International Risk Sharing: Through Equity Diversification or Exchange Rate Hedging?

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Abstract

Well-known empirical puzzles in international macroeconomics concern the large divergence of equilibrium outcomes for consumption across countries from the predictions of models with full risk sharing. It is commonly believed that these risk-sharing puzzles are related to another empirical puzzle – the home-bias in equity puzzle. However, we show in a series of dynamic models, that the full risk sharing equilibrium may not require much diversification of equity portfolios when there is price stickiness of the degree typically calibrated in macroeconomic models. This conclusion holds under a range of assumptions about home bias in preferences, price setting as PCP or LCP, and with or without nominal wage stickiness as long as there is some price rigidity.

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The puzzle of home-bias in equity holdings – that households hold a disproportionate share of their equity portfolios in their own country's equities – is frequently linked with puzzles of international risk sharing. It is natural to make such a link, because diversification in equity holdings seems to be a natural way to diversify income risk.[1] But here we argue that the risk sharing puzzles may be more closely linked to exchange rate hedging by households. In a world in which nominal goods prices are sticky, we make the case that much of the burden of risk sharing is borne by exchange rate hedging behavior through the portfolio of bonds denominated in different currencies, currency forward position or other financial instruments affected by exchange rates. The result is surprising as nominal stickiness and short-term foreign exchange hedges play a key role in international risk sharing instead of permanently-lived equities. We show that when exchange rate hedging is possible, price stickiness plays a dominant role in determining the equity portfolio relative to other effects such as wage setting, currency of pricing, and home bias in preferences.

Foreign exchange risk can be hedged through forward contracts, swaps, or the portfolio of nominal bonds in different currencies, so that equity portfolios hedge risks not associated with foreign exchange rate movements. But, when prices are sticky, much of the differential real income risk of residents in different countries is associated with nominal exchange rate movements. To the extent that exchange rate changes are passed through to import prices, they influence the relative international prices of commodities. If pass-through is incomplete, exchange rate movements influence the profits per unit sold in the exporter's currency. Moreover, relative output across countries is influenced by demand shocks under sticky prices, but these shocks in turn influence exchange rates.

The value of assets is determined by the entire future expected path of payoffs. Equity prices, for example, are determined in most asset-pricing models as a present value of expected discounted dividends. Under typical calibrations of macroeconomic models, prices adjust quickly compared to the rate at which markets discount future dividends. So why should goods price stickiness have much of an influence on asset prices and portfolios?

Indeed, we consider a model in which, were goods prices flexible, the equity portfolio of households could be used to share risk efficiently. In our 2-country model, the real allocations that could be achieved under complete markets will be replicated in

[1] Indeed, a simple Lucas tree model predicts that full risk sharing can be achieved through equity portfolio diversification. (Lucas 1982)

a model in which only equities are traded. There is no role for nominal bonds to hedge risk when goods prices are flexible.

But, when prices are flexible, much of the burden of risk sharing falls not on the portfolio choices of individuals, but on changes in relative goods prices (as Cole and Obstfeld, 1991, have noted.) When a firm experiences a productivity increase, it will increase the supply of its product. The consequent drop in the price of its output can partly offset the income gains from increased productivity. Under standard calibrations of macroeconomic models, Cole and Obstfeld show that the gains from diversification of the equity portfolio might not be very large.

However, this terms-of-trade effect is not operative when prices are sticky. Under sticky prices, the portfolio of assets – stocks and bonds – are therefore an important way to diversify risk. Indeed, even though goods prices may be sticky only temporarily, the asset demand decision might be dominated by the risk-sharing considerations under sticky prices.[2] We demonstrate that only a small amount of price stickiness will have large effects on the portfolio choices of individuals.

We build a two-country dynamic stochastic general equilibrium model with trade in equities and nominal bonds. We consider price stickiness of two types that have been examined in the international macroeconomics literature: producer currency pricing (PCP), in which prices are set in the producers' currencies, and local-currency pricing (LCP) in which prices are set in the currency of consumers. We examine the optimal equity and bond portfolios under nominal wage flexibility and nominal wage stickiness. We also consider the role of home-bias in preferences.

Our main conclusion is that, in a log-linearized version of the model, trade in only equity and bonds will replicate the allocations achieved under trade in a complete set of nominally-denominated state-contingent bonds. For many reasonable parameterizations, the portfolios that replicate the complete-markets outcome do not require much diversification of the equity portfolio. That is, households optimally should hold portfolios that are heavily weighted toward their own country's equities. But the efficient allocation does require that agents take the "correct" open position in foreign exchange. In particular, under all plausible parameterizations, agents should be long in their own currency and short in foreign currency. An unexpected currency depreciation should cause a negative wealth shock to the portfolio. This shock serves to balance the positive income effects from a depreciation under sticky nominal prices.

[2] We have demonstrated this point in detail in a dynamic model of sticky prices that is a special case of the general one considered in this paper. See Engel and Matsumoto (2008).

We begin our analysis in section I with a model that illustrates the economic forces at work. This section extends the example presented in Engel and Matsumoto (forthcoming). It examines the linearized version of a two-country model in a one-period economy. Output is produced using labor by monopolistic firms. Households are endowed with claims to profits of firms in the country where they live and work, but purchase and/or sell a portfolio of equities and a forward position in foreign exchange before the realization of shocks. The model assumes preferences are homothetic in each country, but Home and Foreign residents' preferences may be different. The general result of this model is that when households can trade assets that hedge relative price risk – real exchange rate risk, and risk from the changes of relative prices of Home to Foreign goods in each country – then a full risk-sharing equilibrium can be achieved with no trade in equities.

One application of this result is to the case in which goods prices are flexible and the law of one price holds for all goods. The complete markets allocation can be achieved in this case with only trade in two non-state-contingent bonds – one denominated in units of the Home good and one in units of the Foreign good.[3]

Our focus is primarily on exchange rate hedging in sticky-price models. We show in the static setting that the complete markets equilibrium can be supported with only trade in a forward position in foreign exchange, or nominal bonds denominated in different currency. That is, full risk sharing does not require any diversification of the equity portfolio in this special model. Price setting of goods can be LCP, PCP, or there can be indexation of the price to the exchange rate. But a crucial assumption in this example model is that all goods prices must be set in advance of the realization of shocks. The model puts no restrictions at all on the labor market, or in general, on how firms and workers share revenues.

Section II then sets up a dynamic two-country model. Output is produced using only labor, and firms are monopolistic. In this section, we first assume that some firms set prices in advance, with local-currency pricing, while other firms are flexible-price firms. We consider two different specifications of the labor market. In the first, wages are flexible and the labor market is competitive. In the second, all wages are fixed in advance, and households supply labor as demanded. In both cases, trade in equities and a forward position in foreign exchange (or equivalently, a bond portfolio of zero net value ex ante) will lead to allocations that are identical to those

[3] Our results in this section extend results presented in our earlier paper to allow for different, asymmetric preferences between households in each country.

achieved under trade with a complete set of nominal contingent claims. We show that under plausible parameterizations, the optimal equity portfolio does not reflect much international diversification. That is, there is a high degree of home bias in equities.[4] But this efficient allocation requires agents take sufficiently large positions in foreign exchange to diversify risk fully.

We also repeat the analysis under the assumption that those firms that have sticky prices set them one period in advance according to producer-currency pricing. We show that the equity position under PCP is identical to that under LCP, but the optimal position in foreign exchange is different. Intuitively, the PCP and LCP models are different because of the way exchange rates are passed through to prices. Demand for products from home and foreign firms, and hence profits, will differ under LCP and PCP pricing. But the effects of these exchange rate changes on wealth can be hedged through the forward market in foreign exchange. The optimal equity portfolio is not influenced by LCP vs. PCP because the difference is all in how exchange rate changes affect the economy.

It is important to emphasize that the degree of equity diversification needed to achieve full risk sharing does depend on the degree of price and wage stickiness. We know from the work of Baxter and Jermann (1997) that when goods prices and wages are perfectly flexible, the optimal equity portfolio should be heavily weighted toward the equities of the other country, even to the extent that households may wish to go short in their own country's equities. Our general equilibrium model encompasses the Baxter-Jermann model as a special case, but more generally allows us to examine how the degree of price and wage stickiness influence the composition of the portfolios that optimally share risk.

Finally, even though we find that the optimal risk-sharing equity portfolios under some plausible parameterizations reflect a high degree of home bias in equities, we are not advancing our model as an explanation for the degree of home bias observed in the data. It is important to understand that in the context of our model, home bias in equities is only optimal if agents also hold the "correct" forward position in foreign exchange. In fact, country bond portfolios do not look much like the optimal portfolios we construct. Our point is that the pattern of bond holdings may be as puzzling as the pattern of equity holdings. However, we are also cautious to state that this is indeed a puzzle as what really matters is currency exposure in our model, but

[4] Moreover, home bias in preferences does not generate a discontinuity in the equity portfolio with respect to the degree of home bias in preferences. See the discussion of Obstfeld (2007).

we do not have good data on the actual currency exposure including derivatives. Nonetheless, we do claim that exchange rate hedging and nominal rigidity play an important role in (the lack of) international risk sharing.

In the past few years, there have been many new dynamic models of portfolio choice in general equilibrium. Perhaps these models provide a basis for a more positive analysis of portfolio allocation. Particularly relevant are the studies that use approximation methods, not unlike those used in this paper, to analyze the dynamics of portfolios. Devereux and Sutherland (2006) derive an approximation method for an economy with incomplete markets, but constant portfolio shares, and apply it to a 2-country general equilibrium model with production and trade in equities, and to a 2-country endowment model with trade in real bonds. Devereux and Sutherland (forthcoming) apply this model to a sticky-price monetary model that allows for portfolios of bonds and equity trade. Devereux and Sutherland (2007) extend the approximation method to allow for time-varying portfolios, and apply the method to a two-country endowment model with trade in real bonds. Devereux and Sutherland (2008) examine a similar model, with a focus on the role of changes in valuation for the international distribution of wealth.

Tille and van Wincoop (2008a) use a similar approximation to solve a two-country general equilibrium model with capital and production and trade in equities. Tille and van Wincoop (2008b) use these methods to examine the response of the current account and net foreign assets to changes in saving. Also, Evans and Hnatkovska (2008) examine a similar model with a related solution methodology.

Heathcote and Perri (2008) and Pavlova and Rigobon (2007, 2008) present neoclassical models in closed form using special assumptions on functional forms (such as log preferences) to examine equilibrium portfolios. Like the model of this paper, Heathcote and Perri allow for home bias in preferences. In their model, firms have value because they own capital. Pavlova and Rigobon consider a continous-time version of the two-country endowment model. Kollmann (2006) considers an endowment model with home bias in preferences and trade in equities. Similarly, Coeurdacier (forthcoming) reconsiders the issue of home bias in consumption in the context of an approximated general equilibrium model with endowments. Obstfeld (2007) argues that Coeurdacier's result may not be robust in the presense of non-traded goods. Indeed, Collard, Dellas, Diba, and Stockman (2007) extend an endowment model with traded and nontraded goods of Baxter, Jermann, and King (1998) to incorporate home bias in preferences, and numerically solve it to address the home bias in equity puzzle. Matsumoto (2008) analytically solves a production economy model with traded and nontraded goods to study the role of human capital and nonseparable utility in international portfolio allocations. That paper emphasizes the role of human capital wealth. All of these models assume flexible prices and no exchange rate hedging. As a result, the equilibrium portfolio is often highly sensitive to preference parameters and sometimes super home bias or anti-home bias may arise, depending on the values of parameters.

In closely related work, Coeurdacier and Gourinchas (2008) focus on trade in real bonds denominated in units of output.[5] They consider one-period endowment models with home bias in preferences under flexible goods prices. They study the bond and equity portfolio when there is a terms-of-trade shock and another arbitrary shock. In contrast to the earlier studies, they find that introducing real bonds may make the equilibrium equity portfolios less sensitive to changes in parameters. While both their paper and this paper emphasize the role of the exchange rate, the mechanism of fluctuation of exchange rates is different. Coeurdacier and Gourinchas (2008) adopt home bias in consumption to generate real exchange rate fluctuations. In contrast, we focus on nominal rigidity and nominal exchange rate fluctuations. Indeed, we show that in the presence of nominal rigidity, home bias in preferences does not have much impact on equity portfolios.

van Wincoop and Warnock (2008) empirically study whether home bias in preferences can explain home bias in equity portfolios. In their model (which absents human wealth), the magnitude of the effect of preferences on home bias in equities is small, especially when a real exchange rate hedge is available.

We introduce home bias in preference in our model. As we have shown in our previous paper (Engel and Matsumoto (forthcoming)), when exchange rate hedging is possible, home bias in preferences does not matter at all in a static model. In this paper, we extend our analysis in fully dynamic general equilibrium and we find that it does change the optimal equity portfolio but not much if there is price rigidity and human capital.

Coeurdacier, Kollmann, and Martin (forthcoming) present a 2-country, 2-good model with a number of types of shocks and trade in equities and bonds. They focus

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[5] That paper and ours were written simultaneously and independently. Although some results are very similar, the emphasis of the papers is different. Our focus is primarily on exchange rate hedging under sticky prices. Coeurdacier and Gourinchas (2008) derive some general results about portfolios of equities and bonds in a flexible price endowment model with home bias in preferences when trade in real bonds is permitted.

on "redistributive shocks" – shocks that redistribute income between firm owners and workers – as a source of home bias in equity holdings. Couerdacier, Kollmann, and Martin (2008) consider a similar model, but with investment specific technological change, which they argue can explain the home bias in equity holdings.

I. A General Result in a Static Framework

We build a general-equilibrium model with sticky prices. There are two countries, Home and Foreign, each with population ½. The model of this section is a one-period model in which portfolio choices are made prior to the resolution of uncertainty. We also assume that goods prices must be set prior to the realization of the state.

In section 2, we fully specify dynamic general equilibrium models. In the example model of this section, we use only some of the features of the model – demand functions for goods, market-clearing conditions, and a general assumption about nominal price stickiness – to demonstrate that (1) trade in equities and a foreign exchange hedge will allow households to achieve the "complete markets" equilibrium allocations; (2) this outcome may always be achieved with a portfolio allocation such that households have complete ownership of their own country's firms and hold no foreign equities; and (3), this outcome with complete home bias in equity holdings is the only portfolio that will replicate the complete markets allocation except in the special case in which labor income is perfectly correlated with the nominal exchange rate. This example extends the one presented in Engel and Matsumoto (forthcoming).

To be clear, this demonstration is not a proof that a decentralized market with trade in equities and a forward market in foreign exchange will achieve the efficient allocation. We do not, in this section, show that the portfolio that replicates the complete market allocation is optimal from the household's perspective. The models of sections 2, however, do derive household optimality conditions and derive an equilibrium in which those are satisfied. The purpose of this section is to explore intuitively the economic forces at work that allow a portfolio with complete home bias in equities, but with an appropriate foreign exchange position, to replicate the complete-markets outcome.

Initially, households are endowed with ownership of firms in their own country, and with human capital. The ex post budget constraint of a representative Home household can be written as

(1.1) $P_t C_t = REV_t + X_t$.

Under homothetic preferences, Home expenditure is the product of the exact price index, P_t , and the consumption index, C_t .

We assume here that Home agents have full ownership of Home firms, and no ownership of Foreign firms – there is no trade in equities. Under that assumption, Home households receive all of the revenue of Home firms, REV_t , in the form of profits, capital income, and labor income. We put absolutely no restriction on the mechanism that divides Home revenues between equity owners and workers.

In addition, Home households can make contracts that pay off X_t in Home currency ex post, but cost zero ex ante. The adding up constraint implies that Foreign households must receive $-X_t$ ex post in Home currency terms, or $-X_t / S_t$ in Foreign currency terms.

Next, we use the fact that Home revenue is equal to purchases of the Home good by Home and Foreign residents:

(1.2) $REV_t = P_{h,t} C_{h,t} + S_t P^*_{h,t} C^*_{h,t}$.

$C_{h,t}$ is consumption of the Home good by Home residents, and $C^*_{h,t}$ is consumption of the Home good by Foreign residents. $P_{h,t}$ is the Home currency price of Home goods, and $P^*_{h,t}$ is the Foreign currency price of Home goods. S_t is the exchange rate expressed as the Home currency price of Foreign currency. We do not impose that the law of one price holds. That is, we do not require $S_t P^*_{h,t} = P_{h,t}$. Moreover, $C_{h,t}$ and $C^*_{h,t}$ may be homogeneous-of-degree-one indexes over a set of goods produced in the Home country, in which case the prices should be interpreted as the corresponding exact price indexes.

Using (1.2), we will log-linearize (1.1) around a point of initial trade balance. If trade is initially balanced, $P_0 C_0 - P_{h,0} C_{h,0} = S_0 P^*_{h,0} C^*_{h,0}$, where the 0 subscript denotes the point of linearization. Let $\frac{1+\tilde{\alpha}}{2}$ denote the share of Home expenditure on Home goods at the point of linearization. That is, $\frac{1+\tilde{\alpha}}{2} = \frac{P_{h,0} C_{h,0}}{P_0 C_0}$.[6] Then we can write

[6] $\tilde{\alpha}$ in this section is very closely related to home bias in preference parameter introduced in section II and identical in symmetric case, i.e. $\tilde{\alpha} = \tilde{\alpha}^* = \alpha$, but $\tilde{\alpha}$ is not exactly same as α in general.

(1.3) $p_t + c_t = \tilde{\alpha}(p_{h,t} + c_{h,t}) + (1-\tilde{\alpha})(s_t + p^*_{h,t} + c^*_{h,t}) + x_t$,

where the lower case letters are the deviations of the logs of the upper case letters from the point of approximation. $x_t = \frac{1}{P_0 C_0} X_t$. We use the = sign in this equation to mean equal up to a first-order approximation.

We now summarize some properties of log-linear approximations to consumption functions and demand functions that follow from the assumption of homothetic preferences. We can write the Home aggregate consumption as

(1.4) $c_t = \frac{1+\tilde{\alpha}}{2} c_{h,t} + \frac{1-\tilde{\alpha}}{2} c_{f,t}$, $0 \le \alpha < 1$.

$(1+\alpha)/2$ is the share of Home goods in Home nominal expenditures at the point of approximation. $c_{f,t}$ is the log of the Home household's consumption of Foreign goods. The analogous Foreign household's consumption is given by

(1.5) $c^*_t = \frac{1-\tilde{\alpha}^*}{2} c^*_{h,t} + \frac{1+\tilde{\alpha}^*}{2} c^*_{f,t}$.

In the models of section II, we will have $\tilde{\alpha} = \tilde{\alpha}^*$ as we assume that there is home bias in preferences, but it is symmetric between Home and Foreign households, and that initial wealth is identical. We do not even impose that symmetry in this section. The log of the consumer price indexes for Home and Foreign households are given by, respectively,

(1.6) $p_t = \frac{1+\tilde{\alpha}}{2} p_{h,t} + \frac{1-\tilde{\alpha}}{2} p_{f,t}$, and

(1.7) $p^*_t = \frac{1-\tilde{\alpha}^*}{2} p^*_{h,t} + \frac{1+\tilde{\alpha}^*}{2} p^*_{f,t}$.

$p_{h,t}$ and $p_{f,t}$ are the logs of price indexes, in Home currency, for Home consumption of the aggregates of Home and Foreign goods, respectively. $p^*_{h,t}$ and $p^*_{f,t}$ are defined analogously and are expressed in Foreign currency terms.

Next, let ω be the elasticity of substitution between the aggregates of Home and Foreign goods in Home households, and ω^* be that in Foreign households. Then the (log) nominal demand for Home goods by Home households is given by

(1.8) $p_{h,t} + c_{h,t} = (1-\omega)(p_{h,t} - p_t) + p_t + c_t$.

For the Foreign country, we can write

(1.9) $p^*_{h,t} + c^*_{h,t} = (1-\omega^*)(p^*_{h,t} - p^*_t) + p^*_t + c^*_t$.

Equations (1.8) and (1.9) follow from the homotheticity assumption. Under homotheticity, the demand for Home goods relative to Foreign goods depends only on their relative price and not on the aggregate level of consumption. Rearranging the demand curves using (1.4)-(1.7) gives us (1.8)-(1.9). We assume that the Marshall-Lerner condition holds, so that $\omega, \omega^* \geq 1$.

Substitute (1.8) and (1.9) into (1.3) to rewrite Home budget constraint:

$$(1.10)\quad \begin{aligned} & p_t + c_t \\ & = \frac{1+\tilde{\alpha}}{2}\left[(1-\omega)(p_{h,t} - p_t) + p_t + c_t\right] + \frac{1-\tilde{\alpha}}{2}\left[(1-\omega^*)(p^*_{h,t} - p^*_t) + s_t + p^*_t + c^*_t\right] + x_t \end{aligned}.$$

We can rearrange this Home budget constraint equation, using the price index equations (1.6) and (1.7) to write

$$(1.11)\quad c_t - c^*_t = s_t + p^*_t - p_t + \frac{1+\tilde{\alpha}}{2}(\omega - 1)(p_{f,t} - p_{h,t}) + \frac{1+\tilde{\alpha}^*}{2}(\omega^* - 1)(p^*_{f,t} - p^*_{h,t}) + \frac{2}{1-\tilde{\alpha}}x_t$$

The relative Home to Foreign consumption given in equation (1.11) depends on the real exchange rate, $s_t + p^*_t - p_t$, and the relative price of Foreign to Home goods in each country as well. That is because the real exchange rate and the relative prices influence the relative revenues and profits of Home and Foreign firms.

First, consider the effects of a home depreciation, which is an increase in s_t, holding prices of goods, compensation to workers, and payoffs to their non-equity positions (x_t) constant. With goods prices constant, the demand for the Home and Foreign goods is constant, so the effect of the exchange rate is only on the valuation of revenues. A depreciation raises the Home currency value of Home profits, because the Home currency value of foreign sales increases. Also, the Home currency value of Foreign revenues rises with an increase in s_t for the same reason. Because Home households own Home firms, Home consumption increases.

Relative goods prices also have an effect on relative nominal consumption. An increase in $p_{f,t} - p_{h,t}$ or an increase in $p^*_{f,t} - p^*_{h,t}$ will raise demand for Home goods relative to Foreign goods, and assuming $\omega, \omega^* > 1$, also raise Home revenue relative to Foreign revenue, and so Home consumption rises relative to Foreign consumption.

Can households in the Home and Foreign country make contracts that efficiently allocate risk in this setting? That is, is it possible to write a contract for an asset that pays x_t ex post, and achieves the complete markets allocation? We will assume the curvature of the utility functions of Home and Foreign households is the same. Let ρ be the coefficient of relative risk aversion for households in both countries, evaluated at the point of linearization (where we are assuming utility is separable in aggregate consumption.) It is well known that when asset markets are complete – that is, when households can ex ante trade a complete set of nominal contingent claims – the equilibrium condition that emerges can be log-linearized as

(1.12) $$c_t - c_t^* - \frac{1}{\rho}(s_t + p_t^* - p_t) = 0 .$$

It follows from the relative consumption equation (1.11) that this risk-sharing relationship can be achieved if x_t satisfies

(1.13) $$\frac{1-\rho}{\rho}(s_t + p_t^* - p_t) = \frac{1+\tilde{\alpha}}{2}(\omega - 1)(p_{f,t} - p_{h,t}) + \frac{1+\tilde{\alpha}^*}{2}(\omega^* - 1)(p_{f,t}^* - p_{h,t}^*) + \frac{2}{1-\tilde{\alpha}}x_t .$$

Rearranging equation (1.13), households can attain the complete markets allocation if their non-equity portfolio has a payout given by

(1.14) $$\begin{aligned} x_t &= \left(\frac{\tilde{\alpha}-1}{2}\right)\Bigg\{\left(\frac{\rho-1}{\rho}\right)(s_t + p_t^* - p_t) \\ &+ \frac{(1+\tilde{\alpha})}{2}(\omega - 1)(p_{f,t} - p_{h,t}) + \frac{(1+\tilde{\alpha}^*)}{2}(\omega^* - 1)(p_{f,t}^* - p_{h,t}^*)\Bigg\}. \end{aligned}$$

As in the Lucas (1982) model, households do not need to trade contingent claims to get these payoffs. For example, Home and Foreign households could trade assets ex ante whose payoffs were linear in the real exchange rate, $s_t + p_t^* - p_t$, relative prices in Home, $p_{f,t} - p_{h,t}$, and relative prices in Foreign, $p_{f,t}^* - p_{h,t}^*$. Recall that x_t describes the payoff from "forward positions" – the portfolio of non-equity assets that has an ex ante value of zero. By taking the appropriate forward position in these hedges, Home households could achieve optimal risk sharing.

Many models assume that the law of one price holds for traded goods. Under that assumption, condition (1.14) reduces to

(1.15) $$x_t = \frac{\tilde{\alpha}-1}{4}\left[(\tilde{\alpha} + \tilde{\alpha}^*)\frac{\rho - 1}{\rho} + (1+\tilde{\alpha})(\omega - 1) + (1+\tilde{\alpha}^*)(\omega^* - 1)\right](p_{f,t} - p_{h,t}) .$$

The complete markets allocation can be achieved by trading a hedge on terms of trade movements. This could be achieved by trading two bonds, one denominated in the Home good and one denominated in the Foreign good, as Coeurdacier and Gourinchas (2008) have emphasized. Trade in these bonds constitutes a forward position as we have defined it as long as the initial value of the bond portfolio is zero – the long position in one bond balances the short position in the other.[7]

In reality, markets are not available to hedge terms of trade or even real exchange rate risk explicitly. But when prices are sticky, the log of the real exchange rate and log of the terms of trade in each country are linear in the log of the nominal exchange rate. So households can hedge those risks with only trade in an exchange-rate hedge. Forward markets for foreign exchange do exist, or, a synthetic forward position can be obtained by trading nominal bonds, or with swaps.

We take an agnostic position on the currency of price setting. Prices could all be set in the producer's currency (PCP). Or, prices facing consumers might be set ex ante in the local currency (LCP). Or we might even have prices indexed to the exchange rate, as in Corsetti and Pesenti (2006) or Engel (2007).

We assume $p_{h,t} = 0$, and $p^*_{f,t} = 0$. These assumptions mean simply that the home-currency price of home goods sold in the home currency, and the foreign-currency price of foreign goods sold in the foreign country are constant (independent of shocks) and normalized to one (in levels.) We can assume partial pass-through for traded prices: $p_{f,t} = bs_t$, $p^*_{h,t} = -bs_t$, $0 \le b \le 1$. b is the degree of indexing of consumer prices of imported goods to exchange rates. LCP corresponds to $b = 0$, and PCP to $b = 1$. Under these assumptions, $s_t + p^*_t - p_t = \left[1 - b\left(1 - \left(\frac{\tilde{\alpha} + \tilde{\alpha}^*}{2}\right)\right)\right] s_t$, and $p_{f,t} - p_{h,t} = p^*_{f,t} - p^*_{h,t} = bs_t$.

Substitute into equation (1.14) to get

(1.16) $x_t = \delta s_t$, where

$$\delta = \frac{\tilde{\alpha} - 1}{2}\left\{\left[1 - b\left(1 - \left(\frac{\tilde{\alpha} + \tilde{\alpha}^*}{2}\right)\right)\right]\frac{\rho - 1}{\rho} + b\left(\frac{(1 + \tilde{\alpha})(\omega - 1) + (1 + \tilde{\alpha}^*)(\omega^* - 1)}{2}\right)\right\}.$$

If Home and Foreign agents can take a forward position in foreign exchange, they can achieve the complete-markets allocation. No trade in equities is required. This result holds regardless of degree of home bias in preference, initial wealth, wage

[7] The first version of this paper, June 2008, did not mention how risks could be hedged in the world where the law of one price holds with real bonds. However, the February 2008 version of our earlier paper does show this. That model was simpler in that we assumed preferences were symmetric (but not identical) between Home and Foreign households. We mention this timing because the first version of Coeurdacier and Gourinchas (2008) came out it June 2008. We are not trying to claim priority for our results, but rather declaring that this is a case of simultaneous discovery. In any event, the emphasis of the papers is different. Our focus is primarily on trade in nominal bonds under sticky prices. Coeurdacier and Gourinchas (2008) derive some general results about portfolios of equities and bonds when trade in real bonds is permitted.

setup, or price setup. A forward contract costs one unit of the home currency. Without loss of generality, we can normalize the forward price of foreign exchange to be unity, so the log of the forward rate is zero. If home households purchase δ units of this hedge, they will achieve the payoff given (1.16).

To review: We have assumed a one-period horizon and that all nominal goods prices are fixed ex ante. These two assumptions are critical to the result. We will move on to infinite-horizon models with price adjustment in section II. There, trade in equities is necessary to replicate the complete markets equilibrium, and the equilibrium portfolio will not exhibit complete home bias. But we argue that there may be substantial home bias when a nominal exchange rate hedge is available.

Since we have derived (1.16) from the Home budget constraint alone, it is useful to check the derivation from the standpoint of the Foreign budget constraint. Recalling the definition of x_t, we have shown $x_t \equiv \frac{1}{P_0C_0}X_t = \delta s_t$. X_t is the payoff from a forward position. In levels, we can treat S_0 as the forward rate – it is the ex ante expected spot rate. The payoff to Home agents is given by $X_t = z(S_t - S_0)$, where z is the quantity of contracts that are traded. A log approximation gives us $x_t = \frac{1}{P_0C_0}X_t \simeq \frac{zS_0}{P_0C_0}s_t$, so $z = \frac{P_0C_0}{S_0}\delta$.

The budget constraint for Foreign household, the Foreign equivalent of equation (1.3) is

(1.17) $p_t^* + c_t^* = (1-\tilde{\alpha}^*)(p_{f,t} + c_{f,t} - s_t) + \tilde{\alpha}^*(p_{f,t}^* + c_{f,t}^*) + x_t^*$,

where $x_t^* \equiv \frac{1}{P_0^*C_0^*}X_t^*$. The payoff to the Foreign household in foreign currency terms is $X_t^* = z\left(\frac{S_0 - S_t}{S_t}\right)$. Using a log-linear approximation, we can write

(1.18) $x_t^* \simeq \frac{-z}{P_0^*C_0^*}s_t = -\frac{P_0C_0}{S_0P_0^*C_0^*}x_t$.

Now, multiply both sides of equation (1.17) by $\frac{1-\tilde{\alpha}}{1-\tilde{\alpha}^*}$ and add to equation (1.3). After writing out the expressions for the price indexes, we find that the sum of these two equations reduces to

(1.19) $x_t + \frac{1-\tilde{\alpha}}{1-\tilde{\alpha}^*}x_t^* = 0$.

This condition is equivalent to equation (1.18) given that we have assumed initial balanced trade. Hence, the foreign hedge is given by

(1.20) $x_t^* = -\delta^* s_t$, where $\delta^* = \dfrac{1-\tilde{\alpha}^*}{1-\tilde{\alpha}}\delta$.

The result we have obtained holds, as we have shown, whether price stickiness is of the PCP, LCP, or indexing form. We also have made no assumptions about labor markets. There could be a spot market in labor, with flexible wages, or households could have market power in labor markets and nominal wages could be sticky. There could be bargaining between households and firms over revenues. We have not specified at this stage how revenues are split between firm owners and workers when there is no trade in equities.

The degree of home bias also does not play any role in determining the equity portfolio in this one period model. This is illuminating given that Obstfeld(2007) and Coeurdacier (forthcoming) have shown that home bias in preferences can result in either anti-home bias in equity or super home bias in equity, that is taking a short foreign equity position. We will examine the home bias in preference assumption also in a dynamic model where prices are fixed at most one period. We will show that home bias in preferences does play a minor role in determining the equity portfolio, but price stickiness matters more.

We also have not specified the sources of shocks to the system. There can be real productivity shocks, and nominal monetary shocks. These shocks could influence all of the variables in the system: exchange rates, labor income, profits, consumption, etc.

Simply stated, when households in each country have complete ownership of their own firms (100% home bias in equity holdings), relative consumption risk is translated through relative prices. When there is full price stickiness, the relative prices adjust only with changes in the nominal exchange rate. So a forward position in foreign exchange can fully hedge risk.

The issue we wish to focus on, however, is the implications for the forward position in foreign exchange. Recall that the assumptions of our model require $-1 < \tilde{\alpha}, \tilde{\alpha}^* < 1$, $0 \le b \le 1$, $\omega, \omega^* > 0$. If we make the further, empirically plausible assumption that $\omega, \omega^* \ge 1$, and $\rho > 1$, then equation (1.16) implies $\delta < 0$. This implies that a depreciation in the Home currency has a negative payoff to Home households. This occurs only if Home households take a long position in home currency and a short position in foreign currency. That is, they lend in bonds denominated in home currency and borrow in bonds denominated in foreign currency.

Intuitively, a depreciation of the Home currency (an increase in s_t), as we have discussed above, has a positive effect on the payoffs to Home's equity portfolio. The optimal hedge, then, is to take a foreign exchange position that offsets this exchange rate risk. The intuition, however, is a bit more complicated than this because the optimal portfolio should eliminate purchasing power risk rather than income risk. A depreciation will also raise the Home price of imported goods if some of the depreciation is passed through to import prices ($b > 0$). But the sign of δ is negative for any value of *b*.

We next fully specify a general equilibrium model in a dynamic setting.

II.A Dynamic Sticky-Price Model with Local-Currency Pricing

In this section, we build an infinite-horizon model, which allows us to examine the effects of persistent technology shocks and different degrees of price stickiness as well as the effects of home bias in consumption and different assumptions about wage setting.

The price-setting rule is defined as follows. A fraction τ of firms in each country set prices in advance, and the rest of the firms can adjust their prices in each period after the realization of shocks. This approach allows us to study the portfolio allocation with or without sticky prices, and we can learn how different degrees of price stickiness affect the portfolio. There are different types of firms in each country but we assume the equities of all firms in each country are bundled together. In sections II.A-C, we assume that firms set prices in consumer's currency - that is there is local-currency-pricing (LCP) - and wages are flexible. Then, we examine the sticky wage case in section II.D and the producer-currency-pricing (PCP) case in section II.E.

As in section I, there are two symmetric countries, Home and Foreign, each with population ½.

A. Household Problem

Home households maximize their expected utility:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, \frac{M_t}{P_t}, L_t\right)$$

subject to the following budget constraint:

$$(2.1)\qquad \begin{aligned} &P_t C_t + M_t + Q_t \gamma_{h,t+1} + S_t Q_t^* \gamma_{f,t+1} \\ &= \gamma_{h,t}(Q_t + \Pi_t) + \gamma_{f,t} S_t (Q_t^* + \Pi_t^*) + (S_t - F_t)\tilde{\delta}_t + W_t L_t + M_{t-1} + Tr_t \end{aligned}$$

where Q_t (Q_t^*) denotes the price of Home (Foreign) equities. Households enter time t with money M_{t-1}, equities ($\gamma_{h,t}, \gamma_{f,t}$), and forward contracts $\tilde{\delta}_t$. After the realization of shocks, households choose the consumption level, real money balances, and labor supply. The dividends from firms are paid at time t, and households get the payoff from the forward contract. They receive a transfer from the government as well. Finally, the households choose forward contracts $\tilde{\delta}_{t+1}$ and equity holdings $\gamma_{h,t+1}$, $\gamma_{f,t+1}$, which determine the dividends households receive at time $t+1$.

Home households receive the following income each period: wages ($W_t L_t$, where W_t denotes the wage); dividends; transfers from the government (Tr_t) and the gains or losses from forward contracts. Equity dividends received by a Home household are given by

$$\gamma_{h,t}\Pi_t + \gamma_{f,t} S_t \Pi_t^*,$$

where Π_t is the profit (dividend) of Home firms and Π_t^* is that of Foreign firms in terms of the Foreign currency.[8] S_t is the Home currency price of Foreign currency. Home and Foreign households trade forward contracts in foreign exchange. The forward rate, F_t, is known at the time the forward contract is entered into, prior to the realization of shocks. After the shocks are realized, the Home households receive $\tilde{\delta}_t(S_t - F_t)$ units of Home currency.

We note here that in this section, wages are flexible and determined in a competitive labor market. When we specify firm behavior, we will assume that the labor services of all households are identical, perfectly substitutable. We will later examine a version of this model in which each household's labor services are unique, and each household is a monopoly supplier of that type of labor. There, the different types of labor services are not perfectly substitutable. In that setting, we will also assume nominal wages are set in advance.

Utility is given by

$$(2.2)\qquad U\left(C_t, \frac{M_t}{P_t}, L_t\right) = \frac{1}{1-\rho} C_t^{1-\rho} + \chi \ln\left(\frac{M_t}{P_t}\right) - \frac{\eta}{1+\psi} L_t^{1+\psi},$$

[8] Theoretically, profits can be negative in the case of a loss, but we have to assume that the profits of both Home firms and Foreign firms are positive to take logarithms.

where $\rho > 0$, $\chi > 0$, $\psi \geq 0$, and $\eta > 0$. C_t denotes the consumption basket for Home; M_t denotes Home money; P_t, the price index; and L_t, the labor supply.

C_t is a consumption basket of a representative Home household defined as

$$(2.3) \qquad C_t \equiv \left(\left(\frac{1+\alpha}{2} \right)^{1/(\omega-1)} C_{h,t}^{(\omega-1)/\omega} + \left(\frac{1-\alpha}{2} \right)^{1/(\omega-1)} C_{f,t}^{(\omega-1)/\omega} \right)^{\omega/(\omega-1)}, \quad -1 < \alpha < 1$$

where $\omega > 0$ is the elasticity of substitution between Home produced goods and Foreign produced goods. $C_{h,t}$ is the consumption basket of Home produced goods and $C_{f,t}$ is that of Foreign produced goods:

$$(2.4) \quad C_{h,t} \equiv \left[2^{1/\lambda} \int_0^{1/2} C_{h,t}(i)^{(\lambda-1)/\lambda} di \right]^{\lambda/(\lambda-1)}, \quad C_{f,t} \equiv \left[2^{1/\lambda} \int_{1/2}^{1} C_{f,t}(i)^{(\lambda-1)/\lambda} di \right]^{\lambda/(\lambda-1)},$$

where λ denotes the elasticity of substitution among varieties, with $\lambda > 1$.

There is a representative household in each country, and the household preferences in Home and Foreign are symmetric. When $\alpha > 0$, there is home bias in consumption. That is, the Home household puts relatively more weight on consumption of Home goods, and the Foreign households preferences put relatively more weight on Foreign goods.

We can write the CPI as follows:

$$(2.5) \qquad P_t \equiv \left(\frac{1+\alpha}{2} P_{h,t}^{1-\omega} + \frac{1-\alpha}{2} P_{f,t}^{1-\omega} \right)^{1/(1-\omega)},$$

where

$$(2.6) \qquad P_{h,t} \equiv \left[2 \int_0^{1/2} P_{h,t}(i)^{1-\lambda} di \right]^{1/(1-\lambda)}, \quad P_{f,t} \equiv \left[2 \int_{1/2}^{1} P_{f,t}(i)^{1-\lambda} di \right]^{1/(1-\lambda)},$$

where $P_{h,t}(i)$ is the price of Home goods i sold in Home in terms of the Home currency, and $P_{f,t}(i)$ is the price of Foreign goods i sold in Home in terms of the Home currency.

Foreign households have an analogous utility function for Foreign quantities and prices, which we will denote by superscript asterisks. Foreign prices are denominated in Foreign currency.

Our assumptions on consumption, asset acquisition, etc., follow exactly the standard presentation of the non-stochastic dynamic model (see, for example, Obstfeld and Rogoff (1996)), with one exception: We assume, as in the static model, that households can take a forward position in foreign exchange. Making a contract to buy foreign exchange forward next period, of course, is equivalent to buying a nominal (non-state-contingent) bond denominated in the foreign currency and shorting a nominal bond denominated in the home currency. We could have introduced nominal bonds denominated in each currency separately into the model, rather than forward contracts. However, that would add nothing to our presentation. We shall see below that the (linearized) model with equities and forward contracts reproduces the allocation that would be achieved with trade in a complete set of nominal state-contingent bonds. If we introduced non-state-contingent nominal bonds instead of forward contracts, the position held by each household will exactly reproduce their position in the forward market.

Given prices and the total consumption basket, C_t, the optimal consumption allocations are

(2.7) $$C_{h,t} = (1+\alpha)\left(P_{h,t}/P_t\right)^{-\omega} C_t\,, \qquad C_{f,t} = (1-\alpha)\left(P_{f,t}/P_t\right)^{-\omega} C_t\,,$$

(2.8) $$C_{h,t}(i) = 2\left(P_{h,t}(i)/P_{h,t}\right)^{-\lambda} C_{h,t}\,, \qquad C_{f,t}(i) = 2\left(P_{f,t}(i)/P_{f,t}\right)^{-\lambda} C_{f,t}\,.$$

The dynamic first order conditions for the households are

(2.9) $$\frac{\chi}{M_t} = \frac{C_t^{-\rho}}{P_t} - E_t\beta\frac{C_{t+1}^{-\rho}}{P_{t+1}}\,,$$

(2.10) $$\eta L_t^{\psi} = \frac{C_t^{-\rho}}{P_t} W_t\,,$$

(2.11) $$E_{t-1}\left(\frac{C_t^{-\rho}}{P_t} S_t\right) = F_t E_{t-1}\left(\frac{C_t^{-\rho}}{P_t}\right),$$

(2.12) $$\frac{C_{t-1}^{-\rho}}{P_{t-1}} Q_{t-1} = E_{t-1}\left(\beta\frac{C_t^{-\rho}}{P_t}(Q_t + \Pi_t)\right),$$

(2.13) $$\frac{C_{t-1}^{-\rho}}{P_{t-1}} S_{t-1}Q_{t-1}^{*} = E_{t-1}\left(\beta\frac{C_t^{-\rho}}{P_t} S_t(Q_t^{*} + \Pi_t^{*})\right).$$

First, let $D_{t,t+s} \equiv \left(\frac{C_{t+s}^{-\rho}}{P_{t+s}}\right)\Big/\left(\frac{C_t^{-\rho}}{P_t}\right)$. The no-bubble solution for equity prices implies that

(2.14) $$Q_t = \sum_{s=1}^{\infty} E_t\beta^s D_{t,t+s}\Pi_{t+s}\,, \qquad S_tQ_t^{*} = \sum_{s=1}^{\infty} E_t\beta^s D_{t,t+s}S_{t+s}\Pi_{t+s}^{*}$$

Let

(2.15) $$V_t \equiv \gamma_{h,t+1}Q_t + \gamma_{f,t+1}S_tQ_t^{*}\,,$$

(2.16) $$H_t \equiv \sum_{s=1}^{\infty}\beta^s E_t D_{t,t+s}W_{t+s}L_{t+s}\,,$$

(2.17) $$R_t \equiv \frac{\beta(Q_t + \Pi_t)}{Q_{t-1}}\,,$$

$$R_t^H \equiv \frac{\beta(H_t + W_t L_t)}{H_{t-1}}, \tag{2.18}$$

$$\gamma_{t+1} \equiv \frac{\gamma_{f,t+1} S_t Q_t^*}{V_t} = 1 - \frac{\gamma_{h,t+1} Q_t}{V_t}. \tag{2.19}$$

These are, respectively, financial wealth, human capital, the rate of return on financial wealth and human capital (each multiplied by the utility discount factor for algebraic convenience) and the share of foreign equity in equity portfolio.

We can rewrite the budget constraint (2.1) for time t:

$$P_t C_t + V_t + H_t = V_{t-1}(1-\gamma_t)\beta^{-1} R_t + V_{t-1}\gamma_t \beta^{-1} \frac{S_t}{S_{t-1}} R_t^* + H_{t-1}\beta^{-1} R_t^H + \tilde{\delta}_t (S_t - F_t). \tag{2.20}$$

We will assume below a process for the money supply in which $E_t(M_{t+1}^{-1}) = M_t^{-1}$. We note this now, because under this assumption the first-order condition (2.9) can be simplified directly to get

$$\frac{C_t^{-\rho}}{P_t} = \chi M_t^{-1} + E_t \beta \frac{C_{t+1}^{-\rho}}{P_{t+1}} = \frac{\chi}{1-\beta} M_t^{-1}. \tag{2.21}$$

It follows from this that we can write stochastic discount factor as $D_{t,t+s} = \dfrac{M_t}{M_{t+s}}$. The first order conditions for equity holdings, (2.12) and (2.13), or Euler equations, can be summarized as

$$E_{t-1}\left(\frac{M_{t-1}}{M_t} R_t\right) = E_{t-1}\left(\frac{M_{t-1}}{M_t}\frac{S_t}{S_{t-1}} R_t^*\right) = 1. \tag{2.22}$$

B. Firms

Firms produce output with labor using a linear technology. Labor from each household is perfectly substitutable for labor from any other household.). A firm in this economy monopolistically produces a specific good indexed by i:[9]

$$Y_t(i) = A_t L_t(i), \tag{2.23}$$

where $Y_t(i)$ is the production of firm i, A_t is the country-specific technology parameter and $L_t(i)$ is the labor input of firm i. Home and Foreign markets are segmented, and only the producer can distribute its product.

We have two types of firms in each country. A fraction τ of firms set the price in advance, and the rest set the price after the realization of shocks.

[9] Using a Cobb-Douglas technology with other fixed inputs will not change the result if the returns on the other factors belong to the equity holders.

The profit maximization problem of the Home firm with price flexibility is

$$\max P_{h,t}(i)Y_{h,t}(i) + S_t P^*_{h,t}(i)Y^*_{h,t}(i) - \left(\frac{W_t}{A_t}\right)\left[Y_{h,t}(i) + Y^*_{h,t}(i)\right].$$

Because $Y_{h,t}(i)$ is not a function of $P^*_{h,t}(i)$, and $Y^*_{h,t}(i)$ is not a function of $P_{h,t}(i)$, the problem is easy to solve:

$$P_{h,t}(i) = \frac{\lambda}{\lambda-1}\frac{W_t}{A_t} \equiv P_{flex,h,t}, \qquad P^*_{h,t}(i) = \frac{\lambda}{\lambda-1}\frac{W_t}{A_t S_t} \equiv P^*_{flex,h,t}, \tag{2.24}$$

where $P_{flex,h,t}$ is the optimal price for the Home market of the Home goods produced by the firms that can adjust prices after they observe shocks. $P^*_{flex,h,t}$ is the optimal price for the Foreign market.

Sticky-price firms set prices one period in advance in the consumers' currencies for each country. Firms in each country set prices so as to maximize their expected profits, taking other firms' prices as given, which is equivalent to taking the price level as given since each firm has measure zero on interval [0,1]. The optimal prices are

$$P_{preset,h,t} \equiv \frac{\lambda}{\lambda-1}\frac{E_{t-1}\left[\tilde{D}_t \frac{W_t}{A_t}\left(\frac{1}{P_{h,t}}\right)^{-\lambda}\left(\frac{P_{h,t}}{P_t}\right)^{-\omega} C_t\right]}{E_{t-1}\left[\tilde{D}_t \left(\frac{1}{P_{h,t}}\right)^{-\lambda}\left(\frac{P_{h,t}}{P_t}\right)^{-\omega} C_t\right]}, \tag{2.25}$$

$$P^*_{preset,h,t} \equiv \frac{\lambda}{\lambda-1}\frac{E_{t-1}\left[\tilde{D}_t \frac{W_t}{A_t}\left(\frac{1}{P^*_{h,t}}\right)^{-\lambda}\left(\frac{P^*_{h,t}}{P^*_t}\right)^{-\omega} C^*_t\right]}{E_{t-1}\left[\tilde{D}_t \left(\frac{1}{P^*_{h,t}}\right)^{-\lambda}\left(\frac{P^*_{h,t}}{P^*_t}\right)^{-\omega} C^*_t\right]}, \tag{2.26}$$

where $\tilde{D}$ is the stochastic discount factor of firm owners, and $P_{preset,h,t}$ is the optimal price for the Home market at time t of the goods produced by the firms that set prices in advance. Now we can rewrite the price indexes as follows:

$$P_{h,t} = \left[(1-\tau)P^{1-\lambda}_{flex,h,t} + \tau P^{1-\lambda}_{preset,h,t}\right]^{\frac{1}{1-\lambda}}, \tag{2.27}$$

$$P_{f,t} = \left[(1-\tau)P^{1-\lambda}_{flex,f,t} + \tau P^{1-\lambda}_{preset,f,t}\right]^{\frac{1}{1-\lambda}}. \tag{2.28}$$

Since we have CES sub-utility functions, the market clearing condition can be obtained by equating the output with the sum of the demands for Home goods:

$$A_t L_t = \frac{1+\alpha}{2}\left(P_{h,t}/P_t\right)^{-\omega} C_t + \frac{1-\alpha}{2}\left(P^*_{h,t}/P^*_t\right)^{-\omega} C^*_t. \tag{2.29}$$

Aggregate profits of each country are

$$(2.30)\qquad \Pi_t = \frac{1+\alpha}{2} P_{h,t}\left(P_{h,t}/P_t\right)^{-\omega} C_t + \frac{1-\alpha}{2} S_t P^*_{h,t}\left(P^*_{h,t}/P^*_t\right)^{-\omega} C^*_t - W_t L_t \,,$$

$$(2.31)\qquad S_t\Pi^*_t = \frac{1+a}{2} S_t P^*_{f,t}\left(P^*_{f,t}/P^*_t\right)^{-\omega} C^*_t + \frac{1-\alpha}{2} P_{f,t}\left(P_{f,t}/P_t\right)^{-\omega} C_t - S_t W^*_t L^*_t \,.$$

We assume that the logs of the money supplies and logs of the technology shocks evolve according to (where lower-case letters represent the logs of the upper-case counterparts, i.e., $\ln(X_t)$ is denoted as x_t)

$$(2.32)\qquad m_{t+1} = m_t + v^m_{t+1}\,, \qquad m^*_{t+1} = m^*_t + v^{m^*}_{t+1}\,,$$

$$(2.33)\qquad a^W_{t+1} = \vartheta_W a^W_t + v^W_{t+1}\,, \qquad a^R_{t+1} = \vartheta_R a^R_t + v^R_{t+1}\,,$$

where $\vartheta_W \in [0,1]$, $\vartheta_R \in [0,1)$ are degrees of persistence in world and relative technology levels and where the vector v^x_t ($x = m, m^*, W, R$) is i.i.d.. We denote the world variables as $x^W_t = \frac{1}{2}x_t + \frac{1}{2}x^*_t$, and the relative variables as $x^R_t = x_t - x^*_t$. We assume $Ev^m_{t+1} = Ev^{m^*}_{t+1} = \frac{1}{2}\sigma^2_m$, so that $E_t(M^{-1}_{t+1}) = M^{-1}_t$ as mentioned above. We assume also $\text{var}(v^m) = \text{var}(v^{m^*}) = \sigma^2_m$, and $\text{cov}(v^m, v^{m^*}) = \sigma_{m,m^*}$, $\text{var}(v^W) = \sigma^2_W$, $\text{var}(v^R) = \sigma^2_R$, and $\text{cov}(v^W, v^R) = 0$. We assume initial symmetry between Home and Foreign: that is, $a^R_0 = 0$, and $m^R_0 = 0$.

Note in particular that we have not made any assumptions about the correlation of monetary shocks and productivity shocks. As long as there is some independent component to the money shocks – that is, as long as the correlation between money and productivity shocks lies on the interval (-1,1) – our results go through. In particular, our specification allows for an interpretation in which technology shocks, v^W_{t+1} and v^R_{t+1} are structural, and monetary shocks respond contemporaneously to technology shocks: for example, $v^m_{t+1} \equiv \varepsilon^m_{t+1} + \xi^W v^w_{t+1} + \xi^R v^R_{t+1}$ and $v^{m^*}_{t+1} \equiv \varepsilon^{m^*}_{t+1} + \xi^W v^w_{t+1} - \xi^R v^R_{t+1}$, where ε^m_{t+1} and $\varepsilon^{m^*}_{t+1}$ are structural monetary shocks.

C. Equilibrium Portfolios under LCP and Flexible Wages

We follow the solution method of Engel and Matsumoto (forthcoming), in which we log-linearize the budget constraint, resource constraint, and the non-portfolio first-order conditions. We use a second-order approximation to the asset choice Euler equations. Note that our method of solution is identical to the one proposed by Devereux and Sutherland (2006, forthcoming) for our model.

It is important to note that in this model, and in all of the models that we present, trade in equities and a forward position in foreign exchange is sufficient to replicate the allocations under complete markets. That is because in all cases, once we log-linearize the equations of the dynamics of the model, the payoffs from these assets span the state space. So, as in Engel and Matsumoto (forthcoming), and the simple model of section I, risk sharing condition equation (1.12) holds, which is repeated here for convenience:

$$c_t - c_t^* - \frac{1}{\rho}(s_t + p_t^* - p_t) = 0 \qquad (2.34)$$

The question we wish to address in this paper is what types of portfolios will support this complete market allocation? Our answer is that under a wide range of plausible assumptions, this allocation can be supported with only a small amount of equity trade (and, thus, a lot of home bias in equities), but it requires a bond portfolio in which households lend in their own currency and borrow in foreign currency.

The Appendix derives the following expressions for the optimal portfolios of equities and the forward position in foreign exchange. In equilibrium, because the log-linearized model replicates the complete-markets equilibrium, the Home/Foreign relative marginal utility of nominal wealth is constant. This imparts a stationarity to the economy that supports optimal portfolio positions that are constant over time. The optimal share of Foreign equities in the Home equity portfolio is given by

$$\gamma = \frac{1}{2}\frac{\left[\varpi - 1 + \left(\frac{\rho - 1}{\rho}\right)\alpha\right]\left\{\frac{1-\tau}{1+(1-\tau)\psi\varpi} + \frac{1}{\varpi\psi + 1}\frac{\beta\vartheta_R}{1-\beta\vartheta_R}\right\}}{(1-\zeta)(\varpi - 1)\left\{\frac{1-\tau}{1+(1-\tau)\psi\varpi} + \frac{1}{\varpi\psi + 1}\frac{\beta\vartheta_R}{1-\beta\vartheta_R}\right\} + \zeta\frac{\tau}{1+(1-\tau)\psi\varpi}}, \qquad (2.35)$$

where

$$\varpi \equiv \frac{\alpha^2}{\rho} + \omega(1+\alpha)(1-\alpha).$$

(recalling that ω is the elasticity of substitution among brands of goods produced within each country, ρ is the coefficient of relative risk aversion, and $\frac{1+\alpha}{2}$ is the share of Home goods in Home consumption if all goods prices were equal, so that $\alpha > 0$ implies home bias in consumption.) In this expression, also, ζ is labor's share of revenue in the non-stochastic steady state. Recall also that ψ^{-1} is the elasticity of labor supply, β is the discount factor in utility, ϑ_R is the autocorrelation of relative productivity shocks, and τ is the fraction of firms that set prices in advance.

To understand this expression, it is useful to consider some special cases. First, suppose that all goods prices are flexible, so that $\tau = 0$. Then (2.35) reduces to

$$\gamma = \frac{\varpi - 1 + \left(\frac{\rho - 1}{\rho}\right)\alpha}{2(1-\zeta)(\varpi - 1)}. \tag{2.36}$$

When goods prices are flexible, in general, we find "anti-home-bias" in the optimal equity portfolio. If there were no home bias in preferences ($\alpha = 0$), expression (2.36) simplifies further to $\gamma = 1/[2(1-\zeta)] > 1/2$. That is, Home's holdings of Foreign equities would constitute more than half the equity portfolio. This simply reflects the observation by Baxter and Jermann (1997) that, under flexible prices and wages, the return to Home human capital is positively correlated with the return to Home equities. So, optimally, the portfolio should short Home equities. In equation (2.36), we see that home bias in preferences does not modify this result except under strong home bias in preferences, when $\varpi < 1$. If utility is logarithmic in consumption (as is assumed in Heathcote and Perri (2008) or Pavlova and Rigobon (2007)), so $\rho = 1$, the optimal share of foreign equities is still $\gamma = 1/[2(1-\zeta)]$, as it was without home bias in preferences.

More generally, under the plausible parameterization $\rho > 1$, we can see that a high degree of home bias in preferences, such that $\varpi < 1$, can generate extreme home bias in equity holdings.[10] Suppose there were complete home bias in preferences, so that $\alpha = 1$. Effectively, the two countries are closed economies. Only productivity shocks matter for real allocations under flexible prices and wages. But if Home agents do not wish to buy any foreign goods, than the Foreign equity provides no hedge for Home productivity shocks. To see this, note that when $\alpha = 1$, we have $\varpi = 1/\rho$. In this case, from equation (2.36), we find $\gamma = 0$. It is important to stress that in this case, the portfolio with no equity diversification achieves the same allocation as would occur under complete markets, but in both cases asset markets do nothing to reduce consumption risk. When households in each country produce the only good which they want to consume, it is not possible to hedge productivity shocks.

Now consider the case in which all nominal prices are set one period in advance ($\tau = 1$). Suppose that either agents put no weight on the future $\beta = 0$, or relative productivity shocks are serially uncorrelated ($\vartheta_R = 0$.) Under these assumptions, we

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[10] A detailed discussion of the discontinuity around $\varpi = 1$ can be found in Obstfeld (2007) and Coeurdacier (forthcoming).

effectively replicate the conditions of the model of section I: all prices are sticky, and the future does not matter to agents. We see from equation (2.36) that under these assumptions, $\gamma = 0$. That is, the optimal holdings of Foreign equities by Home households in the portfolio that replicates complete market allocations is zero. This is the result illustrated in section I: equity trade is not needed (under these circumstances) to support efficient risk sharing. We note that in this section we have derived optimal portfolios, taking into consideration all of the first-order conditions of households and firms. That is in contrast to section I where we only asked which sorts of portfolios would support complete market allocations without asking whether the equilibrium could be decentralized.

Suppose there is no home bias in preferences, so $\alpha = 0$. Then expression (2.35) reduces to

$$\gamma = \frac{1}{2}\frac{(\omega-1)\left\{\frac{1-\tau}{1+(1-\tau)\psi\omega}+\frac{1}{\omega\psi+1}\frac{\beta\vartheta_R}{1-\beta\vartheta_R}\right\}}{(1-\zeta)(\omega-1)\left\{\frac{1-\tau}{1+(1-\tau)\psi\omega}+\frac{1}{\omega\psi+1}\frac{\beta\vartheta_R}{1-\beta\vartheta_R}\right\}+\zeta\frac{\tau}{1+(1-\tau)\psi\omega}}. \quad (2.37)$$

This is exactly the formula derived in Engel and Matsumoto (forthcoming), which derives optimal portfolios for the special case of this model (LCP, flexible wages) with no home bias in preferences. Note that in this expression, we will have $\gamma = 0$ when $\omega = 1$. When $\omega = 1$, there are Cobb-Douglas preferences over the Home and Foreign aggregates. In this case, when there is no home bias in preferences, if prices were flexible, the terms of trade changes would fully insure risk (as in Cole and Obstfeld, 1991.) Indeed, were all prices flexible, and $\omega = 1$, the equilibrium equity portfolio would be indeterminate. As Engel and Matsumoto (forthcoming) discuss, in that case, there is no role for the equity portfolio in diversifying real risk since the terms of trade do all of the job, so the composition of the equity portfolio is not determined. On the other hand, when there is some price stickiness, the equity portfolio does help diversify risk. But then, this situation is like the model of section I: all risk can be diversified by the forward position in foreign exchange, so it is optimal to hold no Foreign equities in the Home equity portfolio. Again, the efficient allocation can be achieved without trade in equities.

When there is home bias in preferences, the analogous condition to the one just discussed is $\varpi - 1 + \left(\frac{\rho-1}{\rho}\right)\alpha = 0$ (see (2.35)). Under this condition, full risk sharing is achieved without any diversification in the equity portfolio. But, it is important to point out that this condition holds even under flexible prices (see (2.36)).[11] That is, the equilibrium equity portfolio is determined even under flexible prices, because with home bias, in general, the terms of trade changes do not fully hedge risk.

Table 1 presents values for the share of Foreign equities in the Home portfolio, γ, for calibrated parameter values substituted into equation (2.35). As Engel and Matsumoto (forthcoming) explain, it is not a simple task to calibrate the degree of price stickiness in our model to empirical measures of the speed of price adjustment. The share of the Home household's equity portfolio held in foreign shares, γ, depends on the price stickiness parameter, τ; labor's share, ζ; the elasticity of substitution between Home and Foreign aggregates, ω; the discount factor, β; the persistence of relative productivity shocks, ϑ_R; the elasticity of labor supply, ψ; the coefficient of risk aversion ρ; and, the home-bias in preferences parameter, α.

As in Engel and Matsumoto, and following the literature, we set $\zeta = 2/3$ and $\psi = 1$.

In most calibrations of new-Keynesian models with nominal price stickiness, the expected life of a nominal price (under Calvo price setting) is calibrated to be four quarters. Our model of price stickiness does not translate easily into the Calvo framework, however, where the life of a price follows a Poisson process. In our model, a measure τ of firms set prices for one period, and a measure $1-\tau$ adjust prices instantaneously. So the expected life of a price is τ periods. We calibrate the degree of price stickiness in the following way: we consider different values for τ, ranging from 0.05 to 1. We can then set the length of a period so that the fraction τ of a period equals four quarters, or one period equals $4/\tau$ quarters. In Table 2, we present the equity shares we have calculated for the various values of τ.

As in Engel and Matsumoto (forthcoming), we set $\vartheta_R = (0.855)^{4/\tau}$ and $\beta = (0.99)^{4/\tau}$. So, to be clear, while we vary τ, we change the persistence of relative productivity shocks and the discount factor. We do that in such a way that the expected life of a price is always equal to four quarters. This is our way of dealing with the fact that there is not a unique way of translating our set-up into one in which prices have an expected life of four quarters.

There is no consensus in the literature about the appropriate value of the elasticity of substitution, ω. However, an average value in the international

[11] Except for the case when $\varpi = 1$ and $\rho = 1$.

macroeconomics literature is around 1.2, as Engel and Matsumoto (forthcoming) explain.

The first panel in Table 1 lets τ vary, while setting the home bias parameter at 0, so there is no home bias. That panel shows that for most choices of τ, the portfolio that supports the complete-markets allocation displays a high degree of home bias. That is, as we have stated, not very much equity portfolio diversification is required to achieve this risk-sharing equilibrium. However, it is necessary to have price stickiness in order to have home bias in equity.

The second and third panels of Table 1 look at the effects of home bias in preferences on the portfolio required to support full risk sharing. The second panel fixes τ at 0.5 and lets α vary from 0 to 1, while the third panel fixes τ at 1.0 and lets α vary. There are several things to take away from this table: First, for most of the range of possible values of α, increasing home bias in preferences has little effect on the optimal equity portfolio. Only when $\alpha > 0.9$ do we see much of an effect. Of course, with complete home bias in preferences, $\alpha = 1$, there is complete home bias in equity portfolios, $\gamma = 0$. For all values of α, however, the share of foreign equities in the optimal Home equity portfolio is quite small.

This finding is in strong contrast to that of Obstfeld (2007) and Coeurdacier (forthcoming), which show very large swings in the optimal portfolio with respect to α. Their models are endowment models with flexible prices, where home bias in preferences or trade costs can explain home bias in equity.[12] Under the special case of no labor income,[13] $\zeta = 0$, we find $\gamma = \frac{1}{2}\left[1 + (\rho - 1)\alpha / \rho(\varpi - 1)\right]$, whether or not prices are sticky, which corresponds to Obstfeld's expression. It is the interaction of price stickiness and the role of equities in hedging labor income in the short run that lead to our finding that the optimal equity portfolio is not too sensitive to the assumption of home bias in preferences.

We have focused so far on the composition of the equity portfolio needed to achieve the complete-markets allocation. Now we turn to the forward position in foreign exchange. The Appendix shows:

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[12] They extend their model to include nontraded goods. See also Collard, Dellas, Diba, and Stockman (2007)
[13] van Wincoop and Warnock (2008) also find that empirically home bias in preferences cannot explain home bias in equities, but they abstract from labor income in their study.

$$\delta = -\frac{1}{2}\left(\frac{\rho-1}{\rho}\right)\tau + \frac{1}{2}\left(\frac{\rho-1}{\rho}\right)\tau\frac{\alpha}{\rho}\left(\frac{\alpha(1-\tau)\psi}{1+(1-\tau)\psi\varpi}\right)$$

$$-\frac{1}{2}\tau\frac{\alpha}{\rho}(1-2\gamma)\left\{1-\rho-\left(\frac{\zeta(1+\psi)+(1-\tau)(\varpi-1)\psi}{1+(1-\tau)\psi\varpi}\right)\right\} \tag{2.38}$$

$$-\frac{1}{2}\tau\frac{\alpha}{\rho}\frac{\zeta(1+\psi)}{1+(1-\tau)\psi\varpi}$$

Although this expression could be simplified by canceling some terms, we write it in this somewhat rococo way in order to facilitate the economic explanation. The γ in this expression refers to the optimal value of γ given in equation (2.35).

First, let us consider some special cases. The simplest special case is the one in which all prices are flexible: $\tau = 0$. From equation (2.38), we see that the optimal nominal exchange rate hedge is zero: $\delta = 0$. That result obtains because when prices are flexible, changes in nominal variables do not reflect real risks, so a nominal exchange rate hedge is not necessary. All real risk can be hedged with the equity portfolio.

Second, suppose there is no home bias in preferences, so $\alpha = 0$. Then equation (2.38) reduces to

$$\delta = -\frac{1}{2}\left(\frac{\rho-1}{\rho}\right)\tau.$$

This is exactly the expression derived in Engel and Matsumoto (forthcoming), who only consider the case of LCP and no home bias in preferences. Intuitively, if there is LCP and consumption baskets are identical at home and abroad, then exchange rates have real impacts through their effect on the value of profits earned by firms. As we explained in section I, a depreciation of the Home currency will, for LCP firms, increase the value in Home currency terms of revenues earned by Home and Foreign firms from their sales in the Foreign country. This will increase the purchasing power of Home households. This fluctuation is hedged by taking a short position in Foreign currency when $\rho > 1$.

Third, suppose that all prices are sticky and $\vartheta_R = 0$ or $\beta = 0$. We have already seen in that case that the optimal value of γ given in equation (2.35), is zero. Setting $\gamma = 0$ and $\tau = 1$ in equation (2.38), we get an expression for δ that is the same as the one in section I, equation (1.15), under LCP (so setting $b = 0$ in (1.15)).

In general, when $\alpha \neq 0$, the second term on the right-hand-side of equation (2.38), $\frac{1}{2}\left(\frac{\rho-1}{\rho}\right)\tau\frac{\alpha}{\rho}\left(\frac{\alpha(1-\tau)\psi}{1+(1-\tau)\psi\varpi}\right)$, arises from the partial adjustment of prices and home bias in consumption. This term comes from the fact that exchange rate shocks affect flexible-price prices, which in turn have a differential impact on demand from Home and Foreign households and profits from Home and Foreign firms because of home bias in consumption. When all prices are sticky, $\tau = 1$, this term disappears.

The third term on the right-hand-side of equation (2.38) hedges the effect of exchange rate changes on the relative returns of Home and Foreign equity portfolios. Note that this term would drop out if there were no home bias in equity portfolios ($\gamma = 1/2$).

The final term, in the third line, is the hedge to relative human capital, as this is also affected by exchange rate changes when there is home bias in consumption.

The final column in Table 1 displays various values of δ from equation (2.38) under the various parameterizations described above. In all cases, $\delta < 0$ (unless there is complete home bias in consumption, $\alpha = 1$, in which case $\delta = 0$.) In order to support the optimal risk-sharing allocation, households in the Home country need to take a short position in Foreign currency. That is, they must have a currency portfolio that has zero worth ex ante, but one in which they borrow short-term in Foreign currency and lend short term in Home currency.

Trade in equities and a forward position in foreign exchange can efficiently share risk, in the sense that they can obtain the same outcome as trade in a complete market of contingent claims. Moreover, we have shown that under reasonable parameterizations, the equity portfolio that supports this allocation does not display much international diversification. However, it is crucial for this outcome that agents take an optimal exchange-rate hedge.

D. Equilibrium Portfolios under LCP and Sticky Wages

Obstfeld (2007) has argued that if nominal wages are sticky, the optimal portfolio must exhibit a large amount of international diversification of equity portfolios. However, Obstfeld's calculations are in the context of a model with flexible goods prices. Here we investigate how optimal portfolios are affected when nominal wages are set in advance, under plausible parameterizations of nominal price stickiness.

Intuitively, consider in a static setting, a firm whose price is set in advance, and the firm experiences an increase in productivity. Ceteris paribus, the increase in productivity does not have any effect on revenues of the firm, because the firm cannot alter its price. Given that demand for its product is unchanged but productivity increases, the firm's demand for labor services declines. The firm captures more of the revenue as profit, and worker's payments are reduced. This is true whether or not wages are flexible.

If productivity increases, the marginal cost of the firm's product falls. Under flexible prices, the monopolistic firm will find it optimal to lower price and increase quantity sold. In order to produce more, the firm must hire more labor, so its demand for labor increases. Thus, the firm and workers both benefit from a productivity increase. This is true whether or not wages are flexible.

In the sticky-price case, profits are negatively correlated with labor compensation, while in the flexible-price case they are positively correlated. This suggests that under sticky prices, workers can hedge their labor income risk by owning shares of the firms that they work in. But under flexible prices, profits and labor income are positively correlated, so workers have an incentive to short ownership in the firm in which they are employed.

While this intuition suggests that it is price stickiness, not wage stickiness, that is the key to determining the optimal portfolio position, the logic is far from airtight. The problem is that, in general equilibrium, we cannot perform this "ceteris paribus" experiment. We cannot hold all else constant: changes in other firms' prices, changes in aggregate demand, changes in exchange rates, all of which might be correlated with labor income and profits. So, we formally investigate a version of the LCP model with nominal wage stickiness.

We need to make a couple of modifications of the model presented so far. First, we now assume that the labor input of a given household is imperfectly substitutable in production for labor of other households. Home firms (and, symmetrically, Foreign firms) must use labor input from each household to produce the final output, using a CES production function. That is, equation (2.23) still gives the production function for firm i, but now $L_t(i)$ should be considered an aggregate over the labor input from every household:

(2.39) $$L_t(i) = \left(\int_0^{1/2} L_t(z,i)^{\frac{\lambda_L - 1}{\lambda_L}} \right)^{\frac{\lambda_L}{\lambda_L - 1}}.$$

Households are the monopolistic supplier of their own labor, but are constrained to set their wage one period in advance. We get the familiar equation under sticky wages (where we have imposed the equilibrium condition that all Home households are identical):

(2.40) $$W_t = \frac{\lambda_L}{\lambda_L - 1} \frac{E_{t-1}(L_t^{1+\psi})}{E_{t-1}(C_t^{-\rho} P_t^{-1} L_t)}.$$

We only consider the case in which all wages are sticky. That is, in contrast to how we treat price stickiness, we only consider the cases of all wages are flexible or all wages are set in advance. We make these assumptions because we want to maintain the characteristic that all households are identical in equilibrium.

The Appendix derives the following expression for γ, the share of Foreign equities in the Home equity portfolio:

(2.41) $$\gamma = \frac{1}{2} \frac{\left[\varpi - 1 + \left(\frac{\rho - 1}{\rho}\right)\alpha\right]\left\{\frac{1-\tau}{1+\psi} + \frac{1}{\varpi\psi + 1}\frac{\beta\vartheta_R}{1-\beta\vartheta_R}\right\}}{(1-\zeta)(\varpi - 1)\left\{\frac{1-\tau}{1+\psi} + \frac{1}{\varpi\psi + 1}\frac{\beta\vartheta_R}{1-\beta\vartheta_R}\right\} + \zeta\frac{\tau}{1+\psi}}.$$

Compare this expression to (2.35) under flexible wages: they are very similar. Indeed they are identical if $(1-\tau)\psi\varpi = \psi$. One condition under which that would be achieved is if households had linear disutility of work: $\psi = 0$. Alternatively, for any value of ψ, the equity portfolio under fixed and flexible wages would be identical if $(1-\tau)\varpi = (1-\tau)\left[\alpha^2/\rho + \omega(1+\alpha)(1-\alpha)\right] = 1$.[14] For example, if there were no home bias in preferences $(\alpha = 0)$, this expression reduces to $(1-\tau)\omega = 1$. Our standard calibration has set $\omega = 1.2$, so if $\tau = 1/1.2 \approx .83$, the portfolio is the same under fixed and flexible wages.

The expression for the equilibrium foreign exchange hedge is also not changed very much:

(2.42) $$\begin{aligned}\delta = &-\frac{1}{2}\left(\frac{\rho-1}{\rho}\right)\tau - \frac{1}{2}\left(\frac{\rho-1}{\rho}\right)(1-\tau)\alpha \\ &-\frac{1}{2}(1-\zeta)(1-2\gamma)\left\{(1-\tau)(\varpi-1) - \tau\left(1-\frac{\alpha}{\rho}\right) + \frac{1}{1-\zeta}\tau(1-\alpha)\right\} \\ &-\frac{1}{2}\zeta\left[(1-\tau)(\varpi-1) - \tau\left(1-\frac{\alpha}{\rho}\right)\right]\end{aligned}.$$

The γ in this expression refers to the optimal value of γ given in equation (2.41). We can interpret these terms in the same way as equation (2.38). Again, the second line represents the hedge against the effect of the exchange rate on returns to the equity portfolio, and the third line to returns to human capital.

[14] This is because when $(1-\tau)\varpi = 1$, the relative return on human capital and the relative return on equity will be same under flexible wage and sticky wage once eliminate the effect of exchange rate, which can be hedged by forward contracts.

It is interesting to look at some special cases. First, just as in the case with flexible wages, setting $\gamma = 0$ (complete home bias) and $\tau = 1$ (so all prices are sticky) in equation (2.42), we get an expression for δ that is the same as the one in section I, equation (1.15), under LCP (so setting $b = 0$ in (1.15)).

Second, take the case of complete goods price flexibility, $\tau = 0$. In this case, expression (2.42) becomes $\delta = -(1 - 1/\rho)\alpha/2 - (1-\zeta)(1-2\gamma)(\varpi - 1)/2 - \zeta(\varpi - 1)/2$.

From, (2.41), we have $\gamma = \frac{1}{2}\frac{\varpi - 1 + \left(\frac{\rho - 1}{\rho}\right)\alpha}{(1-\zeta)(\varpi - 1)}$. Substituting into the equation for δ, we find $\delta = 0$. Even if nominal wages are sticky, the optimal exchange rate hedge under flexible prices is zero.

Table 2 presents optimal values of γ and δ under the same parameterizations as Table 1. The notable thing about the values reported in Table 2 is that they are very close to those in Table 1. Nominal wage stickiness does not have a large influence on the portfolio that supports efficient allocation of risk when nominal price stickiness is calibrated realistically.

E. A Dynamic Sticky-Price Model with Producer-Currency Pricing

We now consider the case of producer-currency pricing. Sticky-price firms in each country set a single price in their own currency. Equation (2.25) gives the pricing equation for sticky-price firms in the Home country. That is, PCP firms set prices according to the same formula that LCP firms do for sale in the Home country. The rest of the model is unchanged.

The most important result we find (demonstrated in the Appendix) is that the optimal equity portfolio is the same under LCP and PCP. That is, under PCP and flexible wages, equation (2.35) gives the share of Foreign equities in the Home portfolio, and under PCP and sticky wages, equation (2.41) gives that share.

Why is the equity portfolio not influenced by whether pricing is determined by LCP or PCP? The way in which firms set prices affects only the way the exchange rate influences prices. But the effects of exchange rate changes can be hedged with the foreign exchange hedge. Once δ is chosen optimally, the equity portfolio does not depend on the influence of exchange rates.

Of course, the optimal value of δ will be different under PCP compared to LCP. When nominal wages are flexible, we find

(2.43) $\delta = -\frac{1}{2}\frac{\tau}{1+(1-\tau)\psi\varpi}\left[\left(\frac{\rho-1}{\rho}\right)\alpha + (1-2\gamma)\left[(1-\zeta)(1+\psi)\varpi - (1+\psi\varpi)\right] - \zeta(1+\psi)\varpi\right].$

The γ in this expression refers to the optimal value of γ given in equation (2.35).

Once again, if prices are perfectly flexible ($\tau = 0$), the optimal exchange rate hedge is zero. Also, analogous to what we found under LCP, setting $\gamma = 0$ (complete home bias) and $\tau = 1$ (so all prices are sticky) in equation (2.43), we get an expression for δ that is the same as the one in section I, equation (1.15), under PCP (so setting $b = 1$ in (1.15)).

Table 3 presents values for γ and δ under the same set of parameter assumptions as in Tables 1-2. Of course, the values for γ are the same as in Table 1. It remains the case that for all parameter values, $\delta < 0$ (unless there is complete home bias in consumption, $\alpha = 1$, in which case $\delta = 0$.)

When there is PCP and sticky wages, we find

(2.44) $$\delta = -\frac{1}{2}\left[\left(\frac{\rho-1}{\rho}\right)\alpha + (1-2\gamma)(1-\zeta)(\varpi-1) + \zeta(\varpi-1)\right]$$

The γ in this expression refers to the optimal value of γ given in equation (2.41). If prices are perfectly flexible ($\tau = 0$), we can substitute the optimal value of γ from equation (2.41) into equation (2.44), to find that the optimal exchange rate hedge is zero. Also, analogous to what we have found in the other cases, setting $\gamma = 0$ (complete home bias) and $\tau = 1$ (so all prices are sticky) in equation (2.44), we get an expression for δ that is the same as the one in section I, equation (1.15), under PCP (so setting $b = 1$ in (1.15)).

Table 4 presents values for γ and δ under the same set of parameter assumptions as in Tables 1-3 for the case of PCP pricing and sticky wages. Of course, the values for γ are the same as in Table 2. It remains the case that for all parameter values, $\delta < 0$ (unless there is complete home bias in consumption, $\alpha = 1$, in which case $\delta = 0$.)

III. Conclusion

In sum, we have examined a series of dynamic models with productivity shocks and monetary shocks. In all of these models, if Home and Foreign households can trade equity shares in Home and Foreign firms, as well as take a forward position in foreign exchange, they can replicate the allocations achieved under complete markets. We know that in practice, real world financial markets do not allocate risk well. A major outstanding puzzle in the international macroeconomics literature is the failure of the risk-sharing condition, (1.12). It is often thought that this “consumption-real-exchange-rate anomaly” or “Backus-Smith puzzle” is related to the well-known home bias in equity portfolios – equity portfolios are not well diversified internationally. We examine four models. All allow for home bias in preferences. We consider different types of wage and price stickiness: LCP and flexible wages; LCP and sticky wages; PCP and flexible wages; and, PCP and sticky wages.

The conclusion is the same in all models. Under reasonable parameterizations, the efficient risk-sharing outcome does not require portfolios with much international equity diversification. But they do all require that agents hedge foreign exchange risk by going long in their own currency.

So we conclude that it is a mistake to tie the “consumption-real-exchange-rate anomaly” too closely to the home bias in equities puzzle. Instead, it is more closely tied to a puzzle about bond portfolios – why do we not see portfolios where there is more borrowing in foreign currency terms and lending in domestic currencies? Indeed, since countries have an incentive to inflate and depreciate when debt is denominated in the currency of the borrower, it is especially puzzling that we do not observe the pattern of bond portfolios that would lead to more efficient allocation of risk.

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Table 1

Optimal Portfolios under LCP, Flexible Wages

Panel 1			
α	τ	γ	δ
0	0.05	0.98	-0.013
0	0.1	0.71	-0.025
0	0.15	0.55	-0.038
0	0.2	0.44	-0.050
0	0.25	0.37	-0.063
0	0.3	0.32	-0.075
0	0.35	0.28	-0.088
0	0.4	0.25	-0.100
0	0.45	0.22	-0.113
0	0.5	0.20	-0.125
0	0.55	0.18	-0.138
0	0.6	0.16	-0.150
0	0.65	0.15	-0.163
0	0.7	0.13	-0.175
0	0.75	0.12	-0.188
0	0.8	0.11	-0.200
0	0.85	0.10	-0.213
0	0.9	0.09	-0.225
0	0.95	0.08	-0.238
0	1	0.07	-0.250
Panel 2			
α	τ	γ	δ
0	0.5	0.20	-0.125
0.05	0.5	0.22	-0.124
0.1	0.5	0.24	-0.123
0.15	0.5	0.26	-0.123
0.2	0.5	0.27	-0.124
0.25	0.5	0.29	-0.125
0.3	0.5	0.30	-0.126
0.35	0.5	0.30	-0.128
0.4	0.5	0.31	-0.129
0.45	0.5	0.31	-0.130
0.5	0.5	0.31	-0.130
0.55	0.5	0.31	-0.130
0.6	0.5	0.30	-0.128
0.65	0.5	0.29	-0.125
0.7	0.5	0.27	-0.120
0.75	0.5	0.25	-0.113
0.8	0.5	0.22	-0.103
0.85	0.5	0.18	-0.088
0.9	0.5	0.14	-0.067
0.95	0.5	0.08	-0.039
1	0.5	0.00	0.000

Panel 3			
α	τ	γ	δ
0	1	0.069	-0.250
0.05	1	0.077	-0.242
0.1	1	0.084	-0.235
0.15	1	0.090	-0.228
0.2	1	0.095	-0.222
0.25	1	0.099	-0.216
0.3	1	0.103	-0.211
0.35	1	0.105	-0.205
0.4	1	0.107	-0.200
0.45	1	0.107	-0.194
0.5	1	0.107	-0.187
0.55	1	0.105	-0.180
0.6	1	0.102	-0.172
0.65	1	0.098	-0.162
0.7	1	0.092	-0.150
0.75	1	0.084	-0.136
0.8	1	0.074	-0.119
0.85	1	0.062	-0.099
0.9	1	0.046	-0.073
0.95	1	0.026	-0.041
1	1	0.000	0.000

Notes: Optimal share of Foreign equities in Home equity portfolio, γ, calculated from equation (2.35), and optimal Home position in foreign exchange, δ, calculated from equation (2.38) for various values of home bias in preferences, α, and price stickiness, τ. Other parameters are calibrated as: $\zeta = 2/3$, $\psi = 1$, $\vartheta_R = (0.855)^{4/\tau}$, $\beta = (0.99)^{4/\tau}$, and $\omega = 1.2$.

Table 2

Optimal Portfolios under LCP, Sticky Wages

Panel 1			
α	τ	γ	δ
0	0.05	0.98	-0.012
0	0.1	0.71	-0.025
0	0.15	0.55	-0.037
0	0.2	0.44	-0.048
0	0.25	0.37	-0.057
0	0.3	0.32	-0.066
0	0.35	0.28	-0.075
0	0.4	0.25	-0.082
0	0.45	0.23	-0.090
0	0.5	0.21	-0.097
0	0.55	0.20	-0.104
0	0.6	0.19	-0.111
0	0.65	0.18	-0.118
0	0.7	0.17	-0.124
0	0.75	0.16	-0.131
0	0.8	0.15	-0.137
0	0.85	0.15	-0.144
0	0.9	0.14	-0.150
0	0.95	0.14	-0.156
0	1	0.13	-0.163
Panel 2			
α	τ	γ	δ
0	0.5	0.21	-0.097
0.05	0.5	0.24	-0.092
0.1	0.5	0.26	-0.089
0.15	0.5	0.28	-0.087
0.2	0.5	0.30	-0.085
0.25	0.5	0.31	-0.085
0.3	0.5	0.32	-0.085
0.35	0.5	0.33	-0.085
0.4	0.5	0.34	-0.086
0.45	0.5	0.34	-0.086
0.5	0.5	0.34	-0.087
0.55	0.5	0.34	-0.087
0.6	0.5	0.34	-0.087
0.65	0.5	0.33	-0.086
0.7	0.5	0.32	-0.084
0.75	0.5	0.30	-0.080
0.8	0.5	0.27	-0.074
0.85	0.5	0.23	-0.065
0.9	0.5	0.18	-0.052
0.95	0.5	0.11	-0.031
1	0.5	0.00	0.000

Panel 3			
α	τ	γ	δ
0	1	0.131	-0.163
0.05	1	0.147	-0.146
0.1	1	0.161	-0.131
0.15	1	0.173	-0.119
0.2	1	0.183	-0.109
0.25	1	0.192	-0.100
0.3	1	0.199	-0.092
0.35	1	0.205	-0.086
0.4	1	0.209	-0.080
0.45	1	0.212	-0.076
0.5	1	0.212	-0.072
0.55	1	0.211	-0.069
0.6	1	0.207	-0.066
0.65	1	0.201	-0.062
0.7	1	0.192	-0.059
0.75	1	0.179	-0.055
0.8	1	0.162	-0.050
0.85	1	0.138	-0.043
0.9	1	0.106	-0.034
0.95	1	0.062	-0.020
1	1	0.000	0.000

Notes: Optimal share of Foreign equities in Home equity portfolio, γ, calculated from equation (2.41), and optimal Home position in foreign exchange, δ, calculated from equation (2.42) for various values of home bias in preferences, α, and price stickiness, τ. Other parameters are calibrated as: $\zeta = 2/3$, $\psi = 1$, $\vartheta_R = (0.855)^{4/\tau}$, $\beta = (0.99)^{4/\tau}$, and $\omega = 1.2$.

Table 3

Optimal Portfolios under PCP, Flexible Wages

Panel 1			
α	τ	γ	δ
0	0.05	0.98	-0.034
0	0.1	0.71	-0.053
0	0.15	0.55	-0.064
0	0.2	0.44	-0.073
0	0.25	0.37	-0.081
0	0.3	0.32	-0.088
0	0.35	0.28	-0.096
0	0.4	0.25	-0.103
0	0.45	0.22	-0.110
0	0.5	0.20	-0.118
0	0.55	0.18	-0.125
0	0.6	0.16	-0.133
0	0.65	0.15	-0.140
0	0.7	0.13	-0.148
0	0.75	0.12	-0.156
0	0.8	0.11	-0.164
0	0.85	0.10	-0.172
0	0.9	0.09	-0.180
0	0.95	0.08	-0.188
0	1	0.07	-0.196
Panel 2			
α	τ	γ	δ
0	0.5	0.20	-0.118
0.05	0.5	0.22	-0.132
0.1	0.5	0.24	-0.144
0.15	0.5	0.26	-0.154
0.2	0.5	0.27	-0.163
0.25	0.5	0.29	-0.171
0.3	0.5	0.30	-0.176
0.35	0.5	0.30	-0.181
0.4	0.5	0.31	-0.183
0.45	0.5	0.31	-0.184
0.5	0.5	0.31	-0.183
0.55	0.5	0.31	-0.180
0.6	0.5	0.30	-0.175
0.65	0.5	0.29	-0.168
0.7	0.5	0.27	-0.158
0.75	0.5	0.25	-0.145
0.8	0.5	0.22	-0.128
0.85	0.5	0.18	-0.107
0.9	0.5	0.14	-0.079
0.95	0.5	0.08	-0.045
1	0.5	0.00	0.000

Panel 3			
α	τ	γ	δ
0	1	0.069	-0.196
0.05	1	0.077	-0.219
0.1	1	0.084	-0.239
0.15	1	0.090	-0.255
0.2	1	0.095	-0.268
0.25	1	0.099	-0.278
0.3	1	0.103	-0.285
0.35	1	0.105	-0.289
0.4	1	0.107	-0.289
0.45	1	0.107	-0.287
0.5	1	0.107	-0.281
0.55	1	0.105	-0.271
0.6	1	0.102	-0.258
0.65	1	0.098	-0.242
0.7	1	0.092	-0.222
0.75	1	0.084	-0.197
0.8	1	0.074	-0.169
0.85	1	0.062	-0.135
0.9	1	0.046	-0.097
0.95	1	0.026	-0.052
1	1	0.000	0.000

Notes: Optimal share of Foreign equities in Home equity portfolio, γ, calculated from equation (2.35), and optimal Home position in foreign exchange, δ, calculated from equation (2.43) for various values of home bias in preferences, α, and price stickiness, τ. Other parameters are calibrated as: $\zeta = 2/3$, $\psi = 1$, $\vartheta_R = (0.855)^{4/\tau}$, $\beta = (0.99)^{4/\tau}$, and $\omega = 1.2$.

Table 4

Optimal Portfolios under PCP, Sticky Wages

Panel 1			
α	τ	γ	δ
0	0.05	0.98	-0.034
0	0.1	0.71	-0.053
0	0.15	0.55	-0.063
0	0.2	0.44	-0.071
0	0.25	0.37	-0.075
0	0.3	0.32	-0.079
0	0.35	0.28	-0.081
0	0.4	0.25	-0.083
0	0.45	0.23	-0.085
0	0.5	0.21	-0.086
0	0.55	0.20	-0.087
0	0.6	0.19	-0.088
0	0.65	0.18	-0.088
0	0.7	0.17	-0.089
0	0.75	0.16	-0.089
0	0.8	0.15	-0.090
0	0.85	0.15	-0.090
0	0.9	0.14	-0.091
0	0.95	0.14	-0.091
0	1	0.13	-0.091
Panel 2			
α	τ	γ	δ
0	0.5	0.21	-0.086
0.05	0.5	0.24	-0.096
0.1	0.5	0.26	-0.105
0.15	0.5	0.28	-0.112
0.2	0.5	0.30	-0.119
0.25	0.5	0.31	-0.124
0.3	0.5	0.32	-0.129
0.35	0.5	0.33	-0.132
0.4	0.5	0.34	-0.134
0.45	0.5	0.34	-0.135
0.5	0.5	0.34	-0.135
0.55	0.5	0.34	-0.133
0.6	0.5	0.34	-0.130
0.65	0.5	0.33	-0.125
0.7	0.5	0.32	-0.119
0.75	0.5	0.30	-0.110
0.8	0.5	0.27	-0.098
0.85	0.5	0.23	-0.083
0.9	0.5	0.18	-0.063
0.95	0.5	0.11	-0.037
1	0.5	0.00	0.000

Panel 3			
α	τ	γ	δ
0	1	0.13	-0.091
0.05	1	0.15	-0.102
0.1	1	0.16	-0.111
0.15	1	0.17	-0.119
0.2	1	0.18	-0.125
0.25	1	0.19	-0.131
0.3	1	0.20	-0.134
0.35	1	0.20	-0.137
0.4	1	0.21	-0.138
0.45	1	0.21	-0.138
0.5	1	0.21	-0.136
0.55	1	0.21	-0.132
0.6	1	0.21	-0.128
0.65	1	0.20	-0.121
0.7	1	0.19	-0.113
0.75	1	0.18	-0.102
0.8	1	0.16	-0.089
0.85	1	0.14	-0.074
0.9	1	0.11	-0.054
0.95	1	0.06	-0.031
1	1	0.00	0.000

Notes: Optimal share of Foreign equities in Home equity portfolio, γ, calculated from equation (2.41), and optimal Home position in foreign exchange, δ, calculated from equation (2.44) for various values of home bias in preferences, α, and price stickiness, τ. Other parameters are calibrated as: $\zeta = 2/3$, $\psi = 1$, $\vartheta_R = (0.855)^{4/\tau}$, $\beta = (0.99)^{4/\tau}$, and $\omega = 1.2$.