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Monetary Policy in the Open Economy Revisited: Price Setting and Exchange-Rate Flexibility

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This paper develops a welfare-based model of monetary policy in an open economy. We examine the optimal monetary policy under commitment, focusing on the nature of price adjustment in determining policy. We investigate the implications of these policies for exchange-rate flexibility. The traditional approach maintains that exchange-rate flexibility is desirable in the presence of real country-specific shocks that require adjustment in relative prices. However, in the light of empirical evidence on nominal price response to exchange-rate changes—specifically, that there appears to be a large degree of local-currency pricing (LCP) in industrialized countries—the expenditure-switching role played by nominal exchange rates may be exaggerated in the traditional literature. In the presence of LCP, we find that the optimal monetary policy leads to a fixed exchange rate, even in the presence of country-specific shocks. This is true whether monetary policy is chosen cooperatively or non-cooperatively among countries.

To what extent does independent monetary policy in an open economy require flexibility of the nominal exchange rate? The modern case for flexible exchange rates goes back to Friedman (1953). Real country-specific productivity or demand shocks require adjustment of relative price levels between countries. If nominal prices adjusted quickly, Friedman argues, the choice of exchange-rate regime would be irrelevant because the relative price adjustment could be achieved by nominal price changes:

> If internal prices were as flexible as exchange rates, it would make little economic difference whether adjustments were brought about by changes in exchange rates or by equivalent changes in internal prices. But this condition is clearly not fulfilled. The exchange rate is potentially flexible in the absence of administrative action to freeze it. At least in the modern world, internal prices are highly inflexible.

Freely floating exchange rates allow the needed relative price adjustment to occur instantaneously when nominal goods price adjustment is sluggish. As Feldstein (1992) puts it,

> Either form of adjustment (exchange-rate or price) can bring the real exchange rate to its equilibrium value, but a decline in domestic prices is likely to require a period of increased unemployment. It would certainly be better to have a decline in the nominal exchange rate. The shift to a single currency in Europe would preclude such nominal exchange-rate adjustments and force real exchange-rate reductions to be achieved through lower local wages and prices.

This reasoning has led to the well-known traditional recommendation to monetary policy makers in open economies: an optimal monetary policy in an open economy requires exchange-rate flexibility. But the argument relies on the notion that exchange-rate movements have a large immediate impact on aggregate demand, by allowing instantaneous adjustment of relative prices. Recent empirical work, however, indicates that in the short run there is very little response of consumer prices to changes in nominal exchange rates.[1] The short-run adjustment role of nominal exchange rates is eliminated to the extent that consumer prices are unresponsive to exchange-rate changes: the so-called "expenditure-switching effect" may be negligible.

There is a voluminous literature on optimal monetary policy in open economies under price stickiness.[2] But until recently, little of that analysis has been based on choice-theoretic models, and therefore has lacked an adequate welfare criterion. In this paper we develop a two-country, stochastic, sticky-price general equilibrium optimizing model, drawing on the work of Obstfeld and Rogoff (hereinafter referred to as OR, 1995, 1998).[3] To conduct a realistic analysis of the role of the exchange rate in monetary policy, we allow for both country-specific productivity or supply shocks, and country-specific velocity shocks. We evaluate optimal monetary policy using the welfare of consumers as the criterion. As in OR (1998), our model is simple enough to admit an exact solution for the distribution of prices, exchange rates and all real allocations.

While we are solely concerned with monetary policy under sticky prices, a critical distinction in our analysis is the currency in which prices are set. We allow two different specifications of price setting. Our first specification follows OR (1995, 1998), and implicitly the Friedman and Feldstein analyses, assuming that nominal prices are pre-set in the currency of the producer (denoted producer-currency pricing or PCP). As a result, when exchange rates fluctuate, imported goods prices move one-for-one with exchange rates. But we also allow for another type of price stickiness—prices that are pre-set in consumers' currency (denoted local-currency pricing or LCP).[4] This specification is more in accord with the empirical findings cited above.

The distinction between price-setting specifications is critical for our analysis of the benefits of exchange-rate flexibility in the face of country-specific productivity shocks. Under the Friedman type of price stickiness—PCP—optimal monetary policy relies on nominal exchange-rate adjustment in the presence of real country-specific shocks. The exchange rate must be employed as part of optimal monetary policy in order to achieve a change in relative prices, precisely as described by Friedman. In fact, with exchange-rate flexibility, optimal monetary policy can *replicate* the equilibrium of the economy with fully flexible prices. Flexible exchange rates are a perfect substitute for flexible goods prices.

In contrast, when prices are set in local currency, there is no benefit to exchange-rate flexibility. Under the optimal monetary policy, followed by policy makers in each country, exchange rates are fixed. Intuitively, when the policy maker chooses an optimal monetary rule under LCP, she does not attempt to use monetary policy to alter the relative price of home to foreign goods, because movements in exchange rates do not affect the prices faced by consumers. Unlike the policy rule for the PCP economy, monetary authorities under LCP cannot replicate the flexible price equilibrium. Nevertheless, they still do not want to use the exchange rate as part of an optimal monetary policy.

1. See Mussa (1986), Engel (1993, 1999), Engel and Rogers (1996), and Parsley and Wei (2001), for example.
2. See Devereux and Engel (1998) for a long but partial list of citations.
3. There is by now a substantial literature in open-economy macroeconomics using the OR (1995) structure. See Lane (2001) for a survey. Bacchetta and van Wincoop (2000) examine the impact of alternative exchange-rate regimes in a two-period version of this framework. In addition, a related literature has developed sticky-price monetary models along the lines of Rotemberg and Woodford (1997). See Benigno and Benigno (2001) for an application to open economies.
4. Betts and Devereux (1996, 2000*a*) have also made this assumption in open-economy sticky-price models, but not in a fully stochastic model, which is required to analyse optimal monetary policy.

These results hold true whether optimal monetary rules are determined cooperatively or non-cooperatively among monetary authorities. The optimal monetary policy under LCP is equivalent to that which would be chosen were countries to group together within a cooperative exchange-rate peg. Moreover, an explicit policy of fixing exchange rates is optimal in the following sense. Let the domestic policy maker, instead of following a monetary rule as described in our main results, simply choose a state-contingent exchange-rate rule, while the policy maker in the other country maintains the optimal monetary rule under LCP. Then the domestic policy maker's optimal choice is to keep the exchange rate fixed.

The results therefore imply that the degree to which an optimal monetary policy requires a flexible exchange rate depends critically upon the way in which consumer goods prices are set. Substantial empirical evidence attests to the importance of LCP at the consumer level. In this case, exchange-rate adjustment is not an important part of an optimal monetary policy rule. This would seem to throw doubt on the conventional calculation of the benefits of floating exchange rates. Hence, our results question one of the elements in the traditional assessment of the costs of a single currency.

Section 1 sets out the general equilibrium model. In Section 2, we discuss features of the solution to the model. The optimal monetary policy rules are derived and examined in Section 3. Some conclusions follow. Most of the derivations of our results are contained in an Appendix available on the *Review of Economic Studies* website.

1. THE MODEL

There are two countries: home and foreign. In each country, households maximize expected lifetime utility taking prices and wages as given. Firms are monopolistic and maximize utility for their owners. There are shocks to both velocity and production technologies.

We use two alternative specifications for the pricing of consumer goods. In both specifications, producers must set prices prior to the realization of shocks. This is an institutional constraint in our model, although it can be justified by the presence of menu costs of price adjustment. All prices fully adjust to all shocks after one period; *i.e.* there is no persistence to the price-adjustment process.[5] In the first specification, producers set prices in terms of their own currency. For example, the home currency price of home goods is set and is unresponsive to shocks. The price paid for home goods by foreign consumers then changes when there is an unexpected exchange-rate change. We denote this specification "PCP".

In the other specification, producers set prices in consumers' currencies. For example, home firms set one price for home-country consumers in the home currency and another price for foreign-country consumers in the foreign currency. Unexpected changes in exchange rates have no effect on consumer prices, either at home or in the foreign country. This specification is referred to as one of "LCP".

It is natural to focus on the two extreme cases of price-setting behaviour: complete PCP or complete LCP. PCP is the traditional assumption in open-economy macroeconomics (see for instance OR, 1995). Moreover, it is the central element in the case for flexible exchange rates made by Friedman and many later writers. But there seems to be widespread evidence for LCP, at least in North America and Europe, as discussed in the introduction.

There has been some debate regarding the empirical implications of LCP. Obstfeld and Rogoff (2000) have criticized the LCP assumption on the grounds that it contradicts an empirical

5. This assumption is made so that monetary rules can be derived analytically. To apply our model to the realistic design of optimal monetary policies, it would be necessary to introduce more persistent price stickiness, as in Rotemberg and Woodford (1997), or Chari, Kehoe and McGrattan (2002).

regularity in the terms of trade—that an appreciation of a country's currency tends to be associated with an improvement in its terms of trade. It seems that LCP pricing may imply the opposite relationship. But the Appendix of Devereux, Engel and Tille (2003*b*) shows that the LCP specification used in the present paper is equivalent to one in which producers price in their own currency, but in which importing intermediaries set consumer prices in local currencies. That specification predicts that a nominal appreciation improves in the terms of trade, consistent with observations.

Consumer preferences

The representative consumer in the home country is assumed to maximize

$$U = E_0\left(\sum_{t=0}^{\infty} \beta^s u_t\right), \qquad 0 < \beta < 1$$

where

$$u_t = \frac{1}{1-\rho} C_t^{1-\rho} + \frac{\chi}{1-\varepsilon} V_t \left(\frac{M_t}{P_t}\right)^{1-\varepsilon} - \eta L_t, \qquad \rho > 0, \varepsilon > 0.$$

C is a consumption index that is a geometric average of home and foreign consumption:[6]

$$C = \frac{C_h^n C_f^{1-n}}{n^n (1-n)^{1-n}}.$$

We assume that there are n identical individuals in the home country, $0 < n < 1$. C_h and C_f are indexes over consumption of goods produced at home and in the foreign country, respectively (see the Appendix for further details). The elasticity of substitution between goods produced within a country is λ, where $\lambda > 1$. There is a unit elasticity of substitution between the home goods and foreign goods indexes. M/P are domestic real balances, and L is the labour supply of the representative home agent.

V is a random variable, assumed to follow a first-order Markov process, representing shocks to velocity. The money-in-the-utility function formation is itself an informal means of capturing the benefits of transactions balances in terms of reducing shoe-leather costs. The velocity shock is meant to capture technological innovations that alter the usefulness of money balances. The velocity disturbances follow the processes

$$\ln V_t = \ln V_{t-1} + v_t$$
$$\ln V_t^* = \ln V_{t-1}^* + v_t^*,$$

where v_t and v_t^* are mean-zero i.i.d. normally distributed random variables.[7]

The exact price index P is defined by

$$P = P_h^n P_f^{1-n}, \tag{1.1}$$

where P_h and P_f are price indexes for home and foreign goods, respectively.

The utility function takes a special form, but it allows us to derive exact expressions for welfare across a range of alternative exchange-rate regimes and monetary policies. It is a slight variant of the specification used by OR (1998). While the separability of the utility function may be problematic for analysing some aspects of exchange-rate and current-account dynamics,

6. All variables in this section are implicitly functions of the state of the world $\vartheta_t \in \vartheta^t$, where ϑ^t is a suitably defined set of all possible states at time t (see the Appendix for details). To economize on space, we omit this state notation until we explicitly define the stochastic processes governing money and productivity shocks in the next section.

7. More general models for the stochastic process of velocity unnecessarily complicate the algebra, because as we shall see, optimal policy is a linear function of innovations to velocity shocks and productivity shocks.

it is very helpful in making the model more tractable analytically. Bacchetta and van Wincoop (2000) have emphasized (in a static framework) the importance of non-separabilities of leisure and consumption for the choice of an exchange-rate regime.

There are $1 - n$ identical individuals in the foreign country. Their preferences are identical to those of the home country, except being functions of foreign real balances and foreign labour supply.

Consumers obtain wages W_t from working, receive profits from firms and transfers from government, and get payments on state-contingent bonds. They choose their holdings of money balances, consumption, labour supply and state-contingent bonds.

Asset markets

We assume that residents of each country can purchase a full set of state-contingent nominal bonds. Bonds have payoff in currencies, not goods. This distinction is important in the LCP specification, where the law of one price for goods does not hold. Because the contracts have payoffs in money, consumers are forced to buy goods at prices set for their country. If contracts were denominated in goods, households could effectively circumvent any international price discrimination. When the law of one price holds (as it does under the PCP specification) the distinction between nominal and real payoffs to state-contingent bonds has no importance. We consider the assumption of a complete set of nominal state-contingent bonds to be one of convenience, which approximates the assumption of perfect capital mobility, while ruling out dynamic effects of wealth redistribution, which in both a quantitative and welfare sense are likely to be of second-order importance.

Government budget constraint

Government increases the money supply with direct transfers. The government budget constraint (in *per capita* terms) is simply

$$M_t = M_{t-1} + T_t.$$

In the discussion below, we will be more specific about the form of the optimal monetary rules.

Production technology and firms

Firms are monopolistic competitors. The production function for firm i is given by

$$Y_{it} = \theta_t L_{it},$$

where L_{it} is employment of firm i at time t, and θ_t represents a country-specific productivity shock. We let the productivity disturbances, θ_t and θ_t^*, be governed by the following processes:

$$\ln \theta_t = \ln \theta_{t-1} + u_t$$
$$\ln \theta_t^* = \ln \theta_{t-1}^* + u_t^*$$

where u_t and u_t^* are mean-zero i.i.d. normally distributed random variables.

The objective of the domestic firms is to set prices to maximize the expected utility of the owners. Because there are complete markets, it does not matter whether the owners of firms are home or foreign residents.[8] But prices must be set before information about random money supplies and productivity shocks is known.

8. This is due to (1.5), which ensures that the marginal utility of money is equalized (up to a constant) across countries. Hence a home firm which is maximizing the home currency value of profits weighted by the home agent's marginal utility of money would follow exactly the same plan when evaluating at the foreign agent's marginal utility of home money.

The optimization problem can be expressed as maximizing the expected present value of profits using the market nominal discount factor for the owners of the firm. The objective function of firms is delineated in the Appendix.

Optimal choices

The money demand equation for the representative home-country resident is given by

$$\frac{M_t}{P_t} = \frac{\chi^{1/\varepsilon} V_t^{1/\varepsilon} C_t^{\rho/\varepsilon}}{(1 - E_t d_{t+1})^{1/\varepsilon}}, \tag{1.2}$$

where $E_t d_{t+1}$ is the inverse of the gross nominal interest rate, and $d_{t+1} = \left(\beta \frac{C_{t+1}^{-\rho} P_t}{C_t^{-\rho} P_{t+1}}\right)$. The optimal intratemporal goods demands are described in the Appendix.

Because all shocks are log-normal, the solution of the model will take on a log-normal distribution. All equations in the model save for (1.2) are log-linear, and therefore, in the presence of log-normal distributions, offer exact solutions. But the money market equilibrium conditions in each country require an approximation. Following OR (1998), the Appendix shows that the following approximation can be employed to represent an equilibrium in the home country money market:

$$m_t - p_t = \frac{\rho}{\varepsilon} c_t - \frac{1}{i\varepsilon}(E_t p_{t+1} + \rho E_t c_{t+1} - p_t - \rho c_t) + \frac{1}{\varepsilon} \ln(V_t) + \Gamma_m \tag{1.3}$$

where Γ_m is a constant. An analogous condition may be derived for the foreign country. The accuracy of this approximation may be gauged by the extent to which the parameter ε deviates from unity. It is shown in Devereux and Engel (1998) that in the case $\varepsilon = 1$, the money market clearing equation may be solved exactly, and thus, (1.3) involves no approximation.

The optimal trade-off between consumption and leisure implies

$$\frac{W_t}{P_t C_t^{\rho}} = \eta. \tag{1.4}$$

Risk sharing

Following a number of papers in the recent literature, we assume that consumers have access to a full set of state-contingent bonds (*e.g.* Chari *et al.*, 2002). Our approach can be described as follows. The economy begins at an initial date 0. Before the realizations of the state of the world at this date, agents in each country can trade in a set of state-contingent nominal bonds that specify payoffs in all possible future dates and states. This implies that agents can diversify away all country-specific income risk. In particular, agents will equalize the marginal utility of one unit of the nominal asset in all states of the world. In the Appendix, it is shown that this set-up leads to the following risk-sharing condition:

$$\frac{S_t P_t^*}{P_t} = \Gamma_0 \left(\frac{C_t}{C_t^*}\right)^{\rho}, \tag{1.5}$$

where S_t is the home currency price of foreign currency. The parameter Γ_0 is determined in an equilibrium of the initial market for state-contingent bonds. It is shown in the Appendix that $\Gamma_0 = 1$ will hold in a symmetric equilibrium of the monetary policy game, for both types of price-setting regimes that we analyse.

In the analysis below, we assume that monetary authorities in both countries choose optimal monetary rules taking as given $\Gamma_0 = 1$. That is, the optimal monetary rules are determined after the state-contingent financial markets have closed. But in fact, we would get the same answers

TABLE 1

Optimal prices

$P_{ht}^{\mathrm{PCP}} = \frac{\lambda}{\lambda-1}\frac{E_{t-1}\left(\frac{C_t^{1-\rho}W_t}{\theta_t}\right)}{E_{t-1}(C_t^{1-\rho})}$	$P_{ht}^{\mathrm{LCP}} = P_{ht}^{\mathrm{PCP}}$
$P_{ht}^{*\mathrm{PCP}} = P_{ht}^{\mathrm{PCP}}/S_t$	$P_{ht}^{*\mathrm{LCP}} = \frac{\lambda}{\lambda-1}\frac{E_{t-1}\left(\frac{W_t C_t^{*1-\rho}}{S_t\theta_t}\right)}{E_{t-1}(C_t^{*1-\rho})}$
$P_{ft}^{*\mathrm{PCP}} = \frac{\lambda}{\lambda-1}\frac{E_{t-1}\left(\frac{C_t^{*1-\rho}W_t^*}{\theta_t^*}\right)}{E_{t-1}(C_t^{*1-\rho})}$	$P_{ft}^{*\mathrm{LCP}} = P_{ft}^{*\mathrm{PCP}}$
$P_{ft}^{\mathrm{PCP}} = S_t P_{ft}^{*\mathrm{PCP}}$	$P_{ft}^{\mathrm{LCP}} = \frac{\lambda}{\lambda-1}\frac{E_{t-1}\left(\frac{S_t C_t^{1-\rho}W_t^*}{\theta_t^*}\right)}{E_{t-1}(C_t^{1-\rho})}$

even if we made the reverse assumption. The Appendix shows that our results are entirely unchanged if monetary rules are chosen *before* agents engage in cross-country risk sharing.

Note that from (1.5), consumption will differ across the two countries only to the extent that there are changes in the real exchange rate. In the PCP specification, since purchasing power parity holds, we have, as OR (1998) derive, $C_t = C_t^*$.[9]

Price setting

The solutions for optimal pricing rules are given in Table 1. Under the PCP specification, firms choose a single price, in domestic currency, to maximize the state-contingent value of profits. Take for instance the optimal price set by the home firm, $P_{hti}^{\mathrm{PCP}} = P_{ht}^{\mathrm{PCP}}$, given in the top left-hand entry of Table 1. In a world of certainty, the price would simply be a fixed mark-up over unit labour costs. Here, there is an additional mark-up arising from the covariance of marginal cost with $C_t^{1-\rho}$. The term represents a risk premium arising from the covariance of the firm's profits with the marginal utility of consumption.[10] Specifically, the *ex post* increase in costs from lowering prices by one unit is proportional to $\frac{P_{ht}^{-\lambda-1}C_t W_t}{\theta_t}$, while the increase in revenue is proportional to $P_{ht}^{-\lambda}C_t$. Then weighting each side by the discount rate for period $t-1$, and equating in expectations, we arrive at the optimal price as given by the expression in Table 1.

The law of one price holds for the price charged to foreigners by the home firm, $P_{ht}^{*\mathrm{PCP}}$, in the PCP model. Analogous relationships hold for the prices set by the foreign firms, $P_{ft}^{*\mathrm{PCP}}$ and P_{ft}^{PCP}.

Under the LCP specification, the firm chooses two different prices—one to charge residents of its own country, and one to charge residents of the other country. The price charged by the home firm to the home residents is given by the same expression as in the PCP case. The price charged to foreign residents, $P_{ht}^{*\mathrm{LCP}}$, is explained as follows. In selling to the foreign market, the firm calculates that the *ex post* increase in costs from lowering prices by one unit is proportional to $\frac{P_{ht}^{*-\lambda-1}C_t^* W_t}{\theta_t}$, while the increase in revenue is proportional to $P_{ht}^{*-\lambda}C_t^* S_t$. Weighting both sides by the marginal utility of the home consumer, using the risk-sharing rule (1.5), and equating in

9. OR (1998) have a model in which markets are incomplete, but with PPP and endogenous adjustment of the terms of trade, consumption is equated across countries in all states of the world.

10. See Devereux and Engel (1998) and OR (1998) for further discussion of the pricing equations.

expectation, we obtain the expression for $P_{ht}^{*\mathrm{LCP}}$ in Table 1. Note that with perfect certainty, the price would be set so that its home currency value is a fixed mark-up over unit labour costs. Likewise, under LCP, the price charged by foreign firms to its own residents is the same as under PCP, but the price charged to home-country consumers is not.

Equilibrium

Given equal consumption at home and abroad, the goods-market equilibrium condition in the home country under PCP can be written as

$$\theta_t L_t = \frac{P_t C_t}{P_{ht}}. \tag{1.6}$$

With LCP the goods-market equilibrium in the home country is written as

$$\theta_t L_t = n\frac{P_t C_t}{P_{ht}} + (1-n)\frac{P_t^* C_t^*}{P_{ht}^*}. \tag{1.7}$$

For suitably defined stochastic processes governing M_t, M_t^*, V_t, V_t^*, θ_t, and θ_t^*, the 13 functions (of the state of the world) C, C^*, P_h, P_h^*, P_f, P_h^*, P, P^*, S, W, W^*, L, and L^*, are determined by the 13 equations (1.1) and its foreign equivalent, (1.3) and its foreign equivalent, (1.4) and its foreign equivalent, (1.5), (1.6) (under PCP) or (1.7) (under LCP) and their foreign equivalents and the four pricing equations from Table 1. The full solution is derived in the Appendix.

2. FLEXIBLE PRICE SOLUTION

First, it is useful to see the solution to the model in an environment of fully flexible prices. With flexible prices, the profit maximizing price rules obtain in each state of the world, and not just in expectation. We may then establish that

$$C_t = \left(\frac{\lambda\eta}{\lambda-1}\right)^{-\frac{1}{\rho}} \left(\theta_t^n \theta_t^{*(1-n)}\right)^{\frac{1}{\rho}}. \tag{2.1}$$

Consumption is equalized by perfect risk sharing across countries, and is determined solely by home and foreign productivity shocks. Employment in each country is also equated

$$L_t = \left(\frac{\lambda\eta}{\lambda-1}\right)^{-\frac{1}{\rho}} \left(\theta_t^n \theta_t^{*(1-n)}\right)^{\frac{1-\rho}{\rho}}. \tag{2.2}$$

The terms of trade under flexible prices are given by

$$\frac{P_{ht}}{S_t P_{ft}^*} = \frac{\theta_t^*}{\theta_t}. \tag{2.3}$$

We can derive consumption of home goods and foreign goods, which are also identical for home and foreign households

$$C_{ht} = n\left(\frac{\theta_t}{\theta_t^*}\right)^{1-n} C_t \tag{2.4}$$

$$C_{ft} = (1-n)\left(\frac{\theta_t^*}{\theta_t}\right)^{n} C_t. \tag{2.5}$$

The solutions for consumption and employment do not depend at all on velocity shocks. While aggregate consumption and employment are functions of a geometric weighted average

of productivity shocks, $\theta_t^n \theta_t^{*(1-n)}$, the terms of trade and the consumption of home and foreign goods depend also on relative productivity shocks.

In the next two sections, we investigate optimal monetary policy under commitment, when nominal prices are sticky. We assume the log money supply of each country is given by

$$m_t = m_{t-1} + \mu_t \tag{2.6}$$

$$m_t^* = m_{t-1}^* + \mu_t^*. \tag{2.7}$$

Monetary policy consists of rules for μ_t for the home country and μ_t^* for the foreign country. In addition, monetary rules are designed to respond to unanticipated shocks, so $E_{t-1}(\mu_t) = E_{t-1}(\mu_t^*) = 0$ will hold.[11]

Because prices are reset every period, markets are complete, and there is no other source of persistence such as capital accumulation, the decision rules of all agents are stationary. The logs of the driving variables are random walks, so Jensen's inequality implies that $E_{t-1}C_t^{1-\rho}$ and $E_{t-1}L_t$ incorporate a time trend, e^{zt}, where $z \equiv \exp[\frac{1}{2}(\frac{1-\rho}{\rho})^2(n^2\sigma_u^2 + (1-n)^2\sigma_u^{*2})]$, given from the flexible price solutions to these variables in equations (2.1) and (2.2).[12] The conditional expectations of the detrended variables are constant over time because agents' decision rules are stationary and the monetary policy rules are stationary. The time trend is not affected by monetary policy, so we can analyse monetary policy by considering the objective of maximizing the conditional expectation of utility on a period-by-period basis.

Moreover, we will assume that governments ignore the real balances part of the utility function; *i.e.* we consider optimal policies when $\chi \to 0$.[13] This allows us to represent expected utility as

$$E_{t-1}\hat{U}_t = \frac{E_{t-1}C_t^{1-\rho}}{(1-\rho)} - \eta E_{t-1}L_t. \tag{2.8}$$

Because prices fully adjust after one period, innovations to money supplies prior to time t will have no influence on $E_{t-1}\hat{U}_t$. Optimal monetary policy in the home country then involves choosing a rule for the home money supply in time t to maximize (2.8), and likewise for the foreign monetary policy.

3. PRODUCER-CURRENCY PRICING (PCP)[14]

Exchange rate and consumption under PCP

As we have noted above, under the PCP specification purchasing power parity holds, so that condition (1.5) implies $C_t = C_t^*$. Then taking equation (1.3) less its foreign counterpart, using purchasing power parity and the money supply processes (2.6) and (2.7), we get

$$s_t - s_{t-1} = \mu_t - \mu_t^* - \tfrac{1}{\varepsilon}(v_t - v_t^*). \tag{3.1}$$

Given the random walk assumption of money supplies and money demands, it is not surprising that the nominal exchange rate is a random walk. In addition, the exchange rate is directly affected by money and velocity shocks.

11. The Appendix examines the possibility of more general monetary rules.

12. We assume $\beta z < 1$ to ensure convergence of the sum of discounted utility.

13. We follow OR (1995, 1998, 2000) and Corsetti and Pesenti (2001) in this respect. This assumption allows us to abstract from the traditional considerations that lie behind the Friedman rule of zero nominal interest rate, to which we have nothing new to contribute.

14. The results in this section are very similar to those derived by OR (2002), though our work was done simultaneously and independently. Our presentation, therefore, is terse.

The Appendix shows that innovations to consumption are given by

$$c_t - E_{t-1}c_t = \phi\left(\tilde{\mu}_t - \frac{1}{\varepsilon}\tilde{v}_t\right) + \psi\tilde{u}_t, \tag{3.2}$$

where

$$\phi = \frac{(1+i\varepsilon)}{\rho(1+i)} \qquad \text{and} \qquad \psi = \frac{\varepsilon - 1}{\rho\varepsilon(1+i)}.$$

We define $\tilde{x}_t \equiv nx_t + (1-n)x_t^*$ for $x_t = \mu_t, v_t, u_t$.

Consumption is increasing in shocks to the money supply of both countries. As shown in OR (1995), a foreign monetary expansion will lead to a home country appreciation, a fall in the price level, and an expansion in real balances. As a result, home consumption will rise. A home money expansion on the other hand will directly increase consumption, but part of that is dissipated by exchange-rate depreciation. Money demand shocks affect consumption in exactly the opposite way to money supply shocks.

Unlike the flexible price solution, consumption may either rise or fall in response to productivity shocks, in the absence of monetary rules that target these shocks. A positive technology shock raises expected future consumption, raising the real interest rate, and reduces expected future prices, reducing expected inflation. With $\varepsilon > 1$, the rise in the real interest rate dominates the fall in expected inflation, so that the time t nominal interest rate rises. This leads to an excess supply of real money balances in the present period. With sticky prices, this is eliminated by an increase in current consumption.

Optimal monetary policies

We now turn to the solution for optimal monetary policies. Using the labour market clearing condition (1.6), the wage equation (1.4), and the equation for pricing of home goods from Table 1, we obtain

$$E_{t-1}L_t = \frac{(\lambda-1)E_{t-1}C_t^{1-\rho}}{\lambda\eta}.$$

Combine this into the welfare criterion (2.8) to write

$$E_{t-1}\hat{U}_t = \frac{E_{t-1}C_t^{1-\rho}}{(1-\rho)}\left(\frac{\lambda-(1-\rho)(\lambda-1)}{\lambda}\right).$$

From this expression, it is clear that the objective function facing each monetary authority depends only on the conditional mean and variance of log consumption. The conditional variance of consumption may be obtained from (3.2). The Appendix describes the derivation for the conditional mean. Using the pricing equations from Table 1, and the wage equation (1.4), the Appendix then shows that $E_{t-1}\hat{U}_t$ may be written as

$$E_{t-1}\hat{U}_t = \Theta_t \exp(1-\rho)\left(-\frac{\sigma_c^2}{2} - \frac{n(1-n)}{2\rho}\sigma_s^2 - \frac{\tilde{\sigma}_u^2}{2\rho} + \frac{\tilde{\sigma}_{cu}}{\rho} + n(1-n)\frac{(\sigma_{su}-\sigma_{su*})}{\rho}\right). \tag{3.3}$$

In the notation here, σ_x^2 is the variance of x conditional on $t-1$ information, σ_{xy} is the conditional covariance of x and y, and Θ is a constant function of parameters and the period $t-1$ productivity terms. We define $\tilde{\sigma}_u^2 \equiv n\sigma_u^2 + (1-n)\sigma_{u*}^2$ and $\tilde{\sigma}_{cu} \equiv n\sigma_{cu} + (1-n)\sigma_{cu*}$.

Equation (3.3) indicates that the objective function of the monetary authority under PCP is represented in a very simple form; minimize a *linear* function of consumption variance, exchange-rate variance and the covariance of consumption and the exchange rate with home and

foreign productivity shocks. Given the simple assumptions of our model, the objective function for monetary policy becomes a quadratic.[15]

We focus on monetary policy rules with commitment. In our model, this means that the monetary authority explicitly takes account of the influence of policy rules on expected consumption, working through the effect that the probability distribution of the money supply has on the levels of pre-set prices. Moreover, following the recent literature on optimal rules (*e.g.* OR, 2002), we assume that the monetary authority has complete flexibility to design a state-contingent monetary feedback rule. This means that the monetary rules may be conditioned on the *ex post* realizations of productivity and velocity shocks. Then, because (a) the objective functions depend only on the variance of log consumption and the exchange rate, and their covariances with log productivity shocks, and (b) from (3.1), (3.2), log consumption and the exchange rate are linear functions of the underlying innovations, then the *optimal* monetary rules are log-linear functions of velocity innovations and productivity innovations. Hence, we may write the form of the money rules as follows:

$$\mu_t = a_1 u_t + a_2 u_t^* + a_3 v_t + a_4 v_t^* \tag{3.4}$$

$$\mu_t^* = b_1 u_t^* + b_2 u_t + b_3 v_t^* + b_4 v_t. \tag{3.5}$$

The policy maker chooses the parameters of these monetary rules to maximize (3.3), conditional on (3.1), (3.2). We may implicitly write expected utility as $E\hat{U} = E\hat{U}(a, b)$, where $a = \{a_1, a_2, a_3, a_4\}$ and $b = \{b_1, b_2, b_3, b_4\}$. A Nash equilibrium in monetary policies is defined as the set $\{a^N, b^N\}$ which satisfies the problem P1,

$$\begin{aligned} &\max_{a} \quad E\hat{U}(a, b^N) \\ &\max_{b} \quad E\hat{U}(a^N, b). \end{aligned} \tag{P1}$$

The objective function (3.3) is the *same* for both countries since home and foreign consumption are identical. This means that the solution to P1 is identical to the solution of a *cooperative* monetary policy game, where monetary rules are chosen to maximize joint welfare. Therefore, in the PCP environment, there are no gains to monetary policy coordination.[16]

Proposition 1.

(1) *The solution to problem* (P1) *is*

$$a_1^N = 1 - \frac{n(\varepsilon - 1)}{\varepsilon} \qquad a_2^N = -\frac{(1-n)(\varepsilon - 1)}{\varepsilon} \qquad a_3^N = \frac{1}{\varepsilon} \qquad a_4^N = 0$$

$$b_1^N = 1 - \frac{(1-n)(\varepsilon - 1)}{\varepsilon} \qquad b_2^N = -\frac{n(\varepsilon - 1)}{\varepsilon} \qquad b_3^N = \frac{1}{\varepsilon} \qquad b_4^N = 0.$$

(2) *This solution replicates the flexible price allocations. That is, consumption of all goods in both countries and labour supply are identical to the flexible price levels of these variables.*

15. Woodford (2003) describes an alternative, second-order approximation to the utility function that also gives a quadratic objective function for a monetary authority in an optimizing model. See Benigno and Benigno (2002) for an open-economy approximation.

16. This result is sensitive to the specification. In particular, if it is not optimal to use monetary policy to target the flexible price equilibrium, OR (2002) show that there may be gains to policy coordination. Benigno and Benigno (2002) show circumstances where this is the case, even under PCP and with complete markets. In addition, if we had allowed discretionary monetary policy rather than a system of rules as in (3.4) and (3.5), policy coordination would make a difference—see Betts and Devereux (2000*b*).

Proof. See Appendix. ||

Optimal monetary policy requires the home government to respond positively to home productivity shocks but negatively (positively) to foreign productivity shocks as $\varepsilon > 1$ ($\varepsilon < 1$). Monetary authorities fully accommodate home velocity shocks but ignore foreign velocity shocks. Foreign optimal monetary policy is symmetric to the home policy.

Why is it optimal to target the flexible price equilibrium? The economy has three departures from full efficiency. Prices are sticky, so consumption and employment do not respond to money and productivity shocks as they would in a flexible price environment. There are monopolistic competitive mark-ups. Finally, there is a departure from the Friedman rule of zero nominal interest rates. The second loss cannot be affected by the monetary rules (3.4) and (3.5), since these rules cannot influence the mark-ups obtained in Table 1.[17] The third loss is neglected from the analysis because we ignore the utility of real balances. Thus, the best that the monetary policy rules can do is to attempt to replicate the flexible price equilibrium.[18]

With sticky nominal prices, an optimal monetary response to a positive productivity shock must be expansionary, so as to shift demand up to meet the increased potential aggregate supply. For instance, following a positive home country productivity shock, the response of world money, $na_1 + (1-n)b_2$, must be positive (and equal to $\frac{n}{\varepsilon}$ times the productivity shock in this case). With this rule for the world money supply in (3.2), aggregate consumption will increase by n/ρ times the productivity shock, as required by the flexible price solution in equation (2.1).

But an optimal monetary policy under PCP also requires a change in relative prices. From equations (2.4) and (2.5), a positive home country productivity shock must increase consumption of the home good, relative to consumption of the foreign good.[19] For this to occur, there must be a terms of trade deterioration, as described in equation (2.3), to ensure that world demand is substituted towards the home country's products. The required terms of trade deterioration is generated by nominal exchange-rate depreciation. To achieve this, it is necessary that the home country monetary reaction to a home productivity shock is positive and exceeds that of the foreign country. Optimal monetary policy therefore relies on the *expenditure switching* mechanism of the exchange rate under the PCP pricing regime.

An alternative perspective on the optimal monetary rule under PCP can be obtained by comparison to the results of Woodford (2003) for a closed economy, and Benigno and Benigno (2001), or Clarida, Galí and Gertler (2001) for the open economy. In those papers, it is stressed that under many circumstances, the monetary authority should produce zero inflation. In our model, the optimal rate of inflation is not uniquely pinned down—because prices adjust after one period (see Benigno and Benigno, 2002). But there remains a parallel with this literature. By achieving the desired terms of trade adjustment through movement in the exchange rate, an optimal monetary policy rule obviates the need for nominal price adjustment. Even if all prices were fully flexible, they would be constant if monetary rules were determined as in Proposition 1. Thus, the constraint that firms must set their prices in advance ceases to have any impact on equilibrium outcomes in the economy with optimal monetary policy and PCP.

17. Although monetary policy rules do influence expected output levels in our model, they cannot eliminate the distortion from imperfect competition, because the mark-up depends only on constant elasticities. The welfare impact of deviations from competitive pricing can be eliminated by a production subsidy to firms that operates independently of monetary policy. In our analysis, it is easy to show that an equivalent optimal production subsidy exists. But this does not affect any of our conclusions below.

18. Similar observations in a related model have been made by OR (2002). OR (2002) also illustrate that in an environment of incomplete international assets markets it is not necessarily optimal for monetary policy to sustain the flexible price outcome. See also Benigno and Benigno (2001) for further discussion.

19. If this did not occur, then demand for both home and foreign goods would rise equally, and since home labour productivity has risen, home employment would fall relative to foreign employment. This would clearly be inefficient.

Why is it that monetary authorities must react to foreign productivity shocks? This only occurs when $\varepsilon \neq 1$. In response to a foreign productivity shock, the combined policy rules of the home and foreign country ensure that the home currency appreciates, increasing real balances and raising home consumption. When $\varepsilon > 1$, the foreign productivity shock causes a rise in the nominal interest rate, which acts further to generate excess supply of real balances. To eliminate the effect of this on consumption, the home country money supply must be reduced.

The flexible price solutions for the real variables in equations (2.1)–(2.5) have no role for velocity shocks. Since velocity shocks affect the nominal exchange rate with PCP (equation (3.1)), each monetary authority must exactly offset domestic velocity shocks in order to replicate the flexible price outcome. Intuitively, if there were no productivity shocks, any change in the relative price of home to foreign goods would be undesirable. Only a rule that completely negated the effect of velocity shocks on exchange rates would keep the exchange rate stable and prevent any relative price changes in the PCP model.

4. LOCAL-CURRENCY PRICING (LCP)

Under LCP, PPP will not generally hold. Thus, home and foreign consumption will differ. The price level in each country is entirely predetermined in each period, since prices paid by households for domestic and foreign goods are set in advance in the currency of the households. Using the home money market equation (1.3), we can obtain (see the Appendix for further details of the derivations):

$$c_t - E_{t-1}c_t = \phi\left(\mu_t - \frac{1}{\varepsilon}v_t\right) + \psi\tilde{u}_t. \tag{4.1}$$

Foreign consumption is given by an analogous equation

$$c_t^* - E_{t-1}c_t^* = \phi\left(\mu_t^* - \frac{1}{\varepsilon}v_t^*\right) + \psi\tilde{u}_t. \tag{4.2}$$

Using the risk-sharing condition (1.5) and the money market equation (1.3), we can derive an equation for the exchange rate

$$s_t - s_{t-1} = \rho\phi\left(\mu_t - \mu_t^* - \frac{1}{\varepsilon}(v_t - v_t^*)\right) - \frac{i(\varepsilon - 1)}{1+i}(p_t - p_{t-1} - (p_t^* - p_{t-1}^*)).^{20} \tag{4.3}$$

There is an interesting contrast between the solutions for consumption under LCP in equations (4.1) and (4.2), and the solution under PCP in equation (3.2). With LCP, there is no transmission of foreign money supply or velocity innovations to home consumption (in the absence of monetary rules that target these shocks). Likewise, for the foreign country, consumption is independent of home money or velocity shocks. Intuitively, with LCP, exchange-rate changes are not transmitted to the price level, so there is no channel for monetary shocks in one country to influence consumption in the other country.

Both home and foreign productivity shocks affect home country consumption (again abstracting from the effect of monetary rules that target these shocks) because nominal interest rates are influenced by persistent productivity shocks. The channel is the same as in the PCP case.

Expected employment in the LCP specification is determined by using equation (1.7) above. Again, expected employment is identical for home and foreign countries, and given by

$$E_{t-1}L_t = \frac{n(\lambda - 1)E_{t-1}C_t^{1-\rho}}{\lambda n} + \frac{(1-n)(\lambda - 1)E_{t-1}C_t^{*(1-\rho)}}{\lambda n}. \tag{4.4}$$

20. An expression for the predetermined prices in terms of lagged state variables is given in the Appendix.

Because home and foreign consumption are not equated under LCP, the objective functions for home and foreign governments will differ. Substituting (4.4) into (2.8), we get

$$E_{t-1}\hat{U}_t = E_{t-1}C_t^{1-\rho}\left(\frac{\lambda - n(\lambda-1)(1-\rho)}{\lambda(1-\rho)}\right) - \frac{(1-n)(\lambda-1)}{\lambda}E_{t-1}C_t^{*(1-\rho)}. \tag{4.5}$$

In a symmetric fashion, foreign country expected utility equals

$$E_{t-1}\hat{U}_t^* = E_{t-1}C_t^{*1-\rho}\left(\frac{\lambda - (1-n)(\lambda-1)(1-\rho)}{\lambda(1-\rho)}\right) - \frac{n(\lambda-1)}{\lambda}E_{t-1}C_t^{(1-\rho)}. \tag{4.6}$$

These functions may again be decomposed into conditional means and variances of consumption. Then substituting for these values from the Appendix, we can establish:

$$E_{t-1}C_t^{1-\rho} \propto \exp(1-\rho)\left(-\frac{\sigma_c^2}{2} - \frac{\tilde{\sigma}_u^2}{2\rho} + \frac{\tilde{\sigma}_{cu}}{\rho}\right) \tag{4.7}$$

$$E_{t-1}C_t^{*(1-\rho)} \propto \exp(1-\rho)\left(-\frac{\sigma_{c*}^2}{2} - \frac{\tilde{\sigma}_u^2}{2\rho} + \frac{\tilde{\sigma}_{c*u}}{\rho}\right). \tag{4.8}$$

The optimal monetary rule with LCP requires policy makers to target a combination of two separate functions of consumption variance and the covariance between consumption and productivity shocks. But because under LCP, foreign consumption is independent of the parameters of the home country monetary rule, the home country problem amounts to simply minimizing the expression $\frac{\sigma_c^2}{2} + \frac{\tilde{\sigma}_u^2}{2\rho} - \frac{\tilde{\sigma}_{cu}}{\rho}$. Note that (4.7) and (4.8) indicate that the policy maker should not be directly concerned with the distribution of the exchange rate.

The determination of monetary rules in the LCP case can be described exactly as in the PCP case. The Nash equilibrium in monetary policies is defined as the set $\{a^N, b^N\}$, where again, $a = \{a_1, a_2, a_3, a_4\}$, $b = \{b_1, b_2, b_3, b_4\}$, which satisfies the problem (P2),

$$\begin{aligned} &\max_a \quad E\hat{U}(a, b^N) \\ &\max_b \quad E\hat{U}(a^N, b). \end{aligned} \tag{P2}$$

Proposition 2.

(1) *The solution to problem* (P2) *is*

$$a_1^N = \frac{n}{\varepsilon} \quad a_2^N = \frac{1-n}{\varepsilon} \quad a_3^N = \frac{1}{\varepsilon} \quad a_4^N = 0$$

$$b_1^N = a_2 \quad b_2^N = a_1 \quad b_3^N = \frac{1}{\varepsilon} \quad b_4^N = 0.$$

(2) *Under this solution, the nominal exchange rate is constant.*

Proof. See Appendix. ||

Inspection of these rules (in combination with (4.1) and (4.2)) indicates that they ensure that consumption is identical across countries, and has the same variance as in the flexible price equilibrium. But unlike the case of PCP, these monetary rules do not entail exchange-rate adjustment. Each government responds in an identical way to a home or foreign productivity shock and completely offsets a domestic velocity shock, so the exchange rate does not respond to either shock. Exchange rate adjustment plays no role in altering patterns of expenditure between

home and foreign goods. Therefore, it cannot influence the determination of employment, or *relative* consumption of home and foreign goods, in either country. Optimal monetary policy no longer tries to move the exchange rate so as to achieve an optimal terms of trade adjustment, since the terms of trade do not influence demand or employment.

Because policy cannot influence the terms of trade, policy makers in each country can influence welfare only by affecting the aggregate consumption indexes, as the objective functions (4.5) and (4.6) show. In the flexible price equilibrium, aggregate consumption is a function only of world productivity shocks, $\theta_t^n \theta_t^{*(1-n)}$. So monetary policy makers under LCP follow a rule of completely accommodating money demand shocks so that they have no influence on real consumption levels, and then accommodating the world productivity shock. Netting out their response to money demand shocks, the policy makers respond identically to the world productivity shock, which leads to a constant exchange rate.

The absence of exchange-rate movement is at variance with the Friedman argument for exchange-rate flexibility. In Friedman's view, countries that are subject to idiosyncratic real shocks will benefit from exchange-rate adjustment in face of these shocks. But with LCP an optimal policy response to country-specific shocks does require exchange-rate movement.

Proposition 3. *The monetary rules of Proposition* 2 *do not support the full flexible price equilibrium.*

Proof. See Appendix. ||

While the optimal monetary rules keep the response of aggregate home and foreign consumption equal to that of the flexible price equilibrium, they cannot ensure that the relative consumption of the home and foreign goods equal their values in the flexible price equilibrium. As a consequence, employment in each country is not at the level of the flexible price equilibrium. The combination of these two departures from efficiency implies that the conditional mean of consumption in each country is not equal to its flexible price equilibrium level.

An alternative interpretation is that in contrast to the case of PCP, an optimal monetary rule under LCP does not eliminate the impact of sticky prices. Because the monetary rules of Proposition 2 do not allow for terms of trade adjustment through the exchange rate, then were these rules to be followed in a flexible price economy, *nominal prices would still need to adjust* in response to country-specific productivity shocks. Hence, the fact that nominal prices must be set in advance still imposes a welfare cost in the economy with optimal monetary policy. In the Appendix, it is shown that the impact of this constraint is to raise the level of pre-set nominal prices for all firms. As a result, expected consumption is lower under LCP than under PCP.

Utility is concave in aggregate consumption and linear in leisure. In optimal monetary policy literature based less explicitly on equilibrium optimizing models, the goal of monetary policy is sometimes framed as minimizing aggregate risk. In the context of our model, that would mean the goal of policy is to minimize the variance of aggregate consumption. It is interesting then to note that the optimal monetary policy under LCP achieves the same consumption *variance* as under flexible prices. But welfare is lower under local-currency sticky prices. We may write

$$E_{t-1} C_t^{1-\rho} = (E_{t-1} C_t)^{1-\rho} \exp\left(-\frac{\rho(1-\rho)}{2}\sigma_c^2\right).$$

While optimal policy can achieve the same σ_c^2 (and $\sigma_{c^*}^2$) as under flexible prices, welfare is lower because $E_{t-1} C_t$ (and $E_{t-1} C_t^*$) are lower under LCP. The expected consumption aggregates in each country are lower precisely because the absence of relative price adjustment implies that there is sub-optimal substitution between consumption of home goods and foreign goods.

We saw that with the PCP specification, the monetary policy rules chosen under the non-cooperative equilibrium are identical to those chosen under cooperation. An identical result holds under LCP.

Proposition 4. *The optimal monetary policy rules under cooperation are identical to those described in Proposition 2 in the non-cooperative policy game.*

Proof. See Appendix. ||

Although the result is the same as under PCP, the explanation is quite different. With PCP, perfect risk sharing ensures that the objectives of home and foreign policy makers are identical. That is not true here. But with LCP there is effectively no strategic element to monetary policy choices. Home country monetary policy affects only $E_{t-1}C_t^{1-\rho}$ and not $E_{t-1}C_t^{*1-\rho}$. Aggregate consumption is determined in the money market, so exchange-rate changes do not directly influence money demand (contrast equation (4.1) with (3.2)). Likewise foreign monetary policy affects $E_{t-1}C_t^{*1-\rho}$ but not $E_{t-1}C_t^{1-\rho}$. So, home monetary policy minimizes $\frac{\sigma_c^2}{2} + \frac{\tilde{\sigma}_u^2}{2\rho} - \frac{\tilde{\sigma}_{cu}}{\rho}$ and foreign monetary policy minimizes $\frac{\sigma_{c^*}^2}{2} + \frac{\tilde{\sigma}_u^2}{2\rho} - \frac{\tilde{\sigma}_{c^*u}}{\rho}$ with no interaction between the policies.

In the cooperative LCP equilibrium, the real exchange rate is constant in every period. It follows from equation (1.5) that consumption is perfectly correlated in the home and foreign countries. The optimal monetary policy in the LCP case can do nothing to reproduce the relative price movements that occur under flexible prices (as it could in the PCP case). However, the optimal policy, which fixes the nominal and real exchange rate, can produce perfect risk sharing that otherwise would be absent under LCP if nominal exchange rates fluctuated.

Propositions 2 and 4 show that optimal monetary policy making leads to fixed exchange rates whether monetary policy is chosen cooperatively or non-cooperatively. Alternatively, we can recast the problem so that the instrument of the home policy maker is the nominal exchange rate. Suppose for instance that the home country sets exchange-rate policy while the foreign country pursues optimal monetary policy. Then, home money supply growth, μ_t, is determined endogenously by equation (4.3), while the change in the exchange rate is chosen by the home policy maker as a function of productivity and velocity shocks:

$$s_t - s_{t-1} = \alpha_1 u_t + \alpha_2 u_t^* + \alpha_3 v_t + \alpha_4 v_t^*. \tag{4.9}$$

The foreign policy rule is still given by equation (3.5). We can define a Nash equilibrium for this policy game as the set $\{\alpha^N, b^N\}$, where $\alpha = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$, and $b = \{b_1, b_2, b_3, b_4\}$, that satisfies the problem (P3),

$$\begin{aligned} &\max_{\alpha} \quad E\hat{U}(\alpha, b^N) \\ &\max_{b} \quad E\hat{U}(\alpha^N, b). \end{aligned} \tag{P3}$$

Proposition 5. *The solution to problem* (P3) *has the same values for b as in Proposition* 2. *The solution for optimal home-country exchange-rate policy is* $\alpha_1^N = \alpha_2^N = \alpha_3^N = \alpha_4^N = 0$, *so that the exchange rate is fixed. Equilibrium allocations are identical to those achieved under the policies in Proposition* 2.

Proof. See the Appendix. ||

The force of this proposition is to establish that the home country (foreign country) can use exchange-rate policy in place of monetary policy without the cooperation of the foreign country (home country), which is pursuing optimal self-interested monetary policy. The optimal policy in the LCP setting is then to fix the exchange rate.

A corollary to Propositions 2, 4 and 5 is that optimal monetary policy within a cooperative fixed exchange-rate regime, in which a single monetary authority chooses a policy rule to maximize both country's welfare, is equivalent to an optimal monetary policy under LCP. This follows because under LCP, a cooperative (or non-cooperative) outcome for both countries is to maintain a fixed exchange rate. Thus, if a fixed exchange rate is a pre-existing constraint on the policy maker, we may adapt the approach of either Section 3, under PCP, or this section (under LCP), to show that the monetary rules followed by a single monetary authority under fixed exchange rates will be identical to the rules identified in Proposition 2.

In this sense, our results delineate the extent to which there is a welfare cost in sacrificing nominal exchange-rate movement. If international pricing follows PCP, then the exchange rate is an important part of macroeconomic adjustment, and there are welfare costs associated with lack of adjustment in a single-currency area. But if, as much evidence suggests, international consumer goods pricing is dominated by LCP, then in our model there is no cost at all of giving up nominal exchange-rate adjustment.

5. CONCLUSIONS

The optimal degree of exchange-rate flexibility is an old theme in open-economy macroeconomics. The novel aspect of our paper is to re-set this question in a modern, expected utility maximizing framework, and to investigate the extent to which the answer depends on the manner in which prices are set. Our results are quite sharp. If prices are set in producers' currency, and exchange-rate pass-through from exchange rates to consumer prices is immediate, then there is a strong case for flexible exchange rates; exchange-rate adjustment is a central part of an optimal monetary policy. It would be costly, in welfare terms, for a country or group of countries to commit to fixing exchange rates when there are country-specific productivity shocks. But if prices are set in consumers' currency, and do not respond to movements in exchange rates, then there is no case for flexible exchange rates; optimal monetary policy will keep exchange rates fixed, whether policy is chosen cooperatively, or non-cooperatively.

Our modelling approach is based on special assumptions, both about functional form and other aspects of the environment. One apparent limitation of the analysis is that the specification of price setting, whether PCP or LCP, is taken as exogenous. In one sense, this is deliberate. There is substantial empirical evidence for the LCP case. On the other hand the PCP case is clearly the central assumption of traditional open-economy macroeconomics. Hence, it is important to understand the implications of the different cases for optimal monetary policy. More generally however, one would think that the decision by exporters over currency in which to set prices could be made by the producers themselves. In Devereux, Engel and Storgaard (2003*a*), the choice of currency for price setting is made endogenous. In an environment identical to the present model, it is shown that producers will wish to set their export prices in the currency of the country that has the most stable money growth rate. In a symmetric world economy, where countries' productivities have identical volatility, the variance of optimal money growth rates will be identical. In this case, Devereux *et al.* (2003*a*) show that *both* PCP and LCP are equilibrium configurations in the price-setting decisions of firms. Thus, in the case of full symmetry, there is a strict rationale for following the analysis employed in the current paper; both PCP and LCP are potential price-setting equilibriums. More generally however, we might think that menu cost considerations would play an important role in the decision of which currency that firms should

set prices in. Devereux *et al.* (2003*b*) discuss the role of menu costs, and argue that they are likely to lead firms to follow an LCP pricing rule.

Two further assumptions with regard to the setting of monetary policy rules may be questioned. First, we have assumed that monetary authorities take the risk-sharing rule (1.5) as given when they choose optimal monetary rules. Since all the equilibria of the monetary policy games we focus on are symmetric, agents are *ex ante* identical across countries, and perfect pooling across countries is an equilibrium of the state-contingent bond trading market. Alternatively, we could assume that monetary authorities choose policy rules *before* the agents engage in international risk sharing. In this case, the authorities would take into account the impact of the rules on the trading decisions of agents in state-contingent bond markets. But the Appendix shows that this makes no difference at all for the results. Under PCP, we find that $\Gamma_0 = 1$ is an equilibrium of the *ex ante* risk-sharing market independent of the monetary rules chosen. Under LCP, we find that, while the form of the objective function facing monetary authorities is altered when they take into account endogenous risk sharing, their optimal monetary policy decisions are exactly as before.

The second question about the monetary rules is whether the form of the rules in (3.4) and (3.5) are excessively restrictive—do our results arise only due to the simple family of monetary rules in which the money supply is a linear function of the realized shocks? In the Appendix we show that the answer is no. We do this by focusing on a more general problem of a fictitious world social planner who wishes to maximize an equal-weighted function of home and foreign utility, restricted only by the presence of monopoly pricing and *ex ante* price setting. We show that in both the PCP case and the LCP case, the planner would wish to replicate the allocation given by Proposition 1 (for the PCP case), and Proposition 2 (for the LCP case). Since the planner is restricted only by the fact of *ex ante* pricing, and not limited to using any particular monetary policy rule, it follows that the allocations arising from the monetary rules of Proposition 1 (for the PCP case) and Proposition 2 (for the LCP case) cannot be dominated in welfare terms by any other monetary rules.

Money plays a role in this model because money is directly in the utility function. This follows the recent literature on sticky prices in closed- and open-economy models (*e.g.* OR (1995, 1998), Chari *et al.* (2002)). But there are alternative ways of introducing money in dynamic, general equilibrium models. One obvious alternative is to assume a cash-in-advance constraint (*e.g.* Adao, Correia and Teles, 2001). This would alter the details of our model in some respects, depending on the assumptions made about the timing of purchases and the nature of the cash-in-advance constraint. Whatever the form on money-demand function however, it remains the case that exchange rates cannot alter relative prices in a world of LCP.

A related issue is the question of whether it is always optimal for a monetary authority to set policy rules so as to achieve the flexible price equilibrium allocations. In Section 3, we found that this is the case, under complete markets. This result mirrors that of OR (2000), who establish that, in a two-country model with money in the utility function, and full pass-through from exchange rates to imported goods prices, monetary policy should sustain the flexible price equilibrium outcome, so long as international assets markets are complete.[21] Under LCP however, we have shown that this discussion is moot, since it is not feasible for monetary policy to sustain the flexible price equilibrium when monetary rules cannot generate relative price changes.

Finally, we might ask how plausible are models of these type for understanding exchange rates and international macroeconomic fluctuations? Chari *et al.* (2002) argue that sticky-price

21. Generally however, it is not the case that monetary policy should sustain the flexible price equilibrium in the presence of other distortions, arising from monopolistic competition, or positive nominal interest rates. See Adao *et al.* (2001), and Benigno and Benigno (2002) for instance.

open-economy models have difficulty in matching the properties of real exchange rates. Other writers however (Bergin and Feenstra (2001), Kollmann (2001)), extend the Chari *et al.* analysis in various ways, allowing for variable mark-ups, habit persistence, and nominal wage rigidities, arguing that these extensions can help in reconciling the theoretical models with the observed volatility and persistence of real exchange rates.

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