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TESTS OF MEAN-VARIANCE EFFICIENCY OF INTERNATIONAL EQUITY MARKETS

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1. Introduction

THE mean-variance optimizing model is a popular description of investors' behavior, but one which has received mixed support empirically. As applied to international asset markets, it implies that demand for foreign assets depends on expected returns and variability of returns, which may arise from exchange rate movements and other factors. In this paper we propose some tests of the mean-variance models, and apply the tests to a ten-country asset pricing model of equities.

The behavior of the returns on equities across countries has received considerable attention in recent years. While many studies have been concerned specifically with the transmission of disturbances across markets during and after the crash of October 1987, several papers have addressed the issue of whether asset-pricing models can be used to describe international asset returns. Among those taking this approach are Cumby (1990). Korajczyk and Viallet (1989), Wheatley (1988), and Cho *et al.* (1986).

Our test is closely related to a test of the mean-variance model developed by Frankel (1982b, 1983, 1985) and implemented by Frankel and Engel (1984), Engel and Rodrigues (1989), Ferson *et al.* (1987), Lewis (1988), Attanasio and Edey (1987), Giovannini and Jorion (1989), Bollerslev *et al.* (1988), and Engel *et al.* (1989). Compared to many other tests of the model, the Frankel procedure puts little restriction on the behavior of expected returns and the 'betas' (the co-variance of the rate of return on a particular asset and the rate of return on the portfolio) beyond what is imposed by the model.

A specific alternative hypothesis nesting the mean-variance model is a general linear (Tobin) asset pricing model.[1] As opposed to most other tests of the capital asset pricing model (CAPM) hypothesis, there is an economic interpretation to the alternative hypothesis. Hence, if the restrictions of the mean-variance model are rejected (and we will find that they are), the (unrejected) alternative model stands as an economically meaningful model of asset pricing, and is useful in helping us understand what goes wrong with the mean-variance model.

The techniques we employ retain the desirable properties of the Frankel test but considerably ease the computational burdens imposed by the maximum likelihood estimation (MLE) proposed by Frankel. His method requires

[1] The idea of nesting the mean-variance model in the Tobin model and testing the restrictions can be traced back to Parkin (1970) and Courakis (1975).

simultaneously choosing hundreds of parameters to maximize a likelihood function that describes a linear system with a restriction between the mean and the variance of the errors.

We present two types of tests of the mean-variance model. The first is a Wald test which requires estimation only of the unrestricted (Tobin) asset pricing model. The second is based on a minimum distance estimator of the restricted (mean-variance) model that makes use of parameter estimates and their variances from the unrestricted model. These estimators are much easier to calculate than the MLEs, which allows us to consider asset pricing systems of larger dimensions than is practical with the MLE. Correspondingly, we are also able to allow for much richer dynamics in the covariance structure of the individual asset returns.

Section 2 of this paper reviews the mean-variance hypothesis and discusses the nature of the restrictions that the Frankel procedure tests. Section 3 briefly explains the Wald statistic, and presents the Wald tests of mean-variance efficiency (MVE) under the assumption that variances and coefficients are constant over time, and that variances are allowed to vary over time. Section 4 shows how the parameters of the restricted model can be estimated, and presents additional tests of MVE. In Section 5 we examine the characteristics of the various asset demands implied by our estimates. The implications of our findings are summarized in the concluding section.

2. Testing the mean-variance model

2.1. *The restrictions of the mean-variance model*

A well-known relation that emerges from the mean-variance optimizing model is that for any asset i,

$$E_t(r_{i,t+1} - r^b_{t+1}) = \beta_{it} E_t(r_{m,t+1} - r^b_{t+1}) \tag{1}$$

where

$r_{i,t+1}$ = return on asset i from time t to $t+1$
r^b_{t+1} = riskless rate of return from time t to $t+1$
$r_{m,t+1}$ = return on the value weighted 'market' portfolio of equities between t and $t+1$.[2]

The coefficient β_{it} is, in general, a time-varying coefficient that is defined as

$$\beta_{it} = \mathrm{Cov}_t(r_{i,t+1}, r_{m,t+1})/\mathrm{Var}_t(r_{m,t+1}) \tag{2}$$

[2] The term 'market' here refers to the set of assets in the mean-variance efficient portfolio. If this set of assets coincided with the entire range of assets available in the market, then our tests would be a test of the CAPM hypothesis. The inability to measure returns on the entire market eliminates the possibility of a formal test of CAPM, as Roll (1977) has emphasized.

We could rewrite the equilibrium relation as the 'security market line'

$$E_t(r_{i,t+1} - r^b_{t+1}) = \rho \operatorname{Cov}_t(r_{i,t+1}, r_{m,t+1}) \tag{3}$$

where

$$\rho = E_t(r_{m,t+1} - r^b_{t+1})/\operatorname{Var}_t(r_{m,t+1}) \tag{4}$$

The coefficient ρ is sometimes referred to as the price of risk. It represents the trade-off between the expected return on the market portfolio and the variance of return on that set of assets. A critical assumption we make is that this 'price' is constant over time. In general, this variable could change over time but the assumption of constancy of ρ seems quite innocuous compared to some other common assumptions made to test mean-variance efficiency. (See Frankel 1982, 1983, for a discussion of the drawbacks to some of the common assumptions.) The price of risk in equilibrium is a measure of the degree of risk aversion of investors, and can thus be viewed as a behavioral parameter.

The return on the market portfolio is a weighted average of returns on each of the individual assets:

$$r_{m,t+1} = \sum_{j=1}^{n} \lambda_{jt} r_{j,t+1} \tag{5}$$

where

λ_{jt} = ratio of the value of outstanding shares of asset j to the value of all assets.

Therefore, we can write

$$\operatorname{Cov}_t(r_{i,t+1}, r_{m,t+1}) = \sum_{j=1}^{n} \lambda_{jt} \operatorname{Cov}_t(r_{i,t+1}, r_{j,t+1}) \tag{6}$$

$\operatorname{Cov}_t(r_{i,t+1}, r_{m,t+1})$ could vary over time either because supplies of assets (the λs) change, or because the underlying stochastic process of returns is time-varying, meaning that $\operatorname{Cov}_t(r_{i,t+1}, r_{j,t+1})$ is not constant. We initially assume only the λs move over time, but then we allow the covariances to vary as well.

In matrix form we have

$$E_t r_{t+1} = \rho \Omega_t \lambda_t \tag{7}$$

where

r_{t+1} = vector of excess returns
λ_t = vector of asset shares
$\Omega_t = E_t(r_{t+1} - E_t r_{t+1})(r_{t+1} - E_t r_{t+1})'$.

Tobin's (1958, 1969) general equilibrium portfolio balance theory posits that the demand for assets by individuals should be a function of the vector of expected returns. We can write a linear form of the model which posits $\lambda_t = A_t E_t r_{t+1}$ where A_t is a matrix of coefficients (possibly time-varying) that represents the response of the assets demanded (as a share of the total portfolio) for a change in the expected returns. We can rewrite the system as an equilibrium model for expected returns (as in Tobin, 1969):

$$E_t r_{t+1} = B_t \lambda_t \tag{8}$$

Here, $B_t = A_t^{-1}$. Tobin's model is a general theory that does not specify precisely the determinants of the response of asset demands to expected returns.

We view the mean-variance model as a restriction on the general Tobin model. It tells us exactly the responses of asset demands with respect to changes in expected returns. The response depends upon the covariance of asset returns. Specifically, comparing equations (7) and (8). we can see that the mean-variance model requires that B_t equal $\rho\Omega_t$.

2.2. *Testing the restrictions*

If expectations are rational, we have

$$r_{t+1} = E_t r_{t+1} + \varepsilon_{t+1} \tag{9}$$

where ε_{t+1} is a white-noise error term. Thus, the restricted model (7) can be rewritten as:

$$r_{t+1} = \rho\Omega_t\lambda_t + \varepsilon_{t+1} \tag{10}$$

while the unrestricted model becomes

$$r_{t+1} = B_t\lambda_t + \varepsilon_{t+1} \tag{11}$$

Consider for a moment the case in which Ω_t and B_t are constant over time. Equation (11) then describes a system of linear equations that can be estimated simply by regressing the *ex-post* returns on the values of the asset shares at time t. Such a regression requires data on the returns and the values of each asset as a share of the total market.

Note, however, that the variance-covariance matrix of ε_{t+1} is Ω. Therefore, the restrictions of the mean-variance model on the general system (11) are that the matrix of regression coefficients, B, should be proportional to the covariance matrix of the residuals (with the constant of proportionality equal to ρ).

One way of testing mean-variance efficiency is to compare the likelihood of the system (11) without imposing any restrictions on B to the likelihood obtained from estimating (10), imposing the restriction that B be proportional to Ω. The likelihood ratio test will fail to reject mean-variance efficiency if the estimated likelihood with the restriction imposed is not significantly smaller than the likelihood from the unrestricted regressions. The Wald test, on the other hand, estimates B and Ω from the unrestricted system, then tests the restriction that the two matrices are proportional. Clearly in this case the estimation is quite easy, since only OLS estimation of the system (11) need be performed.

The estimator we propose in Section 4 finds a matrix we can call Ω^*, and a parameter ρ^* which minimizes a weighted average of the distance between Ω^* and the unconstrained estimate of Ω, and the distance between $\rho^*\Omega^*$ and the unconstrained estimate of B. The test of the mean-variance model is simply a test of whether this minimized distance is near zero.

More generally, Ω_t and B_t will vary over time. In particular, we postulate that

$$\Omega_t = \Omega_0 + \Omega_1 z_t^1 + \Omega_2 z_t^2 + \cdots + \Omega_m z_t^m \tag{12}$$

where z_t^j is an economic variable that is known at time t, and Ω_j is constant for all j. This type of variance model has been used recently by Giovannini and Jorion (1987), Engel and Rodrigues (1989), and Shanken (1990). The mean-variance model does not explicitly pinpoint the sources of variability in returns, yet it does not require that returns have constant second moments. In the absence of a specific theoretical model, we hypothesize that the second moments depend on some plausible economic sources of variation in returns.

Correspondingly, we allow the coefficients on the asset-shares in the asset pricing equation (8) to change over time, according to the relation:

$$B_t = B_0 + B_1 z_t^1 + B_2 z_t^2 + \cdots + B_m z_t^m \tag{13}$$

Mean-variance efficiency then imposes the constraints $\Omega_j = \rho B_j$ for all $j = 0, \ldots, m$.

The principle of our test remains the same, even with time-varying B_t and Ω_t. We note that relationship (10) now takes the form

$$r_{t+1} = B_0\lambda_t + B_1 z_t^1 \lambda_t + B_2 z_t^2 \lambda_t + \cdots + B_m z_t^m \lambda_t + \varepsilon_{t+1} \tag{14}$$

This is still a system of regression equations, albeit one with many more variables. Furthermore, it is one that is not efficiently estimated by OLS, because the variance of the vector of error terms, ε_{t+1}, follows a heteroskedastic process given by equation (12).

The unconstrained system can be efficiently estimated using a GLS estimator described in the next section. The likelihood ratio test would also require estimation of the constrained system by maximum likelihood techniques. Instead, we propose a Wald test of the mean-variance restrictions that requires estimation only of the unconstrained system. Then we derive a minimum distance estimator of the constrained system that is considerably less burdensome computationally than the MLE.

3. Testing the mean variance model with the Wald test

We can write the restrictions of the mean-variance model as the conditions that a specific vector of functions of the parameters equal zero. We can then take the estimated coefficients and construct a measure of the distance of the estimated vector of functions from zero. The Wald statistic provides such a measure. (See Silvey, 1975, for a discussion of the Wald test.)

The vector of restrictions can be represented as $h(\theta) = 0$, where θ is the vector of parameters, and h is a vector of functions. The Wald statistic measures how close to zero is $h(\hat{\theta})$, where $\hat{\theta}$ is the vector of estimates of parameters. The Wald statistic is motivated by noting first that $T^{1/2}(\hat{\theta} - \theta)$ is distributed approximately normally with mean zero and variance $\mathscr{I}_\theta^{-1}$, where $\mathscr{I}_\theta$ represents the information matrix of the parameters (and T is the number of observations). We can write the vector of restrictions here as $h(\hat{\theta}) \simeq h(\theta) + H(\hat{\theta} - \theta)$. The matrix H is the matrix of derivatives of the constraints with respect to the parameters. Under the null hypothesis, then, we have $h(\hat{\theta}) \simeq H(\hat{\theta} - \theta)$. Therefore, $T^{1/2}h(\hat{\theta})$ is distributed approximately normally with mean zero and variance $H\mathscr{I}_\theta^{-1}H'$. It follows that a natural test statistic is $T[h(\hat{\theta})]'[H\mathscr{I}_{\hat{\theta}}^{-1}H']^{-1}[h(\hat{\theta})]$. In large samples this Wald statistic is distributed chi-square with n^2 degrees of freedom.

The appendix (available from the authors on request) describes how the test statistics are constructed. We take up first the case when B_t and Ω_t are constant over time.

3.1. *The Wald test with constant variances and coefficients*

Our empirical analysis relates to the demand for equities from ten countries—the US, Belgium, Canada, the UK, France, Germany, Italy, the Netherlands, Japan, and Switzerland. The data on returns are constructed from price and dividend data in monthly issues of *Capital International Perspectives*. Returns on foreign assets are converted into dollar terms using exchange rates from the last day of the month, obtained from the data base of the Federal Reserve Bank of New York. The excess returns are calculated relative to the return on one-month LIBOR deposits, measured as the average of the bid and ask rates on the last day of the month as recorded by DRI. The regressions of equation (8) require data on asset shares, which we construct using the data on capitalization from each of the ten markets, in the monthly issues of *Capital International Perspectives*. The sample period is June 1973 to July 1988. The asset shares are the total capitalization of each market (in dollar terms) as a share of the aggregate capitalization.

The model tested in this section assumes B_t and Ω_t are constant over time. Note that despite this restriction, the β_{it}, and hence the risk premiums, vary over time with asset supplies. From equation (2), β_{it} changes as $\mathrm{Cov}_t(r_{i,t+1}, r_{m,t+1})$ varies. Equation (6) shows how $\mathrm{Cov}_t(r_{i,t+1}, r_{m,t+1})$ depends on the asset supplies. In equation (6), even if $\mathrm{Cov}_t(r_{i,t+1}, r_{j,t+1})$ is constant for all i, j, $\mathrm{Cov}_t(r_{i,t+1}, r_{m,t+1})$ will change as the λ_{jt} change.

For any given value of the parameter for the price of risk, ρ, the set of constraints may be written as $b_{ij} - \rho\omega_{ij} = 0$, where b_{ij} is the ij element of B and ω_{ij} is the ij element of Ω. For example, Frankel and Engel (1984) maximize the constrained likelihood function setting $\rho = 2$. In general, of course, one does not know the true value of ρ. The unconstrained regressions do not provide a unique estimate of ρ. However, the constraints of the mean-variance model can be expressed as the set of proportionality constraints such that b_{ij}/ω_{ij} be equal for all i, j. Expressing the constraints in this fashion does not require knowledge of ρ.

It is well known that algebraically equivalent ways of writing a set of constraints do not give the same numerical answer in finite samples for the

TABLE 1

Regressions of excess returns on equity market shares (equation 4)

	equation shares									
	UK	*US*	*JP*	*GE*	*CA*	*SW*	*NL*	*FR*	*IT*	*BE*
UK	−2.029*	−0.015	0.346*	4.346	0.817	−1.081	2.481	0.743	−1.209	−17.94*
	0.754	0.147	0.150	2.692	1.400	3.885	9.335	2.509	2.071	8.900
US	0.464	−0.060	0.032	3.400*	−0.275	−2.616	−1.494	−0.104	−0.737	−7.705
	0.453	0.089	0.090	1.618	0.841	2.334	5.610	1.508	1.245	5.342
JP	−0.467	0.075	−0.141	−1.207	−1.672†	3.961	6.974	−0.950	3.356	−3.844
	0.545	0.106	0.109	1.944	1.011	2.805	6.741	1.812	1.496	6.419
GE	−0.965†	0.092	0.024	−0.688	−0.232	1.014	5.673	−1.938	1.989	−0.590
	0.554	0.108	0.110	1.976	1.028	2.852	6.854	1.842	1.521	6.527
CA	0.218	0.141	0.014	2.062	−2.042†	2.921	−13.35†	0.516	−0.434	2.771
	0.623	0.122	0.124	2.225	1.157	3.211	7.715	2.073	1.712	7.347
SW	−0.253	0.043	0.175†	4.861*	−0.342	−3.536	−3.831	−1.118	−0.766	−9.529
	0.523	0.102	0.104	1.868	0.972	2.696	6.479	1.741	1.438	6.170
NL	−0.683	0.245*	0.178†	2.902	−1.607†	2.072	−14.32*	0.617	−0.005	−4.698
	0.521	0.102	0.104	1.859	0.967	2.683	6.447	1.732	1.430	6.139
FR	−0.648	−0.177	0.045	4.140†	0.976	1.666	2.979	−7.529*	1.758	3.666
	0.680	0.133	0.136	2.429	1.263	3.505	8.423	2.264	1.869	8.021
IT	−0.051	−0.097	−0.046	1.553	−0.475	1.276	8.663	−0.249	−0.588	−14.24
	0.740	0.145	0.148	2.643	1.374	3.814	9.164	2.463	2.033	8.727
BE	−0.419	0.050	0.173	4.500†	−0.962	−0.706	−3.467	−2.031	0.672	−9.253
	0.557	0.109	0.111	1.987	1.033	2.867	6.890	1.852	1.529	6.561

Coefficient estimates are the first number reported.
Standard errors are the second number reported.
* Significant at the 5% level.
† Significant at the 10% level.

Wald test, although they are asymptotically equivalent. Gregory and Veall (1985) have argued on the basis of Monte Carlo evidence that expressing the constraints to be tested as products (rather than quotients) improves the quality of the approximation to the asymptotic distribution of the test statistic. We test the constraints in the form $b_{ij}\omega_{kl} = b_{kl}\omega_{ij}$, for all i, j, and a particular k, l. The value of the Wald statistic will depend upon the choice of k, l. In the statistics we report below, we chose k, l so that the unconstrained estimates of $b_{k,l}$ and $\omega_{k,l}$ are significantly different from zero. If $b_{k,l}$ and $\omega_{k,l}$ equal zero, then $b_{ij}\omega_{kl} = b_{kl}\omega_{ij}$ trivially.

Table 1 reports the results of the ten OLS regressions of excess rates of return on the asset shares. The performance of the unrestricted model is weak. In particular, few of the coefficients are significant at the 5% level, and (as is typical in equations used to predict returns on equities) little of the variation of the returns can be predicted *ex ante*. Moreover, contrary to what the underlying model leads us to expect (see Frankel, 1982a), the diagonal elements of B are all negative, though only three of these negative coefficients are significant at the 5% level.

These results do not necessarily indicate that the general model should be rejected. Rather, they are characteristic of the imprecision in nearly all models

TABLE 2

Covariance and correlation matrix for residuals from regressions in Table 1

	UK	*US*	*JP*	*GE*	*CA*	*SW*	*NL*	*FR*	*IT*	*BE*
UK	0.6464	0.5092	0.3331	0.4007	0.4737	0.5451	0.6392	0.5224	0.3588	0.4926
US	0.1978	0.2334	0.2710	0.3923	0.6309	0.5330	0.6172	0.4745	0.2164	0.4372
JP	0.1555	0.0760	0.3371	0.3975	0.2374	0.4004	0.4417	0.4153	0.3882	0.5354
GE	0.1902	0.1119	0.1362	0.3484	0.3074	0.7736	0.6839	0.5641	0.3574	0.6377
CA	0.2530	0.2025	0.0916	0.1206	0.4415	0.4898	0.5288	0.4150	0.2580	0.3248
SW	0.2445	0.1437	0.1297	0.2548	0.1816	0.3113	0.7395	0.6062	0.3914	0.6467
NL	0.2853	0.1656	0.1424	0.2241	0.1951	0.2291	0.3082	0.5769	0.3692	0.6314
FR	0.3047	0.1663	0.1749	0.2415	0.2001	0.2454	0.2324	0.5262	0.4454	0.5925
IT	0.2277	0.0825	0.1779	0.1665	0.1353	0.1724	0.1618	0.2550	0.6229	0.3897
BE	0.2350	0.1253	0.1500	0.2234	0.1281	0.2141	0.2080	0.2550	0.1825	0.3521

Covariances ($\times$ 100) appear on and below the diagonal.
Correlations appear above the diagonal.

that try to explain stock returns *ex ante*. It is this very imprecision that leads one to study restricted models such as the mean-variance model. Table 2 reports the unrestricted estimate of Ω. A quick check reveals that there are, in fact, wide variations in the estimates of b_{ij}/ω_{ij}. (Divide the estimates from Table 1 by the corresponding elements from Table 2, bearing in mind that because of our scaling in Table 2, the 'estimated' values of ρ are 100 times these numbers). As before, when the parameters are individually estimated with low precision, reliable inferences are difficult to draw from the individual point estimates.

The first line on Table 3 shows that our Wald tests reject the restrictions of mean-variance efficiency when we test whether the constraints hold and $\rho = 2$. That is, we reject the constraint $b_{ij} - 2\omega_{ij} = 0$. The second test reported on the top line of Table 3 takes the form $b_{ij}\omega_{kl} - b_{kl}\omega_{ij} = 0$. The k, l element was chosen as the coefficient of UK excess returns on UK asset shares, since, in this case, both b_{kl} and ω_{kl} clearly do not equal zero. In contrast to the case in which we choose a value for ρ, we do not reject the constraints of mean-variance efficiency when the constraints take the form $b_{ij}\omega_{kl} - b_{kl}\omega_{ij} = 0$. One possible explanation for this difference in results is that MVE holds, but ρ is not close to 2. However, an alternative explanation, discussed at the end of this section, is that the tests in which ρ is not specified have very low power compared to the linear form of the test.

3.2. *The Wald test with time-varying variances and coefficients*

Until now, we have treated the covariance matrix Ω_t as a constant, indicating homoskedasticity of returns on the equities. In this section we consider the model described by equations (12), (13) and (14), in which the conditional covariances may be a function of currently observed economic data.

TABLE 3
Results of Wald tests

Constraint:	$b_{ij} - 2\omega_{ij} = 0$	$b_{ij}\omega_{kl} - b_{kl}\omega_{ij} = 0$
Constant co-variance		
Wald statistic:	183.45*	97.68
(degrees of freedom)	(100)	(99)
Time-varying co-variance		
Variance a function of:		
LIBOR, price of gold money supply, oil price		
Wald statistic:	1124.23*	82.27
(degrees of freedom)	(500)	(499)
LIBOR, price of gold lagged money, lagged oil price		
Wald statistic:	1051.90*	56.22
(degrees of freedom)	(500)	(499)
4 principal components		
Wald statistic:	1322.98*	188.35
(degrees of freedom)	(500)	(499)
2 principal components lagged money, lagged oil price		
Wald statistic	1058.99*	152.07
(degrees of freedom)	(500)	(499)

* Significant at the 1% level.

We adopt a four-step procedure to estimate the parameters of the unconstrained model. First, we estimate the B_i $(i = 1, \ldots, m)$ with OLS regressions of the system described by equation (14). Noting that the moment matrix, v_{t+1}, of the estimated ε_{t+1} from equation (14) is equal to Ω_t plus a random error term, it follows from equation (12) that:

$$v_{t+1} = \Omega_0 + \Omega_1 z_t^1 + \Omega_2 z_t^2 + \cdots + \Omega_m z_t^m + \upsilon_{t+1} \tag{15}$$

Hence, we can regress this moment matrix on the vector of economic variables to generate consistent estimates of the Ω_i $(i = 1, \ldots, m)$. While this procedure generates consistent estimates of all of the parameters (the B_i and the Ω_i), the estimates are not efficient because the errors are heteroskedastic. So, we use the assumption that the ε_t are normally distributed, and the fitted values of Ω_t to construct a variance matrix for the error terms in the Ω regression (equation 15). Using GLS, we get new estimates of the Ω_i. With these new Ω_i, we get new measures of the fitted values of Ω_t which we use to make the heteroskedasticity correction for the returns regression (equation 14). We then can construct efficient estimates of the B_i using GLS.[3]

We allow the variance to be a function of four different economic variables. In particular, following Giovannini and Jorion (1987) and Shaken (1990), we postulate that the variance of excess returns is a function of short-term

[3] The procedure is described in greater detail in the appendix available from the authors.

interest rates, in this case the one-month LIBOR rate (computed as the average of the bid and ask rates reported on the last day of the month by DRI). In our earlier paper, Engel and Rodrigues (1989), we argued that oil prices and the US money supply may be sources of uncertainty regarding returns. Accordingly, we use the squared growth rates of these series as explanatory variables for the covariances. The oil price series is the producer price index for crude petroleum products, and the money supply variable is US M1. We also use the squared growth rates in the price of gold, measured on the last day of the month, London afternoon fixing. The money supply, oil price and gold price series are from the Federal Reserve Board data base.[4]

Again, the results from the unrestricted regressions (which are not reported) are weak. Few of the coefficients are individually significant, and the adjusted R^2s are low.

The second and third rows of Table 3 report the tests of mean-variance efficiency when the variance depends upon the four economic variables. The left column of Table 3 reports the Wald statistics for the tests of $b_{ij} - 2\omega_{ij} = 0$, while the statistic in the right column is for the case in which ρ is unrestricted. In the latter case, the constraints are tested in their multiplicative form: $b_{ij}\omega_{kl} - b_{kl}\omega_{ij} = 0$. As in the constant variance case, we chose k, l to correspond to the coefficient of UK excess returns on UK shares. The third row in Table 3 reports statistics for models in which the variance depends on the lagged oil price and the lagged money supply (as well as the current squared gold price and Eurodollar rate). The reason these are included is that the variance is supposed to be conditional on information available on the last day of the month. Although investors often know much about the US money supply for a given month on the last day of the month (because the money supply numbers are reported weekly, but with a ten-day lag), they do not know the exact figure until it is reported some time after the last day of the month. The crude oil price index used closely reflects prices on spot markets, but the index itself is not reported until after the end of the month.

The mean-variance constraints when $\rho = 2$ are again rejected at the 5% level (as was the case with the constant Ω model), while the constraints are not rejected when we do not specify a value for ρ. As we will explain below, this failure to reject when the value of ρ is not specified probably arises from the low power of the test when the constraints tested are of the form $b_{ij}\omega_{kl} - b_{kl}\omega_{ij} = 0$.

Our earlier paper (Engel and Rodrigues, 1989) argued that ARCH may be a useful *ad hoc* way to model the time-varying variances. In general it appears that financial models can explain *ex ante* only a small fraction of the variation in asset returns. In practice, there is almost no difference between an ARCH model in which the variance depends on lagged squares (and cross products) of

[4] While the money supplies of all countries may provide useful information to investors, for parsimony we include only the dollar, which has a more dominant role in international financial markets.

regression errors, and a model in which the variance depends upon the lagged squares (and cross-products) of deviations of the returns from their sample means. Here we approximate ARCH by exactly such a model.

We reduce the dimension of the problem by treating the lagged squared deviations of the excess returns from their means as the z_t, while ignoring the possible dependence of elements of Ω_t on lagged cross-products of the returns. We find that there is a high correlation amongst the ten squared returns variables. So, we reduce the dimensions of the z vector further by calculating the principal components of the squared returns, and then constructing a vector z consisting of only the lagged values of the first four principal components. These four components account for 87.87% of the sum of the variances in the squared returns.

The fourth line of Table 3 reports the results of the tests of mean-variance efficiency. As was the case for the models in which the variance depended upon the economic variables, the constraints imposed by mean-variance efficiency are rejected when $\rho = 2$, but are not rejected when ρ is left unrestricted.

This same conclusion emerges when Ω_t is assumed to depend on the lagged values of: the squared changes in the log of $M1$, the squared changes in the log of oil prices, and the first two principal components of the squared deviations from mean of the excess returns. (These two principal components account for 72.13% of the sum of the variances in squared returns.) These tests are reported in the fifth line of Table 3.

As was discussed in Section 3, there are many ways to parameterize the Wald test, and generally the different parameterizations will not yield the same Wald statistic. Even if we stick to the multiplicative form of the constraints, as advocated by Gregory and Veall (1985), so that the constraints are of the form $b_{ij}\omega_{kl} - b_{kl}\omega_{ij} = 0$, the value of the Wald statistic will vary depending upon the choice of k and l. For the model in which the variance is assumed to depend upon lagged squared money, lagged squared oil and the lagged first two principal components of squared returns, we checked the robustness of our conclusions by calculating the Wald statistic when the k, l element corresponds to the coefficient of the excess returns on shares for each of the ten countries. We found that we cannot reject the mean-variance constraints in any of the ten cases. (Note that when we test the mean-variance hypothesis and restrict ρ to equal 2, the constraints are linear, so that there is a unique Wald statistic.)

We believe that the failure to reject the mean-variance restrictions when the value of ρ is not specified is, in fact, a demonstration of the low power of the tests when the constraints are written in the form $b_{ij}\omega_{kl} - b_{kl}\omega_{ij} = 0$. These tests are much less powerful than when the constraints are of the form $b_{ij} - \rho\omega_{ij} = 0$. One reason for this is that the estimates of each of the constraints $b_{kl}\omega_{ij} - b_{ij}\omega_{kl} = 0$ has higher variance because four estimated parameters, rather than two, appear in each constraint. There is also a high correlation among the constraints because each of the constraints involve the estimates of b_{kl} and ω_{kl}. This correlation appears to play the most significant role in lowering the power of the test when the constraints take the form $b_{kl}\omega_{ij} - b_{ij}\omega_{kl} = 0$.

TABLE 4
Average values of B_t
Unconstrained model (B_t defined in Equation 6)

	UK	*US*	*JP*	*GE*	*CA*	*SW*	*NL*	*FR*	*IT*	*BE*
UK	−8.826	0.343	0.743	−0.897	6.645	16.348	−14.25	−17.11	9.784	45.810
US	−2.681	0.035	0.395	−2.592	3.015	3.895	−0.947	−5.896	0.973	24.126
JP	−6.308	0.702	0.387	−13.79	−1.207	19.211	−10.35	5.087	6.735	27.290
GE	−6.350	0.251	0.431	−4.878	3.311	14.966	2.253	−9.945	7.688	25.851
CA	−7.014	0.567	0.640	−10.06	3.229	20.462	−22.73	−3.881	2.387	49.834
SW	−6.157	0.038	0.434	0.355	3.229	8.920	3.442	−8.213	9.520	15.799
NL	−4.961	0.323	0.514	−1.552	1.514	11.872	−13.71	−7.495	4.612	28.787
FR	−7.760	0.407	0.349	−5.539	3.358	23.583	−15.48	−14.64	11.047	55.434
IT	−10.57	0.989	0.817	−14.22	2.902	30.171	−40.46	9.152	2.669	28.491
BE	−4.211	0.594	0.517	−3.071	−1.161	16.672	−27.40	−7.115	3.165	37.236

(z_t includes lagged money, lagged oil prices, and first two principal components of excess returns).

TABLE 5
Average values of Ω_t
Unconstrained model (Ω_t defined in Equation 5)

	UK	*US*	*JP*	*GE*	*CA*	*SW*	*NL*	*FR*	*IT*	*BE*
UK	0.4696	0.1581	0.0983	0.1139	0.1633	0.1827	0.2101	0.2175	0.1372	0.1570
US	0.1581	0.1739	0.0549	0.0784	0.1484	0.1083	0.1233	0.1213	0.0537	0.0845
JP	0.0983	0.0549	0.2192	0.0790	0.0601	0.0777	0.0891	0.0940	0.1032	0.0997
GE	0.1139	0.0784	0.0790	0.2382	0.0722	0.1712	0.1446	0.1757	0.0918	0.1381
CA	0.1633	0.1484	0.0601	0.0722	0.3215	0.1086	0.1230	0.1272	0.0834	0.0707
SW	0.1827	0.1083	0.0777	0.1712	0.1086	0.2109	0.1526	0.1827	0.1063	0.1396
NL	0.2101	0.1233	0.0891	0.1446	0.1230	0.1526	0.2187	0.1622	0.0828	0.1298
FR	0.2175	0.1213	0.0940	0.1757	0.1271	0.1827	0.1622	0.3677	0.1662	0.1737
IT	0.1372	0.0537	0.1032	0.0918	0.0834	0.1063	0.0828	0.1662	0.3906	0.1175
BE	0.1570	0.0845	0.0997	0.1381	0.0707	0.1396	0.1298	0.1737	0.1175	0.2554

(z_t includes lagged money, lagged oil prices, and first two principal components of excess returns)
Reported covariances are 100 × actual values.

So, it appears that in those cases in which the mean-variance model is not strongly rejected, it is because the particular Wald test employed has very low power, and probably not because the null hypothesis is true.

Table 4 reports the sample mean of the fitted values for B_t, for the model in which B_t is assumed to depend upon lagged changes in the log of the money supply, lagged changes in the log of oil prices and the lagged values of the first two principal components of the square of the deviation from mean of the excess returns. Table 5 reports the average of the fitted values for Ω_t from the same model. (Note that the values reported in Table 5 are the elements of Ω_t times 100.)

It is instructive to compare the average coefficients reported in Table 4 to the coefficients reported for the constant B model in Table 1. There are large differences in the two matrices. This indicates the importance of allowing B_t to be time-varying. If the average value of B_t is so much different from the estimated value of B assuming constant coefficients, then clearly there can be even bigger differences at specific dates. The same observation holds for comparison of the estimates of the average value of Ω_t in Table 5 to those for the constant Ω model in Table 2.

Frankel (1986) observed that the mean-variance model was unlikely to explain large values of the foreign exchange risk premium, because the conditional variance of returns was very small. In our context, the implication is that Ω_t is very small relative to B_t. Comparison of Tables 4 and 5 indicate the elements of the matrix of average values of Ω_t are generally three orders of magnitude smaller than the absolute values of the elements of the average values of B_t.

It is not clear yet exactly why the mean-variance restrictions are rejected when the constraint is expressed linearly. The output from Tables 4 and 5 suggests that if the mean-variance constraint were imposed, the estimated value of ρ may not be close to 2. So, have we rejected MVE because we have chosen an inappropriate value of ρ? The non-linear form of the constraint does not require that we specify ρ, and we do not reject MVE in this case, but we have argued that the test has low power under this specification. It appears that examination of the constrained estimates would be helpful in understanding the nature of the mean-variance model. We turn to those estimates in the next section.

4. Estimates of the MVE-constrained model

We can construct minimum distance estimators of the parameters of the constrained model following the suggestion of Rothenberg (1973, p. 24). Call b the vector of coefficients of the estimated B_i matrices ($i = 1, \ldots, m$), where we take the elements by row from each matrix. Let s be the vector of coefficients of the estimated Ω_i matrices ($i = 1, \ldots, m$), where we take the elements by row from the upper triangle of each matrix. The vector of parameters from the constrained model can be estimated by choosing ρ and ω (which will be the estimated vector of coefficients in the constrained Ω_i matrices) to minimize:

$$\chi^2 = \begin{bmatrix} b - \rho h\omega \\ s - \omega \end{bmatrix}' \begin{bmatrix} V_B & 0 \\ 0 & V_S \end{bmatrix}^{-1} \begin{bmatrix} b - \rho h\omega \\ s - \omega \end{bmatrix}. \tag{16}$$

Here, V_B is the covariance matrix of b, V_S is the covariance matrix of s, and h is the matrix that maps the elements of ω into the relevant elements of b.[5]

We have found that in practice this two-stage procedure (first estimate the unconstrained model, then construct the minimum distance estimates of the

[5] The appendix available from the authors describes the estimation in greater detail.

TABLE 6
Average values of Ω_t
Constrained model (Ω_t *defined in Equation* 5)

	UK	*US*	*JP*	*GE*	*CA*	*SW*	*NL*	*FR*	*IT*	*BE*
UK	0.2725	0.0010	−0.0174	−0.0005	0.0109	0.0439	0.0589	0.0595	0.0292	0.0383
US	0.0010	0.0311	−0.0236	−0.0122	0.0178	−0.0028	−0.0000	−0.0012	−0.0220	−0.0120
JP	−0.0174	−0.0236	0.1028	0.0117	−0.0275	−0.0058	0.0002	−0.0115	0.0210	0.0314
GE	−0.0005	−0.0122	0.0117	0.1721	−0.0151	0.0912	0.0582	0.0850	0.0315	0.0727
CA	0.0109	0.0178	−0.0275	−0.0151	0.1971	0.0015	0.0056	0.0059	0.0036	−0.0249
SW	0.0439	−0.0028	−0.0058	0.0912	0.0015	0.1136	0.0467	0.0712	0.0309	0.0581
NL	0.0589	−0.0000	0.0002	0.0582	0.0056	0.0467	0.1031	0.0426	0.0029	0.0408
FR	0.0595	−0.0012	−0.0115	0.0850	0.0059	0.0712	0.0426	0.2355	0.0742	0.0789
IT	0.0292	−0.0220	0.0210	0.0315	0.0036	0.0309	0.0029	0.0742	0.3203	0.0573
BE	0.0383	−0.0120	0.0314	0.0727	−0.0249	0.0581	0.0408	0.0789	0.0573	0.1895

(z_t includes lagged money, lagged oil prices, and first two principal components of excess returns)
Reported covariances are 100 × actual values.
Estimated value of $\rho = 87.29$.
Chi-square (499 d.f.) = 1037.91.

constrained model) requires much less computation than maximum likelihood estimation of the constrained system. Maximizing the likelihood of the systems described in the previous section involves simultaneously choosing 226 parameters to find the maximum of an unusually complicated function. (The complication arises because the means depend upon the variances.) While the minimum distance estimation still requires finding 226 parameters, the minimization of the quadratic form is much more manageable than maximum likelihood estimation.

Table 6 reports the average value of Ω_t (times 100) from the constrained model in which the coefficients and variances are assumed to be functions of the lagged US money supply, the lagged oil price and the lagged first two principal components of the squared deviations from means of the excess returns. It also reports the estimated value of ρ, 87.29.

This very large estimate of ρ is consistent with other empirical investigations of asset pricing relations. It implies that there is a large premium on equity returns. In the context of a general equilibrium model, Mehra and Prescott (1985) also concluded that the size of the equity risk premium was difficult to explain, unless agents were assumed to be extremely risk averse. Frankel (1986) and Engel (1992) have made similar observations about international asset pricing relationships. The mean-variance model appears to be inadequate for explaining these large risk premia.

While we may wish to reject the mean-variance model on the grounds that 87 is an unreasonable estimate of the price of risk, that is not the only reason for doubting the model. The restrictions of MVE on the Tobin asset-pricing model are strongly rejected even when the price of risk is allowed to take on this high value. This should not be too surprising, because examination of the estimates from the unrestricted model (Tables 4 and 5) reveal that there are

TABLE 7
Chi-square tests based on minimum distance estimator

Constant Covariance	
Estimate of ρ	2.05
Chi-square statistic:	183.45*
(degrees of freedom)	(99)
Time-varying Covariance	
Variance a function of:	
LIBOR, price of gold money supply, oil price	
Estimate of ρ	108.36
Chi-square statistic:	1006.51*
(degrees of freedom)	(499)
LIBOR, price of gold lagged money, lagged oil price	
Estimate of ρ	42.87
Chi-square statistic:	1040.36*
(degrees of freedom)	(499)
4 principal components	
Estimate of ρ	283.04
Chi-square statistic:	1097.82*
(degrees of freedom)	(499)
2 principal components lagged money, lagged oil price	
Estimate of ρ	87.29
Chi-square statistic:	1037.91*
(degrees of freedom)	(499)

* Significant at the 1% level.

large differences in the ratios of b_{ij}/ω_{ij}.[6] It is apparent that the model is estimated precisely enough that the restriction of equality of all of the b_{ij}/ω_{ij} can be rejected. In fact, the estimates of the ω_{ij} from the restricted model do not seem very similar to the ω_{ij} of the unrestricted model, or to the b_{ij} of the unrestricted model divided by 87.

Table 7 reports the estimated values of ρ, and the chi-square test of the MVE restrictions for all of the models we have considered (the constant variance model, and the four time-varying variance models). It shows that the restrictions of MVE are strongly rejected in every case. Also, with the exception of the constant-variance model, the estimated values of ρ are very large.

5. Asset demand equations

The models we have estimated have the interpretation of equilibrium asset pricing models: $E_t r_{t+1} = B_t \lambda_t$. However, as we noted in Section 2, this relation

[6] Recall, of course, that Tables 4, 5, and 6 do not report actual values of the estimates of B_i and Ω_i ($i = 1, \ldots, m$), but instead, to save space, report the average values of B_t and Ω_t. However, the statement concerning the b_{ij} and ω_{ij} is true and evident from observation of the (unreported) B_i and Ω_i.

TABLE 8
Coefficients in asset-demand systems
Based on inverse of matrix in Table 4

	UK	*US*	*JP*	*GE*	*CA*	*SW*	*NL*	*FR*	*IT*	*BE*
UK	2.97	−3.65	−1.26	−0.12	8.60	9.93	−24.07	−4.41	−2.07	10.75
US	15.60	−30.04	−3.54	1.72	43.94	35.37	−82.90	−22.64	−12.63	35.31
JP	7.83	−6.73	−3.91	−0.41	23.49	31.37	−73.57	−13.63	−5.24	34.31
GE	0.99	−2.44	−0.52	0.02	3.87	3.45	−7.20	−1.81	−1.03	3.12
CA	1.67	−2.25	−0.64	−0.04	4.56	4.76	−11.54	−2.28	−1.09	4.93
SW	0.60	−1.20	−0.42	0.09	2.38	2.52	−5.72	−1.13	−0.55	2.55
NL	0.29	0.74	−0.14	0.07	1.19	1.02	−2.17	−0.57	−0.33	0.93
FR	0.74	−1.50	−0.34	−0.10	2.74	2.69	−5.80	−1.28	−0.69	2.49
IT	0.63	−0.36	−0.13	−0.20	1.27	1.93	−4.79	−0.70	−0.30	2.14
BE	0.15	−0.38	−0.06	−0.03	0.63	0.53	−1.07	−0.27	−0.18	0.45
	Based on inverse of 2Ω *in Table 5 (rescaled to actual values)*									
UK	237.41	−42.24	−0.77	128.44	−5.97	−110.9	−156.5	−43.06	−18.09	−21.74
US	−42.24	678.56	−4.39	70.01	−192.2	−115.7	−169.6	−38.64	38.19	−23.31
JP	−0.77	−4.39	308.60	−21.10	−7.78	18.61	−66.67	8.70	−49.26	−64.51
GE	128.44	70.01	−21.10	646.21	27.67	−412.3	−209.5	−88.16	5.78	−61.83
CA	−5.97	−192.2	−7.78	27.67	268.95	−28.98	−32.99	−13.94	−22.57	33.35
SW	−110.9	−115.7	18.61	−412.3	−28.98	911.66	−83.53	−64.58	−29.36	−68.32
NL	−156.5	−169.6	−66.67	−209.5	−32.99	−83.53	726.63	2.11	31.69	−38.88
FR	−43.06	−38.64	8.70	−88.16	−13.94	−64.58	2.11	301.57	−51.35	−59.86
IT	−18.09	38.19	−49.26	5.78	−22.57	−29.36	31.69	−51.35	176.36	−25.43
BE	−21.74	−23.31	−64.51	−61.83	33.35	−68.32	−38.88	−59.86	−25.43	375.75
	Based on inverse of $87.29 \cdot \Omega$ *in Table 6 (rescaled to actual values)*									
UK	5.67	1.04	0.82	3.14	0.03	−2.34	−3.35	−0.96	−0.24	−0.50
US	1.04	48.84	9.57	3.76	−2.84	−0.97	−1.71	−0.99	2.68	−0.25
JP	0.82	9.57	14.87	−0.59	0.76	2.04	−0.60	1.24	−0.36	−2.60
GE	3.14	3.76	−0.59	15.26	0.68	−9.46	−4.73	−2.25	0.21	−1.27
CA	0.03	−2.84	0.76	0.68	6.37	−0.53	−0.72	−0.24	−0.42	0.80
SW	−2.34	−0.97	2.04	−9.46	−0.53	21.19	−1.64	−1.23	−0.46	−1.86
NL	−3.35	−1.71	−0.60	−4.73	−0.72	−1.64	16.84	0.05	0.88	−1.02
FR	−0.96	−0.99	1.24	2.25	−0.24	−1.23	0.05	7.21	−1.11	−1.54
IT	−0.24	2.68	0.36	0.21	−0.42	0.46	0.88	−1.11	4.21	−0.71
BE	−0.50	−0.25	−2.60	−1.27	0.80	−1.86	−1.02	−1.54	−0.71	8.80

is based on an inversion of a system of asset demand equations: $\lambda_t = A_t E_t r_{t+1}$. It is interesting to consider the asset demand equations that we have implicitly estimated.

Table 8 reports three different estimates of the demand for equities from the ten countries. They all report the matrix A for the ten-country system of equity demands. The first estimate of A is simply the inverse of the average B_t for the unrestricted model reported in Table 4. The second is $\Omega_t^{-1}/2$ from the unrestricted estimates reported in Table 5. The inverse of Ω_t is divided by 2 on the assumption that the price of risk is 2. The third set of estimates of A is the inverse of the restricted estimate of the average B_t based on Table 6 (that is, the inverse of the matrix in Table 6 divided by 87.29, the estimate of ρ).

First consider the system of demand equations based on the unrestricted average B_t. Note that several of the diagonal elements are negative, which means that demand for an asset declines on average when its expected return increases. While this could occur in principle (for example, if the variance of the return on the asset tended to rise a great deal when the expected return did), it seems likely that this is a mismeasurement of the response of the asset demand to the expected return.

What leads to this mismeasurement? The most probable culprit in this case seems to be mismeasurement of expectations. There are two possible explanations. The first is that expectations are not rational. If we knew agents' true expectations we would find that asset demands rise when the own expected return (measured correctly) rises. But, under this explanation, our assumption that the *ex post* return measures the true expectations up to a white noise error term is incorrect.

The second is that while expectations are rational, there is such a high variance in *ex post* returns, that in any given small sample the *ex post* returns may be a poor measure of individual's true expectations, but these expectations are nonetheless rational. This small-sample, or 'peso' problem haunts all empirical work that imposes the rational expectations assumption. There has not been a satisfactory resolution of the problem.

So, we find that imposing rational expectations leads to an unsatisfactory estimate of the unrestricted (Tobin) system of asset demand equations. Perhaps imposing the MVE restrictions (on the grounds of *a priori* knowledge) leads to more reasonable looking asset demand equations. Consider now the second set of estimates based on the inverse of $\rho\Omega_t$ from the unrestricted regressions (as reported in Table 6, with $\rho = 2$). These are also unconvincing estimates of asset demand equations. Many of the coefficients are too large to be plausible. For example, these estimates imply that a one percentage point increase in the expected return on US equities leads to an increase in the share of US assets in the overall portfolio of 6.78. This is impossible, since the share of any country (in the aggregate portfolio) cannot exceed one. The implication is that no shock could lead to an increase in expected returns on US equities of greater than about 0.15 percentage points in equilibrium, or else the constraint that the share be less than one would be violated.

These large coefficients are in part a result of the assumption that ρ equals 2. The estimates of the constrained model find a ρ equal to 87.29. Examination of the system of asset demand equations based on the constrained model (the third matrix in Table 8) reveals a not unreasonable system of equations. The diagonal coefficients are all positive, and no coefficient is unreasonably large. The own return has the largest influence on asset demands for each of the assets.

The demand for securities also depends heavily on returns on US, UK, Japanese, and German equities.

6. Summary and conclusions

We have presented two sets of tests of the mean-variance efficiency of a portfolio of equities from ten countries. The first set of tests, the Wald tests, do not reject the constraints that MVE places on a general system of asset demand equations when the value of ρ is not specified. This result is true both when the conditional co-variances of individual asset returns are constant over time, and when they are allowed to vary as a function of economic variables.

The second set of tests is based on estimates of a set of asset demand equations on which the MVE constraints are imposed. We estimate the MVE constrained model using a minimum distance estimator, and then test whether the MVE restrictions significantly worsen the fit of the unconstrained model.

The estimated asset demand equations from the regressions that impose the MVE constraints appear to be plausible specifications of equity demand. However, these estimates imply a very large value of ρ, the market price of risk. Furthermore, the constrained model is strongly rejected relative to the general asset-pricing model. In general, our results argue against mean-variance efficiency of the world equity market.

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REFERENCES

Amemiya, T. (1977). 'A Note on a Heteroscedastic Model', *Journal of Econometrics*, **6**, 365–70.

Amemiya, T. (1978). 'Corrigenda: A Note on a Heteroscedastic Model', *Journal of Econometrics*, **8**, 265.

Amemiya, T. (1985). *Advanced Econometrics*, Harvard University Press, Cambridge, MA.

Attanasio, O. and Edey, M. (1987). 'Non-Constant Variances and Foreign Exchange Risk: An Empirical Study', Discussion Paper No. 285, Centre for Labour Economics, London School of Economics.

Bollerslev, T., Engle, R. and Wooldridge, J. (1988). 'A Capital Asset Pricing Model with Time-Varying Covariances', *Journal of Political Economy*, **96**, 116–31.

Cho, D. C., Eun, C. and Senbet, L. (1986). 'International Arbitrage Pricing Theory: An Empirical Investigation', *Journal of Finance*, **41**, 313–30.

Courakis, A. S. (1975). 'Testing Theories of Discount House Portfolio Selection', *The Review of Economic Studies*, **42**, 643–8.

Cumby, R. (1990). 'Consumption Risk and International Equity Returns: Some Empirical Evidence', *Journal of International Money and Finance*, **9**, 182–92.

ENGEL, C. (1992). 'On the Foreign Exchange Risk Premium in a General Equilibrium Model', *Journal of International Economics*, **32**, 305–19.

ENGEL, C and RODRIGUES, A. (1989). 'Tests of International CAPM with Time-Varying Covariances' *Journal of Applied Econometrics*, **4**, 119–38.

ENGEL, C., FRANKEL, J., FROOT, K. and RODRIGUES, A. (1989). 'The Constrained Asset Share Estimation (CASE) Method: Testing Mean-Variance Efficiency of the US Stock Market', mimeo, University of Virginia.

FERSON, W., KANDEL, S. and STAMBAUGH, R. (1987). 'Tests of Asset Pricing with Time-Varying Expected Risk Premiums and Market Betas', *Journal of Finance*, **42**, 201–20.

FRANKEL, J. (1982a). 'A Test of Perfect Substitutability in the Foreign Exchange Market', *Southern Economic Journal*, **49**, 406–16.

FRANKEL, J. (1982b). 'In Search of the Exchange Risk Premium: A Six-Currency Test Assuming Mean-Variance Optimization', *Journal of International Money and Finance*, **1**, 255–74.

FRANKEL, J. (1983). 'Estimation of Portfolio Balance Functions that are Mean-Variance Optimizing: the Mark and the Dollar', *European Economic Review*, **23**, 315–27.

FRANKEL, J. (1984). 'Tests of Monetary and Portfolio Balance Models of Exchange Rate Determination', in J. Bilson and R. Marston (eds), *Exchange Rate Theory and Practice*, University of Chicago.

FRANKEL, J. (1985). 'Portfolio Crowding Out Empirically Estimated', *Quarterly Journal of Economics*, **100**, 1041–65.

FRANKEL, J. (1986). 'The Implications of Mean-Variance Optimization for Four Questions in International Macroeconomics', *Journal of International Money and Finance*, **5**, S53–75.

FRANKEL, J. and ENGEL, C. (1984). 'Do Asset-Demand Functions Optimize over the Mean and Variance of Real Returns? A Six-Currency Test', *Journal of International Economics*, **17**, 309–23.

FULLER, W. (1987). *Measurement Error Models*, Wiley, New York.

GIOVANNINI, A. and JORION, P. (1987). 'Interest Rates and Risk Premia in the Stock Market and in the Foreign Exchange Market', *Journal of International Money and Finance*, **6**, 107–23.

GIOVANNINI, A. and JORION, P. (1989). 'The Time-Variation of Risk and Return in the Foreign Exchange and Stock Markets'. *Journal of Finance*, **44**, 307–26.

GREGORY, A. and VEALL, M. (1985). 'Formulating Wald Tests of Nonlinear Restrictions', *Econometrica*, **6**, 1465–8.

KORAJCZYK, R. and VIALLET, C. (1989). 'An Empirical Investigation of International Asset Pricing', *The Review of Financial Studies*, **2**, 553–85.

LEWIS, K. (1988). 'Inflation Risk and Asset Market Disturbances: The Mean-Variance Model Revisited', *Journal of International Money and Finance*, **7**, 273–88.

MEHRA, R. and PRESCOTT, E. C. (1985). 'The Equity Premium: A Puzzle', *Journal of Monetary Economics*, **15**, 145–61.

PARKIN, M. (1970). 'Discount House Portfolio and Debt Selection', *The Review of Economic Studies*, **37**, 469–97.

ROLL, R. (1977). 'A Critique of the Asset Pricing Theory's Tests Part I: On Past and Potential Testability of the Theory', *Journal of Financial Economics*, **4**, 129–76.

ROTHENBERG, T. (1973). *Efficient Estimation with A Priori Information*, Yale University Press, New Haven.

SHANKEN, J. (1990). 'Intertemporal Asset Pricing: An Investigation', *Journal of Econometrics*, **45**, 99–120.

SILVEY, S. D. (1975). *Statistical Inference*, Wiley, New York.

THEIL, H. (1971). *Principles of Econometrics*. Wiley, New York.

TOBIN J. (1958). 'Liquidity Preference as Behavior toward Risk', *Review of Economic Studies*, **67**, 8–28.

TOBIN, J. (1969). 'The General Equilibrium Approach to Portfolio Balance Theory', *Journal of Money, Credit and Banking*, **1**, 15–29.

WHEATLEY, S. (1988). 'Some Tests of International Equity Integration', *Journal of Financial Economics*, **21**, 177–212.