the famous puzzle, confirmed in the findings from regression (11)), and that the real exchange rate is stationary so it is expected to return to its unconditional mean (established in Table 1.) When the Home real interest rate is high, the Home currency is strong in real terms, and expected to get stronger in the short run. However, eventually it must be expected to depreciate back to its long run level.

One implication of these dynamics is similar to Jorda and Taylor's (2009) findings about forecasting nominal exchange rate changes. They find that the nominal interest differential can help to predict exchange rate changes in the short run: the high interest rate currency is expected to appreciate (contrary to the predictions of uncovered interest parity.) But the forecasts of the exchange rate can be enhanced by taking into account purchasing power parity considerations. The deviation from PPP helps predict movements of the nominal exchange rate as the real exchange rate adjusts toward its long-run level.

Figure 1 presents a slightly different perspective. This chart plots the slope coefficients from regressions of $-\hat{R}_{t+j}$ and q_{t+j} on $r_t - r_t^*$ for the G6 average exchange rate.[13] If interest parity held, the behavior of the real exchange rate should conform to the plot for $-\hat{R}_{t+j}$. That line indicates that the U.S. dollar tends to be strong in real terms when $r_t - r_t^*$ is high, and then is expected to depreciate back toward its long-run mean. The line for the regression of q_{t+j} on $r_t - r_t^*$ shows three things: First, when $r_t - r_t^*$ is above average, the dollar tends to be strong in real terms, and much stronger than would be implied under uncovered interest parity. Second, when $r_t - r_t^*$ is above average, the dollar is expected to appreciate even more in the short run. This is the uncovered interest parity puzzle. Third, when $r_t - r_t^*$ is above average, the dollar is expected to reach its maximum appreciation after around 5 months, then to depreciate gradually. The line labeled "Model" is discussed in the next section.

We turn now to the implications of these empirical findings for models of the foreign exchange risk premium.

3. The Risk Premium

The problem facing most models of the risk premium stem from treating the interest differential as if it contains all relevant information for forecasting the exchange rate. That is, the Fama regression equation (11) is treated as though it determines conditional expectations:

(16) $$E_t q_{t+1} - q_t = \beta_0 + \beta_1 (r_t - r_t^*).$$

If this interpretation were correct, then from equation (2), the risk premium is perfectly correlated to the real interest differential:

(17) $$\lambda_t = r_t^* - r_t + E_t q_{t+1} - q_t = \beta_0 + (\beta_1 - 1)(r_t - r_t^*).$$

[13] The plots for most of the other real exchange rates look qualitatively very similar.

The uncovered interest rate parity puzzle finds $\beta_1 < 1$, so (17) implies λ_t and $r_t - r_t^*$ are perfectly negatively correlated. It follows that, under this approach, $R_t \equiv \sum_{j=0}^{\infty} E_t(r_{t+j} - r_{t+j}^* - \bar{r})$ and $\Lambda_t \equiv \sum_{j=0}^{\infty} E_t(\lambda_{t+j} - \bar{\lambda})$ are perfectly negatively correlated. Since real interest rates are strongly positively serially correlated, so $\text{cov}(r_t - r_t^*, R_t) > 0$, equation (17) must imply $\text{cov}(r_t - r_t^*, \Lambda_t) < 0$ if $\beta_1 < 1$. But the evidence of Table 5 shows the opposite, that $\text{cov}(r_t - r_t^*, \Lambda_t) > 0$. The assumption embodied in equation (16) which is implicit or explicit in much of the literature rules out our key empirical finding – that the correlation of the short-run risk premium and the level risk premium with $r_t - r_t^*$ are of opposite signs. In fact, usually we find models working off the stronger condition that $\beta_1 < 0$. Then there is a stronger implication. Iterating equation (16) forward,

(18) $\quad q_t - \bar{q} = -\beta_1 R_t$.

Then we must have that the real exchange rate, q_t, is perfectly positively correlated with R_t. Given the strong positive serial correlation of real interest rates, this implies we must have $\text{cov}(r_t - r_t^*, q_t) > 0$. That is, we must have a positive correlation of the real interest differential with the real exchange rate.

Thus two strands of the international finance literature are in conflict. The models of the uncovered interest parity puzzle (interpreted as finding $\beta_1 < 0$) that rely on the implicit assumption of equation (16) necessarily are at loggerheads with the literature that finds a currency tends to be stronger in real terms when its relative real interest rate is above the long-run average.

Figure 1 tells the story of this paper. As already noted, this chart plots the slope coefficients from regressions of $-\hat{R}_{t+j}$ and q_{t+j} on $r_t - r_t^*$ for the G6 average exchange rate – these are the lines labeled –R(t+j) and Q(t+j), respectively. The third line, labeled Model, is an example of the theoretical regression coefficients implied by the models of the risk premium built off of the behavior of a single agent in each of the Home and Foreign economies that are discussed below in sections 3.2 and 3.3.[14] The models are built to account for the empirical finding that the Home currency tends to appreciate in the short run when the home interest rate is high. But the models leave the correlation of the level of the real exchange rate and the real interest differential with the wrong sign, and imply monotonic adjustment rather than the hump-shaped dynamics apparent in the data.

Subsequently, we use the notation $d_{t+1} \equiv q_{t+1} - q_t$ for the rate of real depreciation, and $r_t^d \equiv r_t - r_t^*$ for the Home less Foreign short-term real interest differential. We also use $\delta_t \equiv E_t q_{t+1} - q_t$ for the expected real rate of depreciation. In essence, the assumption underlying equation (16) is that a single factor drives δ_t and r_t^d:

(19) $\delta_t = a_1 \phi_{1t}$

(20) $r_t^d = c_1 \phi_{1t}$,

so that

(21) $\delta_t = \frac{c_1}{a_1} r_t^d = \beta_1 r_t$,

as in equation (16).[15]

A model that allows the short-run risk premium and level risk premium to covary with the real interest differential with opposite signs requires a model with at least two factors. However, while two factors are necessary, they are not sufficient.

3.1 A two-factor model

The problem arises from the fact that while $\text{cov}(r_t^d, \lambda_t) < 0$, we have $\text{cov}(r_t^d, E_t \lambda_{t+j}) > 0$ for large enough *j*. In order to account for this, we need a model that allows at least two factors to drive real interest rates and risk premiums:

(22) $\delta_t = a_1 \phi_{1t} + a_2 \phi_{2t}$

(23) $r_t^d = c_1 \phi_{1t} + c_2 \phi_{2t}$,

(24) $\lambda_t = g_1 \phi_{1t} + g_2 \phi_{2t}$.

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[14] This line refers to the implied slope coefficient in the regression of $q_{t+i} - \lim_{j\to\infty}\left(E_t q_{t+j}\right)$ on r_t^d

[15] We drop the constant terms hereinafter because they play no role in explaining the puzzles.

Where the ϕ_{jt}, $j = 1, 2, 3$, are factors with a conditional mean of zero and mutually uncorrelated. Since $\lambda_t = \delta_t - r_t^d$, we have $g_1 = a_1 - c_1$ and $g_2 = a_2 - c_2$.

A projection of d_t onto r_t gives us:

(25) $$\operatorname{cov}(\delta_t, r_t^d) = a_1 c_1 \operatorname{var}(\phi_{1t}) + a_2 c_2 \operatorname{var}(\phi_{2t})$$

For this to be negative as in the familiar uncovered interest parity puzzle, we need $a_1 c_1 \operatorname{var}(\phi_{1t}) + a_2 c_2 \operatorname{var}(\phi_{2t}) < 0$. A weaker condition, which requires only that the slope coefficient in regression (11) be negative is $\operatorname{cov}(r_t^d, \lambda_t) < 0$. We have:

(26) $$\operatorname{cov}(\lambda_t, r_t^d) = g_1 c_1 \operatorname{var}(\phi_{1t}) + g_2 c_2 \operatorname{var}(\phi_{2t}).$$

Then $\operatorname{cov}(r_t^d, \lambda_t) < 0$ implies $g_1 c_1 \operatorname{var}(\phi_{1t}) + g_2 c_2 \operatorname{var}(\phi_{2t}) < 0$. For this covariance to be negative we need at least one of $g_1 c_1$ or $g_2 c_2$ to be negative. Assume (without loss of generality) $g_1 c_1 < 0$. If $g_2 c_2 < 0$ also, then this condition is met. But if $g_2 c_2 > 0$, we need

(27) $$g_2 c_2 \frac{\operatorname{var}(\phi_{2t})}{\operatorname{var}(\phi_{1t})} < -g_1 c_1.$$

Then note that

$$E_t \lambda_{t+j} = g_1 E_t \phi_{1t+j} + g_2 E_t \phi_{2t+j}.$$

To generate $\operatorname{cov}(r_t^d, E_t \lambda_{t+j}) > 0$, we need

$$g_1 c_1 \operatorname{cov}(\phi_{1t}, E_t \phi_{1t+j}) + g_2 c_2 \operatorname{cov}(\phi_{2t}, E_t \phi_{2t+j}) > 0.$$

Assuming positive autocorrelation in the factors, if $g_1 c_1 < 0$ we must have $g_2 c_2 > 0$. In fact, we require:

(28) $$g_2 c_2 \frac{\operatorname{cov}(\phi_{2t}, E_t \phi_{2t+j})}{\operatorname{cov}(\phi_{1t}, E_t \phi_{1t+j})} > -g_1 c_1.$$

Comparing (27) and (28), we need to have:

$$\frac{\operatorname{var}(\phi_{2t})}{\operatorname{var}(\phi_{1t})} < \frac{-g_1 c_1}{g_2 c_2} < \frac{\operatorname{cov}(\phi_{2t}, E_t \phi_{2t+j})}{\operatorname{cov}(\phi_{1t}, E_t \phi_{1t+j})}, \text{ or}$$

(29) $$1 < \frac{-g_1 c_1 \operatorname{var}(\phi_{1t})}{g_2 c_2 \operatorname{var}(\phi_{2t})} < \frac{\operatorname{cov}(\phi_{2t}, E_t \phi_{2t+j}) / \operatorname{var}(\phi_{2t})}{\operatorname{cov}(\phi_{1t}, E_t \phi_{1t+j}) / \operatorname{var}(\phi_{1t})}.$$

We can conclude that two minimal conditions have to be met for the covariance of the interest differential with the period *j* expected depreciation to change signs. First, the interest rate and the risk premium have to load with opposite signs on one factor and the same sign on the other factor, and second, the factor on which the loadings are the same sign has to be more persistent.

It is important to emphasize that there is no condition on the parameters (c_1, c_2, g_1, g_2) that holds for all values of the variances and autocovariances of the factors, ϕ_{1t} and ϕ_{2t} that would deliver both $\text{cov}(r_t^d, \lambda_t) < 0$ and $\text{cov}(r_t^d, E_t\lambda_{t+j}) > 0$. If $\text{cov}(r_t^d, \lambda_t) < 0$ for all possible values of $\text{var}(\phi_{1t})$ and $\text{var}(\phi_{2t})$, then we must have $g_1 c_1 < 0$ and $g_2 c_2 < 0$. Even assuming the factors are positively serially correlated, $\text{cov}(r_t^d, E_t\lambda_{t+j}) > 0$ requires the opposite configuration, $g_1 c_1 > 0$ and $g_2 c_2 > 0$, if $\text{cov}(r_t^d, E_t\lambda_{t+j}) > 0$ is to be true for all possible values of $\text{cov}(\phi_{1t}, E_t\phi_{1t+j})$ and $\text{cov}(\phi_{2t}, E_t\phi_{2t+j})$. If the signs of the parameters are derived from properties of the utility function and the moments of the factors as derived from the stochastic processes of economic variables (as is true in the utility-based models we consider below), the resolution of the empirical facts requires a specific relation between utility parameters and the moments of economic variables, as expressed in equation (29). At best, a utility-based model will explain the anomalies for specific economies that happen to have stochastic processes for the factors that bear the appropriate relation to the utility parameters.

As an example, if we define $\sigma_1^2 = \text{var}(v_{1,t+1})$ and $\sigma_2^2 = \text{var}(v_{2,t+1})$, and then assume:

(30) $\phi_{1t+1} = \rho_1 \phi_{2t} + v_{1t+1}$ and $\phi_{2t+1} = \rho_2 \phi_{2t} + v_{2t+1}$,

we get:

$$\text{cov}(r_t, E_t\lambda_{t+j}) = \text{cov}(c_1\phi_{1t} + c_2\phi_{2t}, g_1\rho_1^j\phi_{1t} + g_2\rho_2^j\phi_{2t}) = \rho_1^j g_1 c_1 \sigma_1^2 + \rho_2^j g_2 c_2 \sigma_2^2 .$$

Then:

(31) $$\text{cov}(r_t, \Lambda_t) = \frac{1}{1-\rho_1} g_1 c_1 \sigma_1^2 + \frac{1}{1-\rho_2} g_2 c_2 \sigma_2^2 ,$$

which according to the empirical work is > 0.

Assuming $g_1 c_1 < 0$, to get $\text{cov}(r_t, \Lambda_t) > 0$, we need at least $a_2 c_2 > 0$,. From (27) that $\frac{\sigma_2^2}{\sigma_1^2} < -\frac{a_1 c_1}{a_2 c_2}$, but from (31) we have $\frac{1-\rho_1}{1-\rho_2}\frac{\sigma_2^2}{\sigma_1^2} > -\frac{a_1 c_1}{a_2 c_2}$. A necessary condition is $\rho_2 > \rho_1$ for this to be satisfied.

We turn now to utility-based models of the risk premium, first recapping the basic theory.

3.2 *Background on the foreign exchange risk premium*

Here we briefly review the basic theory of foreign exchange risk premiums. See, for example, Backus et. al. (2001) or Brandt et. al. (2004).

Let M_{t+1} be the stochastic discount factor for returns in units of Home consumption – the intertemporal marginal rate of substitution between t+1 and t for Home. Let M_{t+1}^* be the corresponding discount factor for returns denominated in units of Foreign consumption. Lower case is log of upper case.

The first-order condition for Home agents for the riskless Home real asset is

(32) $1 = e^{r_t} E_t M_{t+1}$.

Under log normality

$$E_t e^{m_{t+1}} = e^{E_t m_{t+1} + \frac{1}{2}\text{var}_t\, m_{t+1}}, \text{ which gives us}$$

(33) $r_t = -E_t m_{t+1} - \frac{1}{2}\text{var}_t\, m_{t+1}$

We also have

(34) $1 = e^{r_t^*} E_t M_{t+1} D_{t+1}$,

(where $D_{t+1} = Q_{t+1} / Q_t$), and so we get:

(35) $r_t^* = -E_t m_{t+1} - E_t d_{t+1} - \frac{1}{2}\text{var}_t\, m_{t+1} - \frac{1}{2}\text{var}_t\, d_{t+1} - \text{cov}_t(m_{t+1}, d_{t+1})$.

Taking differences, we get

(36) $\lambda_t = r_t^* - r_t + E_t d_{t+1} = \frac{1}{2}\text{var}_t\, d_{t+1} + \text{cov}_t(m_{t+1}, d_{t+1})$.

Following similar steps for the Foreign investor, using the discount factor for Foreign units, we get:

(37) $\lambda_t = r_t^* - r_t + E_t d_{t+1} = -\frac{1}{2}\text{var}_t\, d_{t+1} + \text{cov}_t(m_{t+1}^*, d_{t+1})$.

Equation (36) must hold for Home agents, and (37) for Foreign agents. The intuition of the foreign exchange risk premium, however, is complicated by the Jensen's inequality terms involving $\text{var}_t\, d_{t+1}$, which enter the two equations with opposite signs because Home and Foreign households evaluate returns in different units (Home in terms of the Home consumption basket, Foreign in units of the Foreign consumption basket.) Intuition is aided by taking a simple average of these two equations:

(38) $$\lambda_t = \text{cov}_t\left(\frac{m_{t+1} + m^*_{t+1}}{2}, d_{t+1}\right).$$

As with any asset, the excess return is determined by the covariance of the return with the stochastic discount factor. If the foreign exchange return, d_{t+1} is positively correlated with the Home discount factor, m_{t+1}, Home investors require a compensation for risk so the Foreign security has an excess return relative to the Home bond. For Foreign investors, the foreign exchange return on a Home bond is $-d_{t+1}$. If $-d_{t+1}$ is positively correlated with the Foreign discount factor, m^*_{t+1}, the Home asset is relatively risky, implying a lower excess return on the Foreign bond.

This simplifies under complete markets. Then

(39) $$d_{t+1} = m^*_{t+1} - m_{t+1}.$$

As Brandt et. al. (2004), for example, explain, relation (39) also holds under incomplete markets as long as we interpret m_{t+1} and m^*_{t+1} as the minimum variance (log) discount factors That is, an infinite number of discount factors satisfy the first-order conditions given by (32) and (34) and their Foreign counterparts. But all of those discount factors can be set equal to the respective minimum variance discount factors, m_{t+1} or m^*_{t+1}, plus white-noise error terms.

Substituting into equation (38), we get:

(40) $$\lambda_t = \tfrac{1}{2}(\text{var}_t\, m_{t+1} - \text{var}_t\, m^*_{t+1}).$$

The risk premium is determined for the investor with minimum-variance discount factor by the covariance of the real exchange rate with the discount factor, but the log of the real rate of depreciation in turn is equal to the log of the relative discount factors, giving us (40).

Following the discussion in section 3.1, we can see that a delicate set of conditions on the stochastic discount factors must be met in order to generate our finding that $\text{cov}(r_t^d, \lambda_t) < 0$ and $\text{cov}(r_t^d, \Lambda_t) > 0$. Using (33), (35), and (39) (or simply (33) and its foreign counterpart) we have:

(41) $$r_t^d = -(E_t m_{t+1} - E_t m_{t+1}^*) - \frac{1}{2}(\text{var}_t\, m_{t+1} - \text{var}_t\, m_{t+1}^*) .$$

Considering (40) and (41), it is clear that a model that focuses on the behavior of the stochastic discount factors must produce appropriate behavior for the variances of the logs of the discount factors. Almost all models in the literature treat the Home and Foreign countries as symmetric, because the empirical findings on the uncovered interest parity puzzle are so ubiquitous and do not seem to depend on the economic characteristics of a particular country. In that case, a two-factor model of the risk premium would take the form:

(42) $$\lambda_t = \tfrac{1}{2}(\text{var}_t\, m_{t+1} - \text{var}_t\, m_{t+1}^*) = g_1\phi_{1t} + g_2\phi_{2t} .$$

The factors ϕ_{1t} and ϕ_{2t} derive from shocks to the variances of the stochastic discount factors. We might have, for example, $\phi_{1t} = u_t - u_t^*$ and $\phi_{2t} = v_t - v_t^*$, where u_t and v_t are factors driving $\text{var}_t\, m_{t+1}$ while u_t^* and v_t^* determine $\text{var}_t\, m_{t+1}^*$. We might have correlation between u_t and u_t^* or between v_t and v_t^*, but u_t and v_t (and u_t^* and v_t^*) are mutually independent. The parameters g_1 and g_2 in (42) are determined in the model and will depend in part on taste parameters. Without loss of generality, assume $g_1, g_2 > 0$.

For there to be any correlation between r_t^d and λ_t, it is necessary that $E_t m_{t+1} - E_t m_{t+1}^*$ have some correlation with ϕ_{1t} and ϕ_{2t}. Write the model as:

(43) $$\delta_t = -(E_t m_{t+1} - E_t m_{t+1}^*) = \ell_t + a_1\phi_{1t} + a_2\phi_{2t} .$$

ℓ_t aggregates the factors that drive the expected rate of depreciation that are independent of ϕ_{1t} and ϕ_{2t}. Taste parameters help to determine a_1 and a_2.

Since $r_t^d = \delta_t - \lambda_t$, then $\text{cov}(r_t^d, \lambda_t) < 0$ requires $(a_1 - g_1)g_1 \text{var}(\phi_{1t}) + (a_2 - g_2)g_2 \text{var}(\phi_{2t}) < 0$. For this relationship to hold for all realizations of the variances of the factors, we must have $a_1 - g_1 < 0$ and $a_2 - g_2 < 0$. But this configuration of parameters would not admit the possibility that $\text{cov}(r_t^d, \Lambda_t) > 0$. In order to have both

$\text{cov}(r_t^d, \lambda_t) < 0$ and $\text{cov}(r_t^d, \Lambda_t) > 0$, there needs to be a particular configuration of the parameters and a relationship between the parameters and the persistence of the factors (see section 3.1). A necessary condition is that $a_1 - g_1$ and $a_2 - g_2$ have opposite signs. Assume without loss of generality that $a_1 - g_1 < 0$ and $a_2 - g_2 > 0$. Then, additional requirements are that the innovation variance in ϕ_{1t} be sufficiently larger than the innovation variance of ϕ_{2t} and that ϕ_{2t} be sufficiently more persistent than ϕ_{1t} (see (29).)

The approach to explaining the data puzzles examined in this section relies on some relationships that hold under very general conditions. As Cochrane (2005) and Cochrane et. al. (2006) emphasize, the first of the two key building blocks, equations (32) and (34), are simply no-arbitrage conditions. They do not require rational expectations or efficient markets, for example. Violations of those assumptions only affect the stochastic discount factors. The second building block, (39), could be a strong condition if that were interpreted to hold for all Home and Foreign agents, because it requires complete markets. But if we reinterpret the stochastic discount factors to refer to particular agents – the ones with minimum-variance stochastic discount factors – then the underlying building blocks of this model are quite general. As Backus et. al. (2010) say, "It is almost a tautology that we can represent exchange rates as ratios of nominal pricing kernels in different currency units: It is less a tautology that we can write down sensible stochastic processes for variables that are consistent with the carry trade evidence." Even harder is to reconcile both the carry trade evidence and the evidence on the level risk premium. Focusing only on the stochastic discount factors and their statistical properties may be an unproductive approach, or at least an approach that does not provide enough insight. The required conditions are delicate and necessitate some strong restrictions between the parameters of preferences (which help determine the coefficients g_1, g_2, a_1 and a_2) and the dynamic behavior of the economy (which helps determine the volatility of the stochastic discount factors, as determined by ϕ_{1t} and ϕ_{2t}.)

3.3 Models of foreign exchange risk premium based on the stochastic discount factor

In this section, we examine models of the risk premium, λ_t, that are based on specifications of the stochastic discount factors derived from underlying models of preferences.

In particular, Verdelhan (2010) builds a model based on the Campbell-Cochrane (1999) specification of external habit persistence to explain the familiar uncovered interest parity puzzle. Also, Bansal and Shaliastovich (2010) and Backus, Gavazzoni, Telmer and Zin (2010) demonstrate how the "long-run risks" model based on Epstein-Zin (1989) preferences can account for the forward-premium anomaly. We will see, however, that neither model is able to account for the empirical findings of Section 2 of this paper.

Before proceeding to models of the discount factor based explicitly on a specification of preferences, it is worthwhile to highlight some points demonstrated by Backus, Foresi, and Telmer's (2001) study of affine pricing models. The models that are considered there encompass those of, for example, Frachot (1996) and Bakshi and Chen (1997). Backus et. al. take an adaptation of Duffie and Kan's (1996) general class of affine models to a discrete-time setting appropriate for examining the interest parity puzzle. The starting point is the specification of state variables, which are assumed to follow independent stochastic processes:

(44) $$z_{it+1} = (1-\varphi_i)\theta_i + \varphi_i z_t + \sigma_i z_{it}^{1/2}\varepsilon_{it+1},$$

where $0 < \varphi_i < 1$, θ_i, and $\varepsilon_{it} \sim NID(0,1)$. In turn, the log of the discount factors, m_{t+1} and m^*_{t+1}, are assumed to be linear functions of these state variables.

Backus et. al. demonstrate the difficulties in explaining for the interest parity anomaly in this setting. They show an impossibility result – if we insist that interest rates are always positive, then we cannot account for the negative correlation of d_{t+1} and r^d_t. Moreover, if we are willing to allow for a small probability of negative interest rates, these models still have troubles matching such things as the unconditional distribution of interest rates and exchange rates, and the slope of the yield curve.

The point to be made here is that the models that Backus et. al. consider that are able to match the interest parity puzzle are still one-factor models of the risk premium, so they run afoul of the problems discussed earlier in this section. This is true even though they consider models in which the discount factors and the variances of the discount factors are multi-factor.

First, they lay out an "independent factors" model of the discount factors:

(45) $$-m_{t+1} = (1+\lambda_0^2/2)z_{0t} + (-1+\lambda_1^2/2)z_{1t} + \lambda_0 z_{0t}^{1/2}\varepsilon_{0t+1} + \lambda_1 z_{1t}^{1/2}\varepsilon_{1t+1}$$

(46) $$-m^*_{t+1} = (1+\lambda_0^2/2)z_{0t} + (-1+\lambda_2^2/2)z_{2t} + \lambda_0 z_{0t}^{1/2}\varepsilon_{0t+1} + \lambda_2 z_{2t}^{1/2}\varepsilon_{1t+1}$$

The interest differential is given by $r_t^d = z_{2t} - z_{1t}$. The risk premium is $\lambda_t = (\lambda_1^2/2)z_{1t} - (\lambda_2^2/2)z_{2t}$. Backus et. al. consider the symmetric case of $\lambda_1 = \lambda_2$, which gives us $\lambda_t = [(\lambda_1^2 - \lambda_2^2)/2](z_{1t} - z_{2t})$. In this case, effectively there is a single factor driving the interest differential and the risk premium, $z_{1t} - z_{2t}$, which we have seen cannot reproduce our finding that $\text{cov}(r_t^d, \lambda_t) < 0$ and $\text{cov}(r_t^d, \Lambda_t) > 0$.

If the symmetry assumption is not imposed then we can view this as a genuine two-factor model of the interest differential and risk premium. However, this model still cannot account for $\text{cov}(r_t^d, \Lambda_t) > 0$. In order for that covariance to be positive, we need at least for some horizons j that $\text{cov}(r_t^d, E_t\lambda_{t+j}) > 0$. However, in this model,

$\text{cov}(r_t^d, E_t\lambda_{t+j}) = -(\lambda_1^2/2)\varphi_1^j z_{1t}^2 - (\lambda_2^2/2)\varphi_2^j z_{2t}^2 < 0$.

The second model considered is the "interdependent factors" model:

(47) $$-m_{t+1} = (1+\lambda_1^2/2)z_{1t} + (\gamma^* + \lambda_2^2/2)z_{2t} + \lambda_1 z_{1t}^{1/2}\varepsilon_{1t+1} + \lambda_2 z_{2t}^{1/2}\varepsilon_{2t+1}$$

(48) $$-m_{t+1}^* = (\gamma^* + \lambda_2^2/2)z_{1t} + (1+\lambda_1^2/2)z_{2t} + \lambda_2 z_{1t}^{1/2}\varepsilon_{1t+1} + \lambda_1 z_{2t}^{1/2}\varepsilon_{2t+1}.$$

The interest differential from this model is $(1-\gamma^*)(z_{1t} - z_{2t})$, and the risk premium is $\lambda_t = [(\lambda_1^2 - \lambda_2^2)/2](z_{1t} - z_{2t})$. Again, this is effectively a one-factor model for the interest differential and risk premium, so cannot account for our empirical results.

Even though these affine models are relatively unconstrained in the sense that they do not generate the stochastic discount factors from an underlying equilibrium model of utility maximization, they still are still inconsistent with having both $\text{cov}(r_t^d, \lambda_t) < 0$ and $\text{cov}(r_t^d, \Lambda_t) > 0$. Even though more than one factor drives the discount factor, we may still have a single factor driving the interest differential and risk premium. Moreover, when this formulation does allow for a multi-factor model of the interest differential and risk premium, the sign restrictions on the parameters still produce a model unable to account for the empirical puzzles raised here.

Ultimately, we would like to understand the excess return in the context of an equilibrium model, in which the discount factor is generated from the marginal utility of household/investors. Two recent papers accomplish this goal using non-standard preferences. Verdelhan (2010) builds a model of the stochastic discount factor based on Campbell-Cochrane preferences, and Bansal and Shaliastovich (2009) and Backus, Gavazzoni, Telmer and Zin (2010) show how a model of the discount factor based on Epstein-Zin preferences can explain the uncovered interest parity puzzle.

These papers directly extend equilibrium closed-economy models to a two-country open-economy setting. The closed economy models assume an exogenous stream of endowments, with consumption equal to the endowment. The open-economy versions assume an exogenous stream of consumption in each country. These could be interpreted either as partial equilibrium models, with consumption given but the relation between consumption and world output unmodeled. Or they could be interpreted as general equilibrium models in which each country consumes an exogenous stream of its own endowment and there is no trade between countries. Under the latter interpretation, the real exchange rate is a shadow price, since in the absence of any trade in goods, there can be no trade in assets that have any real payoff.

In both studies, the consumption streams are taken to follow unit root processes, as in the closed economy analogs, but with no assumption of cointegration. That implies that relative consumption levels and real exchange rates have unit roots, implications which do not have strong empirical support. However, relative real interest differentials and excess returns in these models are stationary. The moments that are of concern to us, $\text{cov}(r_t^d, \lambda_t)$ and $\text{cov}(r_t^d, \Lambda_t)$, are well defined, and the analysis of the necessary conditions for $\text{cov}(r_t^d, \lambda_t) < 0$ and $\text{cov}(r_t^d, \Lambda_t) > 0$ of section 3.1 applies.

In Verdelhan (2010) there are two symmetric countries. The objective of Home household *i* is to maximize

(49) $$E_t \sum_{j=0}^{\infty} \beta^j (C_{i,t+j} - H_{t+j})^{1-\gamma} / (1-\gamma),$$

where γ is the coefficient of relative risk aversion, and H_t represents an external habit. H_t is defined implicitly by defining the "surplus", $s_t \equiv \ln\left((C_t - H_t)/C_t\right)$, where C_t is aggregate consumption, and s_t is assumed to follow the stochastic process:

(50) $$s_{t+1} = (1-\phi)\overline{s} + \phi s_t + \mu(s_t)(c_{t+1} - c_t - g).$$

Here, ϕ and $\overline{s}$ are parameters, and $c_t \equiv \ln(C_t)$ is assumed to follow a simple random walk:

(51) $c_{t+1} = g + c_t + u_{t+1}$, where $u_{t+1} \sim i.i.d.\, N(0,\sigma^2)$.

$\mu(s_t)$ represents the sensitivity of the surplus to consumption growth, and is given by:

(52) $\mu(s_t) \equiv \frac{1}{\overline{S}}\sqrt{1-2(s_t - \overline{s})} - 1$, when $s_t \le s_{\max}$, 0 elsewhere.

The log of the stochastic discount factor is given by:

(53) $m_{t+1} = \ln(\beta) - \gamma\left[g + (\phi - 1)(s_t - \overline{s}) + (1+\mu(s_t))(c_{t+1} - c_t - g)\right]$

When the parameters $\overline{S}$ and $s_{\max}$ are suitably normalized, Verdelhan shows we can write the Home relative to Foreign interest differential as:

(54) $r_t^d = -B(s_t - s_t^*)$,

where s_t^* is the foreign surplus, and $B \equiv \gamma(1-\phi) - (\gamma^2\sigma^2 / \overline{S}^2)$. The excess return is given by:

(55) $\lambda_t = -(\gamma^2\sigma^2 / \overline{S}^2)(s_t - s_t^*)$.

If the parameters are such that $B < 0$, then this model can account for the empirical finding of $\text{cov}(r_t^d, \lambda_t) < 0$. However, since r_t^d and λ_t are proportional to a single factor, $s_t - s_t^*$, then the model will not simultaneously be able to produce $\text{cov}(r_t^d, \Lambda_t) > 0$, as long as interest rates are positively serially correlated.

Bansal and Shaliastovich (2010) apply the "long-run risks" model to the uncovered interest parity puzzle. In each country, households are assumed to have Epstein-Zin (1989) preferences. The Home agent's utility is defined by the recursive relationship:

(56) $U_t = \left\{(1-\beta)C_t^{\rho} + \beta\left[E_t\left(U_{t+1}^{\alpha}\right)^{\rho/\alpha}\right]\right\}^{1/\rho}$.

In this relationship, β measures the patience of the consumer, $1-\alpha$ is the degree of relative risk aversion, and $1/(1-\rho)$ is the intertemporal elasticity of substitution. Bansal and Shaliastovich focus on the case of $\alpha < \rho$, which corresponds to the case in which agents prefer an early resolution of risk, and in which the intertemporal elasticity of substitution is greater than one, $0 < \rho < 1$.

Like Verdelhan, Bansal and Shaliastovich assume an exogenous path for consumption in each country. In the Home country (with $c_t \equiv \ln(C_t)$):

(57) $c_{t+1} - c_t = \mu + l_t + \sqrt{u_t}\,\varepsilon_{t+1}^x$.

The conditional expectation of consumption growth is given by $\mu + l_t$. The component l_t represents a persistent consumption growth modeled as a first-order autoregression:

(58) $$l_{t+1} = \varphi_l l_t + \sqrt{w_t}\varepsilon^l_{t+1}.$$

Conditional variances are stochastic and follow first-order autoregressive processes:

(59) $$u_{t+1} = (1-\varphi_u)\theta_u + \varphi_u u_t + \sigma_u \varepsilon^u_{t+1}$$

(60) $$w_{t+1} = (1-\varphi_w)\theta_w + \varphi_w w_t + \sigma_w \varepsilon^w_{t+1}.$$

The innovations, ε^x_{t+1}, ε^l_{t+1}, ε^u_{t+1}, and ε^w_{t+1} are assumed to be uncorrelated within each country, distributed $i.i.d.\ N(0,1)$, but each shock may be correlated with its Foreign counterpart.

When the first-order conditions are log-linearized (as in Hansen, Heaton, and Li (2005)), the following expression emerges for the log of the stochastic discount factor:

(61) $$-m_{t+1} = \delta + \gamma^r_l l_t + \gamma^r_u u_t + \gamma^r_w w_t + \lambda^r_x \sqrt{u_t}\varepsilon^x_{t+1} + \lambda^r_l \sqrt{w_t}\varepsilon^l_{t+1} + \lambda^r_u \sigma_u \varepsilon^u_{t+1} + \lambda^r_w \sigma_w \varepsilon^w_{t+1}.$$

The parameters in this log-linearization take some space to define, but these parameters are crucial) for this model's ability to explain the data (with the exception of δ, whose complicated definition we skip):

$$\gamma^r_l = 1-\rho \qquad \gamma^r_u = \alpha(\alpha-\rho)/2 \qquad \gamma^r_w = \alpha(\alpha-\rho)\omega_l^2/2$$

$$\lambda^r_x = 1-\alpha \qquad \lambda^r_l = -(\alpha-\rho)\omega_l \qquad \lambda^r_u = -(\alpha-\rho)\omega_u \qquad \lambda^r_w = -(\alpha-\rho)\omega_w$$

$$\omega_l = \beta/(1-\beta\varphi_l) \qquad \omega_u = \alpha\beta/2(1-\beta\varphi_u) \qquad \omega_w = \alpha\beta\omega_l^2/2(1-\beta\varphi_w)$$

Bansal and Shaliastovich assume that the long-run expected growth component of consumption, l_t, is the same in the Home and Foreign countries on the grounds that long-run growth prospects are nearly identical across countries.[16] Under this assumption, Bansal and Shaliastovich find:

(62) $$r^d_t = \left(\gamma^r_u - (\lambda^r_x)^2/2\right)(u_t - u^*_t)$$

(63) $$\lambda_t = \left((\lambda^r_x)^2/2\right)(u_t - u^*_t)$$

One configuration of parameters that will deliver the result that $\text{cov}(r^d_t, \lambda_t) < 0$ is that agents have a preference for early resolution of risk, $\alpha < \rho$, and that the intertemporal elasticity

[16] As noted above, Bansal and Shaliastovich do not assume cointegration of the consumption processes, so shocks to the level of consumption in each country result in permanent level differences.

of substitution is greater than one, which requires $0<\rho<1$. As Bansal and Shaliastovich (2010) explain, these parameter choices are needed in order for this model to account for variance asset pricing facts, such as the term structure of interest rates.[17] However, because r_t^d and λ_t are proportional to a single factor, $u_t-u_t^*$, the model cannot account for $\text{cov}(r_t^d,\Lambda_t)>0$ under the same assumptions on parameters.

Backus et. al. (2010) do not impose the restriction that long-run expected growth in the Home country, l_t, is identically equal to the corresponding variable in the Foreign country, l_t^*. In this case, equations (62) and (63) generalize:

(64) $$r_t^d=\gamma_l^r(l_t-l_t^*)+\left(\gamma_u^r-(\lambda_x^r)^2/2\right)(u_t-u_t^*)+\left(\gamma_w^r-(\lambda_l^r)^2/2\right)(w_t-w_t^*)$$

(65) $$\lambda_t=\left((\lambda_x^r)^2/2\right)(u_t-u_t^*)+\left((\lambda_l^r)^2/2\right)(w_t-w_t^*).$$

Our discussion in section 3.1 shows that a model in which r_t^d and λ_t are determined by at least two factors has the potential for reconciling the findings that $\text{cov}(r_t^d,\lambda_t)<0$ and $\text{cov}(r_t^d,\Lambda_t)>0$. A necessary condition is that the coefficients on one of the factors has the same sign in the solutions for r_t^d and λ_t, and the coefficients are of opposite sign on the other factor. But this condition is not satisfied here. Clearly in (65), both $u_t-u_t^*$ and $w_t-w_t^*$ have positive effects on the risk premium. Under the sign restrictions $\alpha<\rho$ and $0<\rho<1$, the coefficients on $u_t-u_t^*$ and $w_t-w_t^*$ are both negative in equation (64) for the real interest differential. This can be seen because $\gamma_u^r-(\lambda_x^r)^2/2=-(1-2\alpha+\alpha\rho)/2<0$ and $\gamma_w^r-(\lambda_l^r)^2/2=-\rho(\rho-\alpha)\omega_l^2/2<0$.

Even though the long-run risks model is a flexible model of preferences, under the set of parameter restrictions that have been used to account for other asset-pricing puzzles, the model cannot handle our finding that $\text{cov}(r_t^d,\lambda_t)<0$ and $\text{cov}(r_t^d,\Lambda_t)>0$. Effectively, this model, like Verdelhan's, is constructed so that any shock that increases the variance of the Home discount factor will lower the Home interest rate. These shocks also, by equation (40), must increase the risk premium. So the models are constructed in a way that they are able to explain the familiar uncovered interest parity puzzle, $\text{cov}(r_t^d,\lambda_t)<0$. But for the models to account for

[17] See Bansal and Yaron (2004).

$\text{cov}(r_t^d, \Lambda_t) > 0$, there must be a component of the variance of the Home discount factor that covaries positively with the Home real interest rate. Referring to equations (42) and (43) above, and the subsequent discussion, the model of Backus et. al. (2010) does not satisfy the sign restrictions on the factors driving r_t^d and λ_t that are necessary conditions for explaining both the uncovered interest parity puzzle and the level risk premium evidence.

4. Other Issues

4.1 Whose price index?

The empirical approach taken in section 2 requires taking a stand on the appropriate price index used to deflate nominal returns for the Home and Foreign investor. In each country, we deflated nominal returns using the consumer price index measure of inflation. The theory of the risk premium discussed in section 3.3, however, applies to the mythical investor that has the minimum variance stochastic discount factor. If markets are not complete, it is not possible to identify that investor, so perhaps it is too presumptuous to assume these investors deflate using the CPI of their respective countries. That is, perhaps our empirical approach has gone astray just because it is not using the appropriate price deflator.

However, Engel (1993,1999) presents evidence that there is very little within-country variation in prices compared to the variation of the real exchange rate, at least for the U.S. relative to other advanced countries. The real exchange rate is given by $q_t = s_t + p_t^* - p_t$. In turn each log price index is a weighted average of individual consumer goods prices: $p_t = \sum_{i=1}^{N} w_i p_{it}$, $p_t^* = \sum_{i=1}^{N} w_i^* p_{it}^*$. The papers show, in essence, that there is very high correlation between $s_t + p_{it}^* - p_{it}$ for almost all goods, and these are very highly correlated with q_t. On the other hand, relative prices of goods within a country, $p_{it} - p_{jt}$, generally have much lower variance than $s_t + p_{it}^* - p_{it}$. The implication is that if we consider price indexes that use different weights than the CPI weights, the constructed real exchange rate will still be highly correlated with q_t.

This suggests that there probably is not much to be gained by ascribing some other price index to the investor with the minimum variance stochastic discount factor. That is, changing the weights on the goods in the price index is unlikely to have much effect on the measurement of real returns on Home and Foreign assets for Home and Foreign investors.

4.2 The method when real exchange rates are non-stationary

If the real exchange rate is non-stationary, the empirical method used here can be adapted. The forward iteration that is the foundation of the empirical study, $q_t - \lim_{j\to\infty}\left(E_t q_{t+j}\right) = -R_t - \Lambda_t$, does not require that the real exchange rate be stationary. Instead, we could measure $\lim_{j\to\infty}\left(E_t q_{t+j}\right)$ as the permanent component of the real exchange rate. The level risk premium, Λ_t, could then be constructed as the sum of the prospective real interest rate, R_t, and the transitory component of the real exchange rate, $q_t - \lim_{j\to\infty}\left(E_t q_{t+j}\right)$.

Section 2.1 presented evidence that the real exchange rate is stationary, so there is no permanent component. Another approach, potentially, is to measure the permanent component using the Beveridge-Nelson (1981) decomposition, or some related method.[18] However, Engel (2000) discusses the problem of near observational equivalence of stationary and non-stationary representations of the real exchange rate. Suppose the real exchange rate is the sum of a pure random walk component, ω_t, and a transitory component, ρ_t. Engel (2000) argues, based on an economic model and evidence from disaggregated prices, that it is plausible that U.S. real exchange rates contain a transitory component that itself is very persistent (though stationary) and very volatile (high innovation variance.) There may be a random walk component related to the relative price of nontraded goods, but this component has a low innovation variance. The transitory component, ρ_t, dominates the forecast variance of real exchange rates even for reasonably long horizons because it is so persistent and volatile.

In this case, there are two dangers in trying to separate a transitory component from the permanent component. On the one hand, even if the real exchange rate were stationary, so that $\omega_t = 0$, the econometrician may not reject a random walk because of the high persistence of ρ_t. The permanent-transitory decomposition might mistakenly determine that there is a permanent

[18] See Morley, Nelson, and Zivot (2003) for a discussion of the relationship between the Beveridge-Nelson decomposition and more restrictive state-space decompositions.

component that accounts for most of the variation of the real exchange rate, with little role for a transitory component.

The other danger is the opposite – that the econometrician uses powerful enough methods to detect the stationarity of ρ_t, but does not tease out the random-walk component, ω_t. In this case, the econometrician might conclude that all movements in the real exchange rate are transitory (though quite persistent).

We have rejected a unit root in the real exchange rate, and so conclude that there is only a ρ_t component. However, if the ω_t component has such innovation variance that it is undetectable, then for our purposes it is reasonable to measure the transitory component, ρ_t, by the actual real exchange rate. If Engel's (2000) characterization of U.S. real exchange rate dynamics is correct, then there is not much to be gained by trying to undertake a permanent-transitory decomposition, and analyzing the transitory component rather than the actual real exchange rate.

4.3 The term structure

There are two possible ways to see connections between this study and studies of the term structure. First, is there a relationship between the findings here, and those of Alexius (2001) and Chinn and Meredith (2004) that interest-parity holds better at long horizons using long-term interest differentials? (Note that Bekaert, Wei, and Xing (2007) do not find evidence to support this claim.) The answer is no, not directly. Our study does not derive any relationship between long-term interest rates and exchange rate changes. It is critical to realize that the prospective real interest rate, R_t, is not a long-term rate but instead an infinite sum of expected short-term rates. The two differ because long-term interest rates incorporate a term premium. Our study has not offered any insights into the relationship between the term premium and exchange rates.[19]

Another connection is that there is an analogy to the uncovered interest parity puzzle in the term structure literature. The long-short yield differential can predict excess returns. However, the literature on the term structure does not have evidence such as that in Figure 2 – that the expected excess return reverses signs at some horizon. We can draw an analogy between

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[19] To be sure, many papers, such as those of Verdelhan (2010) and Bansal and Shaliastovich (2010) build models that are meant to account for both the term premium and the uncovered interest parity puzzle.

the Fama regression for exchange rates and a version of the empirical work that establishes the term structure anomaly. Let $p_{t,n}$ be the log of the price of a bond with *n* periods to maturity at time *t*, that has a prayoff of one (in levels) at maturity. If an investor holds that bond for one period, the return is $p_{t+1,n-1} - p_{t,n}$. The expected excess return is given by $\lambda_t^b = E_t p_{t+1,n-1} - p_{t,n} - r_t$, where r_t is the return on a one-period bond. The yield to maturity of the bond with *n* periods to maturity is given by $y_{t,n} = -p_{t,n} / n$. Then consider the regression:

(66) $$y_{t+1,n-1} - y_{t,n} - = \alpha + \beta \frac{1}{n-1}(y_{t,n} - r_t) + u_{t+1}.$$

If the expectations hypothesis of the term structure held, we would find $\alpha = 0$ and $\beta = 1$. Instead, the empirical literature tends to find $\beta < 1$, and sometimes $\beta < 0$.[20] Equation (66) is equivalent to:

(67) $$p_{t+1,n-1} - p_{t,n} - r_t = \alpha' + (1-\beta)(y_{t,n} - r_t) + u_{t+1}',$$

where $\alpha' = -\alpha(n-1)$, and $u_t' = -(n-1)u_t$. The expected value at time *t* of the left-hand side of this regression is λ_t^b, so when $\beta < 1$ which implies $\text{cov}(r_t - y_{t,nt}, \lambda_t^b) < 0$. This is analogous to the finding in the foreign exchange literature that $\text{cov}(r_t^d, \lambda_t) < 0$.

We can rewrite the equation for the risk premium and iterate forward to get:

(68) $$p_{t,n} = -E_t \sum_{j=0}^{n-1} r_{t+j} - E_t \sum_{j=0}^{n-1} \lambda_{t+j}^b .$$

The equivalent to our finding in foreign exchange markets that $\text{cov}(r_t^d, \Lambda_t) > 0$ would be evidence that $\text{cov}(r_t - y_{t,nt}, E_t \sum_{j=0}^{n-1} \lambda_{t+j}^b) > 0$. Just as $\text{cov}(r_t^d, \Lambda_t) > 0$ requires $\text{cov}(r_t^d, E_t \lambda_{t+j}) > 0$ for some *j*, a necessary condition for $\text{cov}(r_t - y_{t,nt}, E_t \sum_{j=0}^{n-1} \lambda_{t+j}^b) > 0$ is that $\text{cov}(r_t - y_{n,t}, E_t \lambda_{t+j}^b) > 0$ for some *j*. However, so far such evidence has not been established in the term structure literature.

[20] See for example Campbell and Shiller (1991) and Dai and Singleton (2002).

5. Conclusions

When a country's interest rate is relatively high, the country's short-term bonds tend to have a higher return than their counterpart in another country, at least among the U.S.-G7 country pairs. At the same time, when that country's interest rate is high, its currency tends to be very strong – stronger than would be implied under uncovered interest parity. The first empirical regularity suggests that the returns on the high-interest rate currency incorporate a risk premium, but the second empirical finding suggests the opposite – that the overall effect of currency risk is to strengthen the high-interest rate currency. These two facts are difficult to understand by looking only at the behavior of a particular risk-averse agent in each country. Section 3 demonstrates the difficulties to this approach.

There have been several recent studies that attempt to build theoretical models to account for the uncovered interest parity that do not rely on modeling the preferences of agents, but instead model the interaction of more than one group of agents. These papers build models that are designed to explain the first empirical fact – the uncovered interest rate parity puzzle.

Frankel and Froot (1990) introduce a model with two groups of agents, chartists and fundamentalists, each following ad hoc behavioral rules. The chartists base their asset choice on extrapolation of the exchange rate returns from the past, while the fundamentalists form their expectations based on a model of macroeconomic fundamentals. Neither group of agents have expectations that are rational – the fundamentalists expectations do not take into account the effect of the chartists on the equilibrium exchange rate.

Alvarez, Atkeson and Kehoe (2008) build a two-country model in which the fraction of agents in each country that participate in financial markets varies over time. In each country, agents are constrained to use their own local currency to purchase their consumption good which is also produced locally. In asset markets, agents can trade interest earning bonds denominated in their own currency for interest earning foreign currency bonds. They can also trade currencies in the asset market. The household is split into two parts – one that deals in the goods market and the other that deals in the asset market. For a fixed cost each period, the household can transfer money from the asset market to the goods market to be used for goods purchases. The fraction of households that transfer money each period is endogenous in equilibrium, and depends on the money growth rate. The risk premium in this model is linked to the variance of the stochastic discount factor of the "active" households – the ones that have paid the fixed cost to transfer money between asset and goods markets.

The model of Bacchetta and van Wincoop (2010) in which some agents are slow to adjust their portfolio produces dynamics that accord with some of the empirical findings of this paper. In the model, there is a group of agents that does not change its portfolio when the state of the economy changes. Portfolios adjust only when new agents enter the financial market in the overlapping generation model developed here. When the Home short-term interest rate is high, the Home currency appreciates, but only new entrants to the market choose portfolios based on the higher short-term interest rate. Over time, more agents enter the market, and the currency then is expected to appreciate further for several months before depreciating.

None of these papers explicitly delineates how the level of the real exchange rate is related to interest rates because they are primarily concerned with the short-run excess returns. However, the mechanism in these papers seems intuitively to be incompatible with our findings that the high-interest rate currency has a low level risk premium. These models essentially rely on underreaction to explain the exchange rate dynamics in response to an interest rate increase. When the interest rate rises, one group of agents does not react – the chartists in Frankel and Froot, and the inactive agents in Alvarez et. al. and in Bacchetta and van Wincoop. The second group of agents does not take full advantage of the profit opportunity that might arise, either because this group is itself not fully rational (the fundamentalists in Frankel and Froot) or because it is risk averse (the active agents in Alvarez et. al. and Bachetta and van Wincoop.) These dynamics imply that in the short run, the currency appreciates less than it would if all agents were fully active. Apparently, the models cannot account for the fact that the currency initially appreciates more than it would under uncovered interest parity, which is a key finding of this paper. Bachetta and van Wincoop do compare the dynamics of the real exchange rate in their model compared to one in which all the portfolios are actively managed. The currency never appreciates in their model at any horizon as much as it would under active management, so the model does not deliver the key fact that the high-interest rate currency incorporates a level risk premium that strengthens the currency beyond its no-risk-premium level. Since none of the papers presents directly the implications of their models for the statistics derived in this paper, it is not clear how they perform in this regard, but at an intuitive level they do not seem well equipped to deal with the level risk premium puzzle.

Several recent papers have explored the implications for rare, large currency depreciations for the uncovered interest parity puzzle. Farhi and Gabaix (2009) present a full general equilibrium model of rare disasters and real exchange rates. Their model implies that when the Home real interest rate is high, the Home currency is weak in real terms, and so cannot account for the levels puzzle presented here.[21] This correlation occurs during "normal times" in their model – the anticipation of a future disaster leads to a simple positive correlation between the real interest differential and the real exchange rate. Nonetheless, there are two caveats that must be considered in light of Farhi and Gabaix and the related literature. The first is that if rare disasters are important, than the linear VAR technology used in this paper may not correctly capture the stochastic process for real exchange rates and real interest rates. Farhi et. al. (2009) and Burnside et. al. (2010a) extract information from options to infer expectations about rare large movements in exchange rates. Moreover, if these large rare events are important, then the lognormal approximations that lie behind our analysis of the risk premium in sections 3.3 and 3.4 are not correct. Higher order cumulants matter for the risk premium in that case.[22]

It may be that it is necessary to abandon the assumption that all agents have fully rational expectations. Some version of the model proposed by Hong and Stein (1999) may account for the empirical results uncovered here, which perhaps could be described as a combination of overreaction and momentum trading. That is, the short-term behavior of the real exchange rate under high interest rates incorporates overreaction in that the currency appreciates more than it would under interest parity. But perhaps momentum trading leads to expectations of further appreciation in the short run when the interest rate is high. Burnside et. al. (2010b), Gourinchas and Tornell (2004), Ilut (2010), Li and Tornell (2008) are recent approaches that have relaxed the assumption of full rationality in some way.

Finally, Table 3b may contain a clue toward a model that helps to resolve the empirical puzzles. In that table, we find that an increase in Home relative to Foreign expected inflation tends to be associated with an expected real appreciation of the Home currency, holding the real interest differential constant. This regularity for the G7 country pairs is very different than what has been found to hold in relatively high inflation emerging economies.[23] In those countries, higher expected inflation in a country tends to be reflected in a higher expected nominal currency

[21] See also Guo (2009), Gourio et. al. (2010) and Gourinchas et. al. (2010).
[22] See Martin (2010).
[23] See Bansal and Dahquist (2000).

depreciation and a higher local nominal interest rate. The net effect is to make uncovered interest parity, in nominal terms, hold better empirically for emerging economies than developed countries. To contrast, here we find that higher expected Home inflation leads to an appreciation, not a depreciation, of the Home currency. Moreover, this expected appreciation is in real terms, and does not work through the effect on the current real interest differential (since the regression controls for the latter variable.) This suggests some sort of interaction between monetary policy and real exchange rates, but the connection to excess returns is not straightforward.

High real interest rates tend to strengthen a currency. That is common wisdom in foreign exchange markets. It fits the textbook description of exchange rate behavior, and is consistent with the empirical evidence in this paper and in other recent studies. This regularity cannot be ignored when we try to explain the uncovered interest parity puzzle. The high interest rate country may have short run excess returns (the uncovered interest parity puzzle), but it has a strong currency as well.

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Table 1
Tests for Unit Root in Real Exchange Rates

Univariate Unit Root Tests, 1973:3-2009:10

Country	*ADF*	***DF-GLS***
Canada	-1.771	-1.077
France	-2.033	-2.036*
Germany	-2.038	-2.049*
Italy	-1.888	-1.914†
Japan	-2.071	-0.710
United Kingdom	-2.765†	-2.076*
G6	-2.052	-1.846†

* significant at 5% level, † significant at 10% level

Panel Unit Root Test, 1973:3-2009:10

Model	*Estimated Coefficient*	*1%*	*5%*	***10%***
No Covariates	-0.01705*	-0.02199	-0.01697	-0.01485
With Covariates	-0.01688*	-0.02126	-0.01638	-0.01411

* significant at 5% level, † significant at 10% level

Table 2
Fama Regressions: $s_{t+1} - s_t = \beta_0 + \beta_1(i_t - i_t^*) + u_{t+1}$
1979:6-2009:10

Country	$\hat{\beta}_0$	*s.e.*($\hat{\beta}_0$)	$\hat{\beta}_1$	***s.e.*($\hat{\beta}_1$)**
Canada	-0.045	(-0.224,0.134)	-1.271	(-2.564,0.220)
France	-0.028	(-0.316,0.260)	-0.216	(-1.298,0.866)
Germany	0.192	(-0.109,0.493)	-1.091	(-2.335,0.153)
Italy	0.032	(-0.309,0.373)	0.661	(-0.206,1.528)
Japan	0.924	(0.465,1.383)	-2.713	(-4.141,-1.285)
United Kingdom	-0.410	(-0.752,-0.068)	-2.198	(-3.646,-0.750)
G6	0.054	(-0.165,0.273)	-1.467	(-2.783,-0.151)

Notes: * significant at 5% level, † significant at 10% level for test of null $\beta_1 = 1$

Table 3A

Fama Regression in Real Terms: $q_{t+1} - q_t = \beta_0 + \beta_1(\hat{r}_t - \hat{r}_t^*) + u_{t+1}$

1979:6-2009:10

	Model 1				***Model 2***			
<u>*Country*</u>	$\hat{\beta}_0$	90% c.i.($\hat{\beta}_0$)	$\hat{\beta}_1$	90% c.i.($\hat{\beta}_1$)	$\hat{\beta}_0$	90% c.i.($\hat{\beta}_0$)	$\hat{\beta}_1$	s.e.($\hat{\beta}_1$)
Canada	0.021	(-0.170,0.212)	0.138	(-1.222,1.498)	0.022	(-0.170,0.214)	0.149	(-1.225,1.523)
		(-0.159,0.166)		(-1.908,1.632)		(-0.159,0.168)		(-1.827,1.683)
		(-0.135,0.157)		(-1.800,1.676)		(-0.136,0.165)		(-1.739,1.668)
France	-0.080	(-0.399,0.239)	-0.576	(-2.269,1.117)	-0.075	(-0.397,0.247)	-0.526	(-2.265,1.213)
		(-0.281,0.052)		(-2.240,0.719)		(-0.286,0.064)		(-1.964,0.733)
		(-0.294,0.054)		(-2.602,1.125)		(-0.290,0.064)		(-2.485,1.260)
Germany	-0.042	(-0.341,0.257)	-0.837	(-2.689,1.015)	-0.043	(-0.341,0.255)	-0.912	(-2.753,0.929)
		(-0.209,0.082)		(-3.458,0.313)		(-0.206,0.086)		(-3.622,0.252)
		(-0.207,0.081)		(-3.419,0.411)		(-0.205,0.079)		(-3.531,0.373)
Italy	0.075	(-0.234,0.384)	0.640	(-1.056,2.336)	0.082	(-0.229,0.393)	0.733	(-0.960,2.426)
		(-0.149,0.274)		(-1.136,2.087)		(-0.141,0.278)		(-1.038,2.115)
		(-0.124,0.261)		(-1.328,2.358)		(-0.116,0.276)		(-1.228,2.465)
Japan	0.108	(-0.201,0.417)	-1.314	(-2.860,0.232)	0.110	(-0.197,0.418)	-1.358	(-2.911,0.195)
		(0.008,0.336)		(-3.300,0.254)		(0.006,0.325)		(-3.167,0.111)
		(0.015,0.326)		(-3.441,0.379)		(0.012,0.326)		(-3.485,0.263)
United Kingdom	-0.241	(-0.603,0.121)	-1.448	(-3.042,0.146)	-0.228	(-0.588,0.132)	-1.347	(-2.915,0.221)
		(-0.574,-0.074)		(-3.614,-0.127)		(-0.555,-0.073)		(-3.369,0.002)
		(-0.611,-0.067)		(-3.846,-0.039)		(-0.588,-0.069)		(-3.725,0.027)
G6	-0.048	(-0.287,0.191)	-0.933	(-2.548,0.682)	-0.047	(-0.286,0.192)	-0.914	(-2.511,0.683)
		(-0.210,0.080)		(-2.932,0.409)		(-0.189,0.082)		(-2.909,0.447)
		(-0.202,0.068)		(-3.005,0.527)		(-0.181,0.077)		(-2.861,0.588)

Notes:

Table 3B

Fama Regression in Real Terms: $q_{t+1} - q_t = \beta_0 + \beta_1(\hat{r}_t - \hat{r}_t^*) + \beta_2(\hat{E}_t(\pi_{t+1} - \pi_{t+1}^*)) + u_{t+1}$

1979:6-2009:10

	Model 1					
Country	$\hat{\beta}_0$	*90% c.i.(* $\hat{\beta}_0$ *)*	$\hat{\beta}_1$	*90% c.i.(* $\hat{\beta}_1$ *)*	$\hat{\beta}_2$	*90% c.i.(* $\hat{\beta}_2$ *)*
Canada	-0.001	(-0.210,0.208)	-0.666	(-2.005,0.673)	-3.362	(-5.318,-1.406)
		(-0.161,0.239)		(-2.279,1.385)		(-5.364,-1.214)
		(-0.164,0.224)		(-1.973,1.089)		(-5.714,-1.330)
France	-0.078	(-0.400,0.244)	-0.600	(-2.316,1.116)	-0.560	(-2.332,1.212)
		(-0.320,0.076)		(-2.183,0.779)		(-2.462,1.816)
		(-0.310,0.045)		(-2.559,1.090)		(-2.424,1.548)
Germany	0.280	(-0.110,0.670)	-0.598	(-2.422,1.226)	-3.009	(-5.179,-0.839)
		(-0.092,0.631)		(-2.523,1.314)		(-5.715,-0.197)
		(-0.046,0.675)		(-2.415,1.257)		(-5.702,-0.480)
Italy	0.020	(-0.335,0.375)	0.581	(-1.192,2.354)	-0.306	(-1.332,0.720)
		(-0.280,0.366)		(-1.249,2.014)		(-1.499,1.200)
		(-0.246,0.315)		(-1.544,2.411)		(-1.390,0.996)
Japan	0.975	(0.465,1.485)	-2.612	(-4.140,-1.084)	-3.956	(-6.060,-1.852)
		(0.510,1.527)		(-4.131,-0.890)		(-6.057,-1.889)
		(0.574,1.505)		(-4.029,-0.989)		(-5.955,-1.668)
U.K.	-0.408	(-0.763,-0.053)	-2.170	(-3.943,-0.397)	-3.203	(-7.334,0.928)
		(-0.738,-0.096)		(-3.791,-0.448)		(-5.486,-0.438)
		(-0.746,-0.149)		(-3.918,-0.376)		(-7.626,1.493)
G6	0.080	(-0.190,0.350)	-1.299	(-3.102,0.504)	-3.017	(-5.567,-0.467)
		(-0.143,0.271)		(-3.008,0.343)		(-5.270,-0.335)
		(-0.132,0.268)		(-3.221,0.655)		(-5.844,-0.191)

Country	Model 2					
	$\hat{\beta}_0$	*90% c.i.*($\hat{\beta}_0$)	$\hat{\beta}_1$	*90% c.i.*($\hat{\beta}_1$)	$\hat{\beta}_2$	*90% c.i.*($\hat{\beta}_2$)
Canada	0.000	(-0.209,0.209)	-0.642	(-1.983,0.699)	-3.405	(-5.386,-1.424)
		(-0.162,0.250)		(-2.201,1.419)		(-5.401,-0.874)
		(-0.154,0.233)		(-1.943,1.139)		(-5.851,-0.934)
France	-0.074	(-0.398,0.250)	-0.560	(-2.322,1.202)	-0.631	(-2.350,1.088)
		(-0.313,0.084)		(-2.034,0.794)		(-2.406,1.402)
		(-0.312,0.086)		(-2.476,1.300)		(-2.346,1.487)
Germany	0.267	(-0.121,0.655)	-0.674	(-2.487,1.139)	-2.900	(-5.058,-0.742)
		(-0.057,0.598)		(-2.711,1.102)		(-5.586,-0.398)
		(-0.021,0.630)		(-2.577,1.146)		(-5.611,-0.674)
Italy	0.020	(-0.337,0.368)	0.668	(-1.095,2.431)	-0.351	(-1.387,0.685)
		(-0.276,0.382)		(-1.149,2.091)		(-1.402,1.204)
		(-0.254,0.343)		(-1.409,2.424)		(-1.386,0.840)
Japan	0.961	(0.451,1.471)	-2.635	(-4.173,-1.096)	-3.881	(-5.977,-1.785)
		(0.510,1.493)		(-4.119,-1.029)		(-5.920,-1.820)
		(0.551,1.484)		(-4.126,-0.964)		(-5.955,-1.668)
U.K.	-0.407	(-0.761,-0.053)	-2.154	(-3.922,-0.386)	-3.247	(-7.325,0.831)
		(-0.711,-0.105)		(-3.750,-0.185)		(-5.647,-0.586)
		(-0.729,-0.148)		(-3.866,-0.176)		(-7.720,1.685)
G6	0.081	(-0.190,0.352)	-1.293	(-3.086,0.500)	-3.034	(-5.589,-0.479)
		(-0.141,0.270)		(-3.020,0.353)		(-5.240,-0.317)
		(-0.124,0.277)		(-3.168,0.564)		(-5.648,0.257)

Notes:

Table 4

Regression of q_t on $\hat{r}_t - \hat{r}_t^*$: $q_t = \beta_0 + \beta_1(\hat{r}_t - \hat{r}_t^*) + u_{t+1}$

1979:6-2009:10

	Model 1		***Model 2***	
Country	$\hat{\beta}_1$	90% c.i.($\hat{\beta}_1$)	$\hat{\beta}_1$	90% c.i.($\hat{\beta}_1$)
Canada	-48.517	(-62.15,-34.88) (-94.06,-31.41) (-140.54,-27.34)	-48.962	(-62.73,-35.19) (-92.51,-33.11) (-139.93,-29.36)
France	-20.632	(-32.65,-8.62) (-44.34,-1.27) (-54.26,1.75)	-20.388	(-32.42,-8.35) (-42.53,-3.73) (-52.83,-0.46)
Germany	-52.600	(-67.02.-38.18) (-85.97,-25.35) (-105.29,-19.38)	-52.738	(-67.10,-38.37) (-85.87,-25.88) (-105.62,-19.06)
Italy	-39.101	(-51.92,-26.28) (-67.63,-16.36) (-90.01,-13.70)	-39.550	(-52.46,-26.64) (-67.29,-17.78) (-87.39,-15.45)
Japan	-19.708	(-29.69,-9.72) (-42.01,-1.05) (-46.53,-4.33)	-19.669	(-29.72,-9.61) (-42.79,-0.92) (-46.23,-3.94)
United Kingdom	-18.955	(-31.93,-5.98) (-40.19,-3.08) (-55.94,4.08)	-18.387	(-31.01,-5.76) (-38.63,-3.61) (-52.82,4.95)
G6	-44.204	(-55.60,-32.80) (-73.17,-23.62) (-82.87,-21.74)	-44.032	(-55.34,-32.72) (-74.89,-22.06) (-82.93,-22.75)

Notes:

Table 5

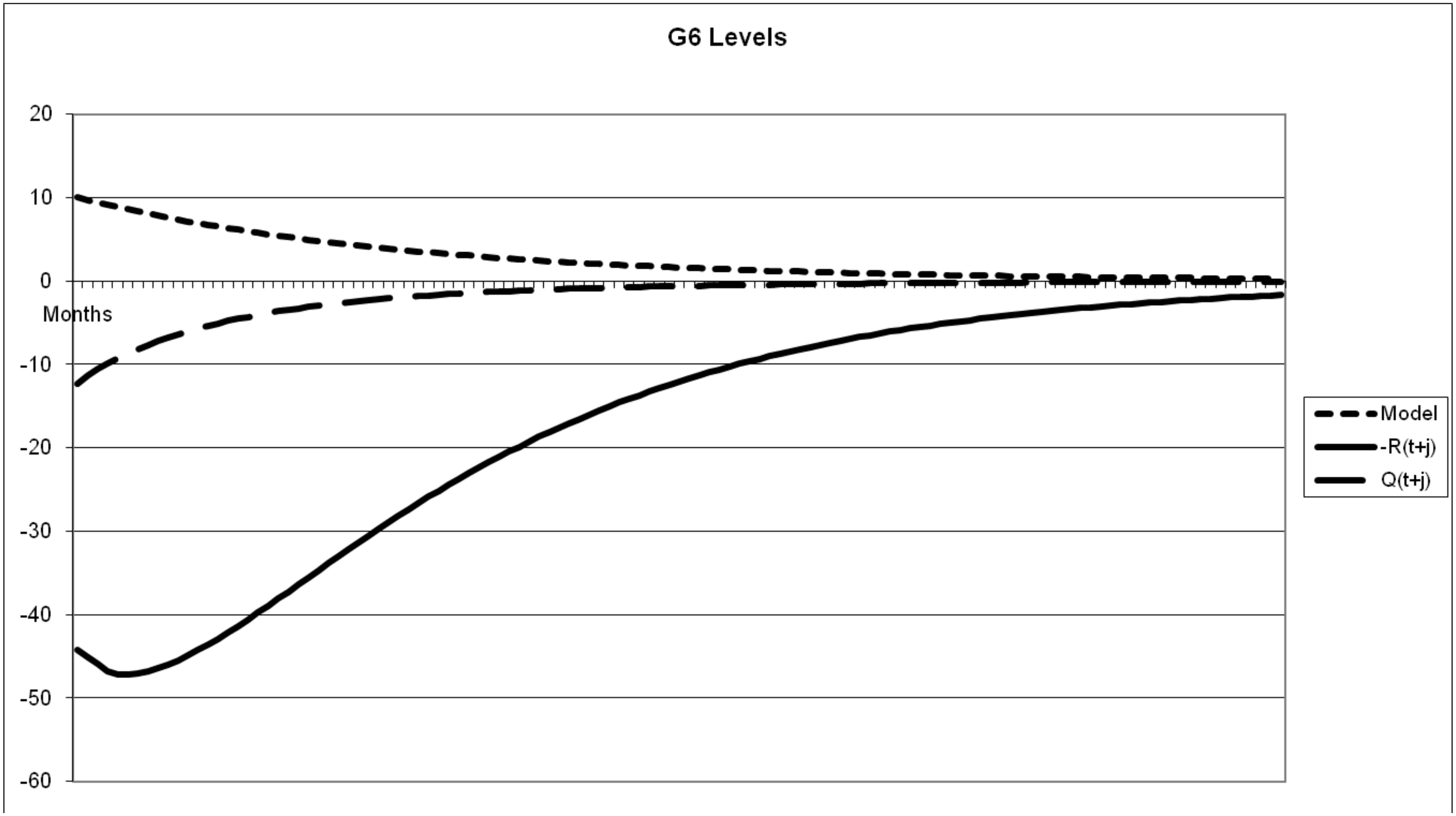
Regression of $\hat{\Lambda}_t$ on $\hat{r}_t - \hat{r}_t^*$: $\hat{\Lambda}_t = \beta_0 + \beta_1(\hat{r}_t - \hat{r}_t^*) + u_{t+1}$

1979:6-2009:10

	Model 1		***Model 2***	
Country	$\hat{\beta}_1$	90% c.i.($\hat{\beta}_1$)	$\hat{\beta}_1$	90% c.i.($\hat{\beta}_1$)
Canada	23.610	(15.12,32.10)	24.192	(15.64,32.75)
		(12.62,51.96)		(13.35,53.16)
		(11.96,63.71)		(12.99,71.13)
France	13.387	(1.06,25.72)	14.045	(1.84,26.25)
		(-2.56,36.25)		(0.80,35.39)
		(-6.98,42.40)		(-3.60,41.27)
Germany	34.722	(19.66,49.78)	34.816	(19.77,49.87)
		(9.34,57.59)		(10.30,59.11)
		(3.68,69.36)		(5.701,73.54)
Italy	27.528	(17.58,37.48)	28.400	(18.40,38.40)
		(14.98,48.32)		(16.00,48.83)
		(12.51,58.54)		(13.26,57.41)
Japan	15.210	(4.76,25.66)	15.208	(4.71,25.70)
		(-0.45,37.08)		(-0.99,37.77)
		(0.91,38.87)		(1.50,38.48)
United Kingdom	14.093	(0.33,27.86)	13.575	(0.17,26.98)
		(0.39,34.46)		(-0.11,33.13)
		(-8.70,46.45)		(-8.70,44.32)
G6	31.876	(20.62,43.13)	31.876	(20.78,42.97)
		(16.89,54.62)		(17.39,55.49)
		(16.78,60.89)		(16.33,59.36)

Notes:

Figure 1



The line labeled –R(t+j) plots estimates of the regression coefficient of $-R_{t+j}$ on r_t^d

The line labeled Q(t+j) plots estimates of the regression coefficient of q_{t+j} on r_t^d

The line labeled Model plots the regression coefficient of q_{t+j} on r_t^d implied by a class of models discussed in Section 3

Figure 2

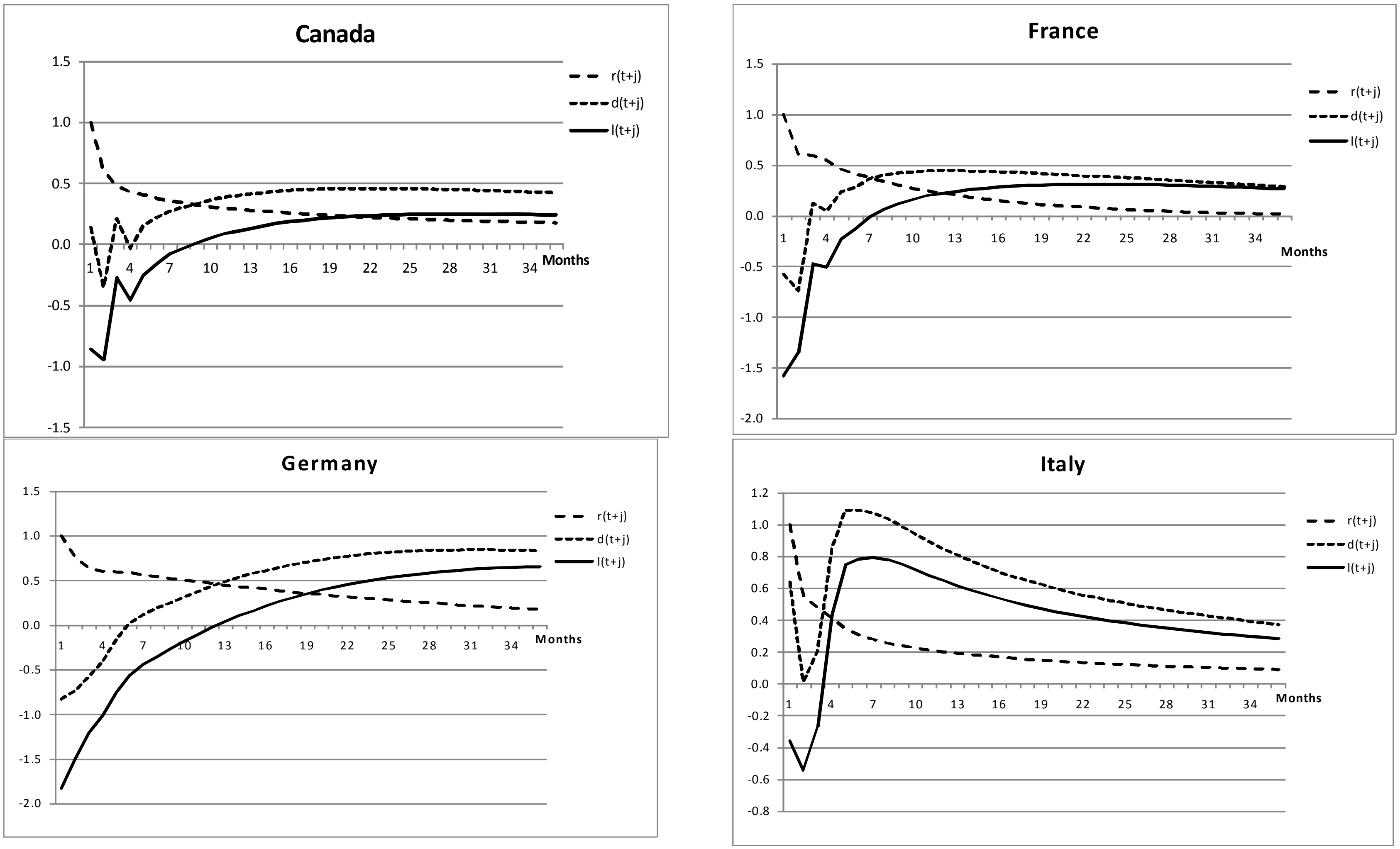
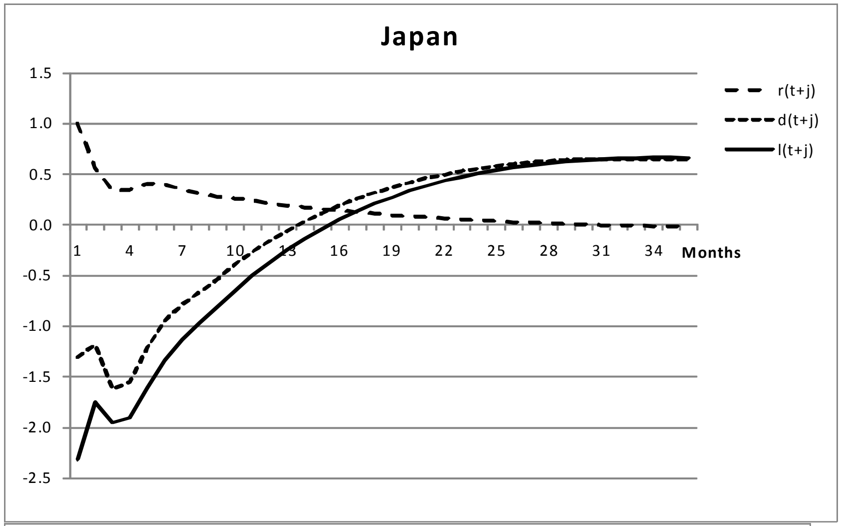
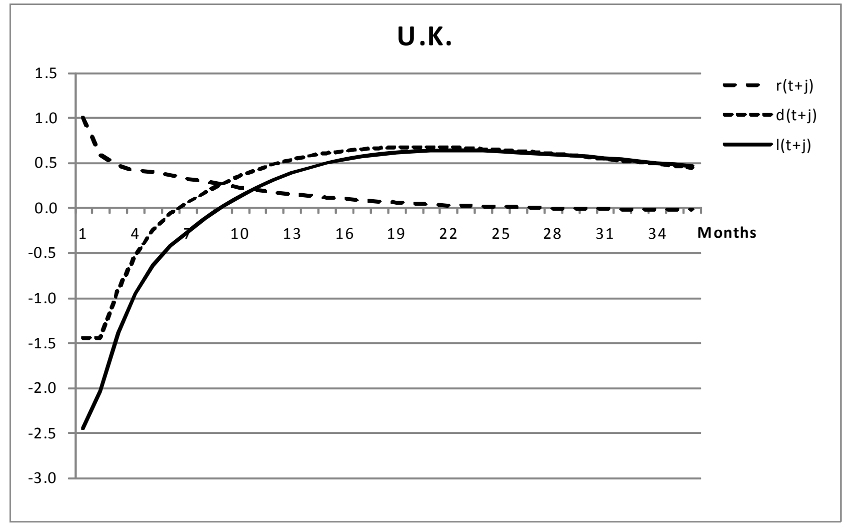
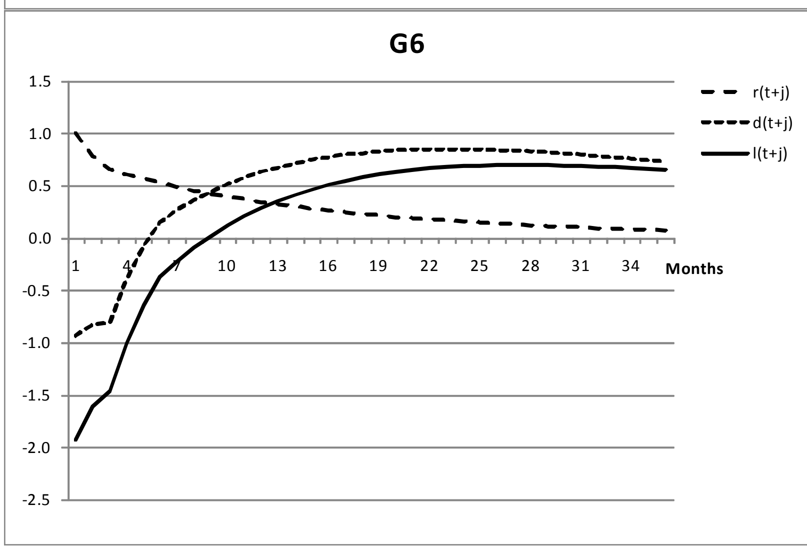


Figure 2







Figures plot the slope coefficient of regressions of variables listed in legends on r_t.

r(t+j) in legend refers to $E_t r_{t+j}$

d(t+j) in legend refers to $E_t d_{t+j} = E_t(q_{t+j+1} - q_{t+j})$

l(t+j) in legend refers to $E_t \lambda_{t+j}$