INTERNATIONAL ECONOMIC REVIEW
Vol. 33, No. 4, November 1992

THE RISK PREMIUM AND THE LIQUIDITY PREMIUM IN FOREIGN EXCHANGE MARKETS*

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The forward exchange rate may include a liquidity premium as well as a risk premium. The nature of these premiums is investigated in a general equilibrium model. The more obvious point the paper makes is that the forward rate will be affected by the liquidity of financial assets. The subtler point the paper makes is that the liquidity premium of the forward rate is also influenced by the liquidity of goods.

1. INTRODUCTION

A wide range of empirical tests have rejected the proposition that forward exchange rates are conditionally unbiased predictors of future spot exchange rates. Possible explanations for this result are that foreign exchange markets are not efficient, that there is a risk premium, or that the statistical tests have incorrectly rejected a true hypothesis.

This paper examines how liquidity costs may affect the equilibrium forward rate and lead to rejection of tests of uncovered interest parity. Liquidity costs refer to costs of converting assets (or goods) into money so that they can be used to buy goods.

More liquid assets (assets that can be more easily traded for goods) can yield a lower explicit return, and that feeds back into the forward rate. Since by covered interest arbitrage (here assuming no transactions costs) $F_t = S_t(1 + i_{t+1}^0)/(1 + i_{t+1}^1)$—where F_t is the one-period ahead forward exchange rate, S_t is the spot exchange rate (both expressed as domestic currency per unit of foreign currency), i_{t+1}^0 the nominal return on a one-period domestic asset, and i_{t+1}^1 the return on a one-period foreign asset—the value of the forward rate depends on the liquidity of the corresponding assets. Einzig (1937) discusses the importance of the liquidity of assets in determining the forward rate.

However, the liquidity of assets is not the only way in which liquidity issues affect the equilibrium forward rate. The fact that goods are not as liquid as money—that is, typically goods cannot be traded for goods as easily as money can be traded for goods—also has an impact on foreign exchange prices.

This paper explores the determination of equilibrium spot and forward exchange rates in general equilibrium cash-in-advance models similar to those of Svensson (1985a, 1985b).

* Manuscript received November 1990; revised September 1991.

[1] I would like to thank Maxim Engers, Craig Hakkio and Ron Michener for helpful suggestions.

2. THE FORWARD RATE WHEN ASSETS AND GOODS ARE LIQUID

This section presents results for the equilibrium forward exchange rate in the model of Lucas (1982). Lucas' model is a useful benchmark, because in that model money is essentially no more liquid than assets and goods. That is because agents can observe the output realization before acquiring balances.

In Lucas' model there are two countries equally populated by individuals who have identical preferences and initial wealth. Residents of each country are endowed with property rights to the proceeds of an asset that yields a stochastic return each period. The goods that are the output of these assets are different in the two countries. Each country also has a government whose sole function is to distribute random amounts of a currency it has created to residents of its own country. The four stochastic processes—the two output processes and the two money processes—are first-order Markov.

Individuals in each country are infinitely lived, and maximize a time-separable lifetime utility function. Individuals can trade shares in claims to each of the two output processes and each of the two money endowment streams.

Individuals get utility from consumption of both the home good and the foreign good. In order to buy the home good, consumers must first acquire units of the home currency. For the foreign good, they must obtain foreign money ahead of time. The nominal value of consumption of each good must not exceed the consumers' holdings of the respective currencies.

The timing of events is critical to the difference between the Lucas and Svensson models. In the Lucas model, individuals enter a period with some wealth. They then observe the current output and money transfer realizations. Then, individuals acquire the money they need to make the goods purchases they desire. Then, the goods market closes, and individuals take their remaining wealth and use it to acquire shares in the four assets that have a payoff. The wealth they carry into the next period consists of any money that was acquired but not spent, and the value of their assets. Dividends on shares acquired this period are paid at the end of the period and money transfers occur at the beginning of the next period.

The key aspect of the timing that is relevant for the determination of the relative liquidity of the assets and money is that individuals observe the realizations of the random variables prior to the acquisition of any assets. Hence, consumers can figure out what equilibrium prices and quantities will be. They do not acquire any excess money balances (as long as nominal interest rates are positive). By the same token, there is never a shortage of cash—that is, the value of an additional unit of money is just the same as an additional unit of any other form of wealth. The marginal unit of money does not have an added value above other forms of wealth because of its greater liquidity. At the margin all assets have the same liquidity value—zero.

In the Svensson model money must be acquired before the realization of the stochastic processes becomes known (in fact, it is acquired a period ahead of time). In that case, money might be valued for its liquidity on the margin.

Table 1 introduces the notation that will be used throughout this paper.

Individuals maximize $E_t\ \Sigma_{j=0}^{\infty}\ \beta^j u(c^0_{t+j},\ c^1_{t+j})$. Engel (1992) shows that in equilibrium in the Lucas model

$$F_t = E_t[u_1(c^0_{t+1},\ c^1_{t+1})/P^1_{t+1}]/E_t[u_0(c^0_{t+1},\ c^1_{t+1})/P^0_{t+1}], \tag{1}$$

where the subscripts on u represent partial derivatives.

Engel (1992) argues that a plausible definition of risk neutrality that follows from the work of Stiglitz (1979) or Kihlstrom and Mirman (1981) is that an investor who is risk neutral has a utility function that is an affine transformation of a linear homogeneous utility function. It follows that the risk averse investors we are able to consider are only those whose utility can be expressed as a concave function of linear homogeneous utility functions—that is, those with homothetic utility. Only for such individuals can we meaningfully ask how their behavior would differ if they were not risk averse.

For an individual with concave homothetic utility,

$$u(c^0_t,\ c^1_t) = v(h(c^0_t,\ c^1_t)) \tag{2}$$

where $h(\ )$ is homogeneous of degree one, and v is a concave function. When agents are risk neutral, $v'(\ )$ is constant.

Let us define nominal expenditure, Z_t, in a given period by

$$Z_t = P^0_t c^0_t + S_t P^1_t c^1_t.$$

The first two first-order conditions may be rewritten as

$$\frac{u_1(c^0_t,\ c^1_t)}{u_0(c^0_t,\ c^1_t)} = \frac{S_t P^1_t}{P^0_t}. \tag{3}$$

These two equations allow us to write felicity as a function of nominal expenditures and nominal prices:

$$u(c^0_t,\ c^1_t) = v(Z_t,\ P^0_t,\ S_t P^1_t). \tag{4}$$

In the special case of homothetic utility, the indirect utility function can be written in the form

$$v(Z_t,\ P^0_t,\ S_t P^1_t) = v(Z_t/P_t)$$

where P_t is a nominal price index. The function $v()$ here is the same as the one defined in equation (2) above.

Using the fact that $v'(Z_t/P_t)/P_t = u_0(c^0_t,\ c^1_t)/P^0_t$, Engel (1992) shows that the equilibrium forward rate may be written as

$$F_t = E_t\left[\frac{v'(Z_{t+1}/P_{t+1})S_{t+1}}{P_{t+1}}\right]\Bigg/E_t\left[\frac{v'(Z_{t+1}/P_{t+1})}{P_{t+1}}\right]. \tag{5}$$

When agents are risk neutral, $v'(Z_t/P_t)$ is a constant, so the forward rate equals simply

$$F_t = E_t\left[\frac{S_{t+1}}{P_{t+1}}\right] \Big/ E_t\left[\frac{1}{P_{t+1}}\right]. \tag{6}$$

Equation (3) is the familiar relation that the marginal rate of substitution between goods equals their relative price. This equation holds in the Lucas model in spite of the fact that goods are not traded for goods. It holds as a consequence of the fact that individuals observe the realization of the stochastic variables at time t prior to acquiring money. In the Svensson model of the next section, equation (3) does not obtain. Hence, we cannot write an indirect utility function as in equation (4).

3. THE FORWARD RATE WHEN MONEY MIGHT HAVE LIQUIDITY VALUE

The model of this section closely follows that of Svensson (1985a). In all aspects except for timing it is identical to the model of Lucas (1982) set out in the previous section.

In the Svensson model, money for expenditure in period t must be acquired at time $t-1$. At the beginning of period t, the consumer observes the state of the world, then makes his consumption purchases, using the currencies he acquired in the previous periods. He then receives his money transfers and dividends. Then the asset market opens, and he acquires new money and claims to the output and money transfer processes.

The investor's utility entering period t will depend upon the amount of currencies that he chose to acquire in period $t-1$, as well as his level of wealth. We can write the value function as

$$H(w_t, M_t^0, M_t^1; s_t) = \max \{u(c_t^0, c_t^1) + \beta E[H(w_{t+1}, M_{t+1}^0, M_{t+1}^1; s_{t+1})]\}.$$

The contemporaneous budget constraint is given by

$$w_t - c_t^0 - \rho_t c_t^1 - q_t^0\theta_t^0 - q_t^1\theta_t^1 - \xi_t^0\psi_t^0 - \xi_t^1\psi_t^1 - M_{t+1}^0/P_t^0 - S_t M_{t+1}^1/P_t^0 = 0.$$

The cash-in-advance constraints are given by

$$M_t^0/P_t^0 - c_t^0 = 0, \text{ and}$$

$$M_t^1/P_t^1 - c_t^1 = 0.$$

The transition equation for wealth is given by

$$\begin{aligned} w_{t+1} = {} & (q_{t+1}^0 + \varepsilon_{t+1}^0)\theta_t^0 + (q_{t+1}^1 + \rho_{t+1}\varepsilon_{t+1}^1)\theta_t^1 + \frac{(\omega_{t+1}^0 - 1)\bar{M}_t^0}{P_{t+1}^0}\psi_t^0 \\ & + \xi_{t+1}^0\psi_t^0 + \frac{S_{t+1}(\omega_{t+1}^1 - 1)\bar{M}_t^1}{P_{t+1}^0}\psi_t^1 + \xi_{t+1}^1\psi_t^1 + \frac{M_{t+1}^0}{P_{t+1}^0} + \frac{S_{t+1}M_{t+1}^1}{P_{t+1}^0}. \end{aligned} \tag{7}$$

In the notation used here, money that is spent in period $t+1$ (but acquired in period t) has a period $t+1$ subscript. Hence M_{t+1}^j and ω_{t+1}^j are in the time t information set.

The relevant first-order conditions may be written as

$$u_0(c_t^0, c_t^1) = \lambda_t + \mu_t^0 \tag{8}$$

$$u_1(c_t^0, c_t^1) = \rho_t\lambda_t + \mu_t^1 \tag{9}$$

$$\beta E_t[(\lambda_{t+1} + \mu_{t+1}^0)/P_{t+1}^0] = \lambda_t/P_t^0 \tag{10}$$

$$\beta E_t[(\rho_{t+1}\lambda_{t+1} + \mu_{t+1}^1)/P_{t+1}^1] = S_t\lambda_t/P_t^0. \tag{11}$$

λ is the multiplier for the contemporaneous budget constraint, and the μ^i are for the cash-in-advance constraints.

From the first-order conditions we can see immediately that as long as at least one of the cash-in-advance constraints is binding, and at least one of the μ_t^j is nonzero, that the marginal rate of substitution does not equal the relative price of the goods.

Although the discussion can be carried on in terms of the returns from forward contracts, it will be clearer to use the covered interest parity relation to derive the expression for the forward rate. Covered interest parity is an arbitrage condition which states

$$F_t = [S_t(1 + i_{t+1}^0)]/(1 + i_{t+1}^1).$$

The nominal interest rates, i_{t+1}^j, require some explanation in the context of the Svensson model. The domestic nominal interest rate, i_{t+1}^0, is the percentage rate of return in terms of the domestic currency on an asset that is purchased at the end of period t and pays one unit of domestic currency (with certainty) at the end of period $t + 1$. The foreign nominal interest rate is defined in an analogous manner. The key point here is that the asset pays off in the asset market—the money that the asset earns cannot be spent on goods at time $t + 1$. Note that the i_{t+1}^j are in the time t information set.

Expressions for the nominal interest rates can be derived as in Svensson (1985a, 1985b):

$$1 + i_{t+1}^0 = [\lambda_t/P_t^0]/\beta E_t[\lambda_{t+1}/P_{t+1}^0]$$

$$1 + i_{t+1}^i = [\rho_t\lambda_t/P_t^1]/\beta E_t[\rho_{t+1}\lambda_{t+1}/P_{t+1}^1].$$

Since $S_t = \rho_t P_t^0/P_t^1$, we have, using covered interest parity,

$$F_t = E_t[\rho_{t+1}\lambda_{t+1}/P_{t+1}^1]/E_t[\lambda_{t+1}/P_{t+1}^0].$$

Using equations (8) and (9), this expression can be rewritten as

$$F_t = \frac{E_t\{[u_1(c_{t+1}^0, c_{t+1}^1) - \mu_{t+1}^1]/P_{t+1}^1\}}{E_t\{[u_0(c_{t+1}^0, c_{t+1}^1) - \mu_{t+1}^0]/P_{t+1}^0\}}. \tag{12}$$

Recalling that a risk-neutral agent's utility is given by the linear homogeneous function $h(c_t^0, c_t^1)$, the forward rate in this case is

$$F_t = \frac{E_t\{[h_1(c^0_{t+1}, c^1_{t+1}) - \mu^1_{t+1}]/P^1_{t+1}\}}{E_t\{[h_0(c^0_{t+1}, c^1_{t+1}) - \mu^0_{t+1}]/P^0_{t+1}\}}. \tag{13}$$

A measure of the risk premium would be the deviation of the forward rate from the right-hand side of equation (13).

There are apparently two reasons why the forward rate given in equation (13) is different from the corresponding equation from the Lucas model, equation (6) (or, analogously, why equation (12) is different from equation (5)). First, when the liquidity constraint is binding in the Svensson model, the marginal liquidity value of money is not zero, as in the Lucas model. The μ^j_t are not zero. These terms arise essentially because money is more liquid than other assets.

The second reason for the difference, is that when the liquidity constraint is binding in the Svensson model, it is no longer the case that $u_0/P^0 = v'/P$, or $u_1/P^1 = v'S/P$. This is because goods are less liquid than money—that is, goods cannot be traded for goods. With a binding liquidity constraint, the marginal rate of substitution between the goods does not equal the relative price, ρ, as can be seen from the first-order conditions (8) and (9). There is not an expenditure function of the form derived in equation (4). Section 4 shows that the standard representation of the risk premium that would follow from equation (6) is valid with a specific reinterpretation of the price index, P.

4. THE VALUE OF LIQUIDITY

In this section, we offer a reinterpretation of the value of liquidity. A market for liquidity is introduced. Agents who did not acquire enough cash a period ahead of time may acquire additional money before going to the goods market. The aggregate amount of liquidity is zero, so the price of a unit of liquidity in equilibrium will turn out to be just the price that keeps demand for additional liquidity equal to zero.

Agents enter period t with an amount of wealth given by the wealth transition equation (7) from the model of Section 3, some of which is composed of the two currencies that were acquired in period t. Before the goods market opens, but after agents learn the state of the economy, they can acquire N^j_t units of currency j, at a nominal price of g^j_t per unit of the currency. The cash-in-advance constraints are now given by

$$P^0_t c^0_t \leq M^0_t + N^0_t$$

$$P^1_t c^1_t \leq M^1_t + N^1_t.$$

The budget constraint facing the individual at time t is given by

$$w_t + N^0_t/P^0_t + S_t N^1_t/P^0_t \geq c^0_t + \rho_t c^1_t + q^0_t\theta^0_t + q^1_t\theta^1_t + \xi^0_t\psi^0_t$$
$$+ \xi^1_t\psi^1_t + M^0_{t+1}/P^0_t + S_t M^1_{t+1}/P^0_t + g^0_t N^0_t/P^0_t + g^1_t S_t N^1_t/P^0_t.$$

Note that if in equilibrium additional liquidity is not desired, then the prices of liquidity, g_t^j, will equal one—additional money acquired has value only as additional wealth. The N_t^j will drop out of the budget constraint.

The consumer's optimization problem is altered, in that he must now choose values for N_t^j in addition to the other variables. The first order conditions given by equations (8) through (11) still hold. In addition, we will have

$$\mu_t^0 = (g_t^0 - 1)\lambda_t \tag{14}$$

$$\mu_t^1 = \rho_t(g_t^1 - 1)\lambda_t. \tag{15}$$

From equations (8) and (9), using (14) and (15), we have

$$\frac{u_1(c_t^0, c_t^1)}{u_0(c_t^0, c_t^1)} = \frac{S_t Q_t^1}{Q_t^0}.$$

The Q_t^j represent the price of goods inclusive of the cost of acquiring liquidity: $Q_j^t = g_t^j P_t^j$. The relative price, $S_t Q_t^1/Q_t^0$, represents the price of acquiring a unit of the domestic good with a unit of the foreign good. If we define nominal expenditures inclusive of liquidity costs,

$$\tilde{Z}_t = Q_t^0 c_t^0 + S_t Q_t^1 c_t^1,$$

we can then derive an indirect utility function,

$$u(c_t^0, c_t^1) = v(\tilde{Z}_t, Q_t^0 S_t Q_t^1).$$

In the case of homothetic utility, that can be written as

$$v(\tilde{Z}_t, Q_t^0, S_t Q_t^1) = v(\tilde{Z}_t/Q_t),$$

where Q_t is a price index of nominal prices inclusive of liquidity costs. With risk-neutral consumers with linear homogeneous utility functions, v' is constant, as before.

Using equations (7), (8), (14) and (15) we can now write equation (12) as

$$F_t = \frac{E_t[u_1(c_{t+1}^0, c_{t+1}^1)/Q_{t+1}^1]}{E_t[u_0(c_{t+1}^0, c_{t+1}^1)/Q_{t+1}^0]}. \tag{16}$$

We can use the facts that $u_0/Q^0 = v'/Q$, and $u_1/Q^1 = v'S/Q$ to obtain under risk neutrality

$$F_t = E_t\left[\frac{S_{t+1}}{Q_{t+1}}\right] \Bigg/ E_t\left[\frac{1}{Q_{t+1}}\right]. \tag{17}$$

The expression for the forward rate in equation (17) shows that the standard measure of the foreign exchange risk premium derived from equation (6) would be appropriate if the price indices were interpreted correctly. Nominal prices should include liquidity costs. Hence, the approach to empirically testing risk neutral efficiency of Engel (1984) and Hodrick (1989) would be accurate if prices were measured correctly and included liquidity costs.

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TABLE 1

NOTATION

c_t^j = Consumption of good j at time t
E_t = Expectations conditional on information available at time t
F_t = One-period ahead forward rate at time t
g_t^j = Price of a unit of liquidity at time t
H = Value function
i_{t+1}^j = Nominal interest rate earned between t and $t+1$
M_t^j = Individual's holdings of currency j at time t
$\bar{M}_t^j$ = Aggregate supply of currency j at time t
N_t^j = Individual's acquisition of currency j at time t for spending at t
P_t^j = Nominal price, in own currency, of good j at time t
P_t = Nominal price index in domestic currency at time t
q_t^j = Price of equity share j at time t in terms of domestic good
Q_t = Nominal price index inclusive of liquidity costs in domestic currency at time t
s_t = State vector at time t
S_t = Domestic currency price of foreign currency at time t
w_t = Wealth at time t in terms of the domestic good
Z_t = Nominal expenditure at time t
$\bar{Z}_t$ = Nominal expenditure at time t inclusive of liquidity costs
β = Discount factor in utility
ε_t^j = Dividend paid by equity j at time t, or output in country j at t

θ_t^j = Share of equity j purchased at time t
ξ_t^j = Price of share of claim to money transfer j at time t in terms of domestic good
ρ_t = Real exchange rate at time t, equal to $S_t P_t^1 / P_t^0$
ψ_t^j = Share of claim to money transfer j at time t
ω_t^j = Rate of growth of money j between $t - 1$ and t