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TESTS OF INTERNATIONAL CAPM WITH TIME-VARYING COVARIANCES

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SUMMARY

We perform maximum-likelihood estimation of a model of international asset pricing based on CAPM. We test the restrictions imposed by CAPM against a more general asset pricing model. The 'betas' in our CAPM vary over time as the supplies of assets change and as the conditional covariances or returns on those assets change. We let the covariances change over time as a function of macroeconomic data, and an alternative model allows the covariances to follow a multivariate ARCH process. We also can identify a modified CAPM model with measurement error. We find that the estimated CAPM performs much better when variances are not constant over time. Nonetheless, CAPM is rejected in favour of the less-restricted model of asset pricing.

1. INTRODUCTION

One branch of the 'asset market approach' to exchange rates has focused on the demand by residents of an open economy for foreign currency denominated assets. In this view, people diversify their portfolios to hold a variety of domestic and foreign assets with the aim of getting the maximum return on their portfolios while taking into consideration the riskiness of the assets. In particular, foreign currency denominated assets are subject to exchange rate risk—or, perhaps more accurately, they might be subject to different purchasing power risk than domestic assets. In the general equilibrium of such a 'portfolio balance' model, the supplies of outside assets affect macroeconomic variables, including the exchange rate. The portfolio balance approach to flexible exchange rate was pioneered by Black (1973), Kouri (1976), Branson (1977) and Girton and Henderson (1977).

Some general forms of the portfolio balance model have been treated empirically by, among others, Frankel (1982a, 1984); Branson, Halttunen and Masson (1977, 1979); Dooley and Isard (1979, 1983) and Lewis (1988a). These models typically postulate that demand for domestic assets relative to foreign assets is a function of the expected value of $i - i^* - \delta$, where i is the return on home assets, i^* is the return on assets from abroad, and δ is the rate of depreciation of the domestic currency. The anticipated level of $i - i^* - \delta$ is often referred to as the risk

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premium in this literature. It represents the excess expected return the domestic asset must pay to compensate for its riskiness. (Of course, the risk premium as defined here could be negative, implying foreign assets pay a greater expected rate of return.) These papers proceed by making some assumption on how expectations of the future exchange rate are formed, and then estimate bond demand equations as functions of the risk premium, or else estimate a reduced form in which the exchange rate depends on asset supplies.

The portfolio balance approach can be seen as one possible explanation of the finding by many authors (including Geweke and Feige (1979), Frankel (1980), Hansen and Hodrick (1980) and Cumby and Obstfeld (1981)) that uncovered interest parity does not hold. That is, in the sample periods tested, the conditionally expected return differential between comparable assets across countries is non-zero ($i_{t+1} - i^*_{t+1} - \delta_{t+1}$ has a non-zero mean conditional on information available at time t). Although the presence of a risk premium is one explanation for this finding, others include the possibility of inefficient markets or a peso problem.

Numerous studies have tested whether the rejection of uncovered interest parity is attributable to a risk premium, without resorting directly to estimating bond demand equations. Various ingenious approaches have been taken by Hansen and Hodrick (1983), Hodrick and Srivastava (1984), Domowitz and Hakkio (1985), Mark (1985), Cumby (1987, 1988), Giovannini and Jorion (1987a), and Kaminsky and Peruga (1987). These studies typically exploit the time series properties of asset returns (and sometimes other variables such as consumption) without relying on asset supply data.

One branch of this literature can be viewed as a refinement on the portfolio balance models, in that it derives asset demand equations (rather than asset pricing equations as in most of the literature cited in the previous paragraph) from an underlying utility-maximization approach. Frankel (1982b) proposed that a popular and reasonable model of asset diversification—the capital asset pricing model (CAPM)—can be implemented for international asset data and estimated. Furthermore, the restrictions that CAPM places on more general bond demand equations can be tested. Papers that employ this type of international CAPM test include Frankel (1983), Frankel and Engel (1984), Engel and Rodrigues (1986), Lewis (1988b), Attanasio and Edey (1987) and Giovannini and Jorion (1987c).[1]

These papers demonstrate that CAPM implies an equation of the form:

$$E_t z_{t+1} = c + \rho \Omega_t \lambda_t \tag{1}$$

where z_{t+1} is a vector of real rates of return between time t and $t+1$ relative to the real return on some numeraire asset (the jth real return is calculated as $[(1+i^j)(1+a^j)/(1+\pi)] - 1$, where i is the nominal rate of return of an asset in its own currency, a is the rate of appreciation of the currency relative to the currency of the numeraire asset and π is the rate of inflation of prices in the currency of the numeraire asset), c is a vector of constants[2], ρ is a constant that is a measure of relative risk aversion of the typical market participant, Ω_t is the conditional variance at time t of z_{t+1}, and λ_t is a vector whose jth element is the value of the total outstanding supply of the jth asset as a share of the value of all assets. The derivation of this equation assumes that all investors have the same consumption basket, and that the law of one price holds.

Frankel's key observation is that if $E_t z_{t+1}$ is replaced by realized values of z_{t+1} then, under

[1] Attanasio and Edey (1987), and Giovannini and Jorion (1987c), are quite similar to this paper and were written independently.

[2] In general, c is a covariance that could change over time (see Frankel and Engel, 1984). We cannot, however, impose any constraints between c and the vector of errors. We treat c as a constant for convenience.

rational expectations, the variance of the vector of forecast errors $(z_{t+1} - E_t z_{t+1})$ should equal Ω_t. This suggests an empirical test of CAPM. Regress each relative ex-post rate of return on all of the asset shares in λ. If there are N assets (not counting the numeraire), this would yield an $N \times N$ matrix of regression coefficients that under the CAPM hypothesis should be proportional to the covariance matrix of the regression errors—with the constant of proportionality equal to ρ.

In the international finance context this idea has been implemented by constructing aggregate asset data comprising the outstanding obligations of governments from each of several countries. Dollar assets are chosen as numeraire, and average real returns for assets from each of the other countries relative to the real return on dollar assets are calculated. In the CAPM tests that have been performed using this technique there has been little support for the restrictions imposed by the CAPM theory (see Frankel (1986) for a discussion of this literature).

Typically the conditional variance, Ω_t, has been treated as a constant. However, several authors, including Hodrick and Srivastava (1984), Cumby and Obstfeld (1985), Hsieh (1984) and Diebold and Nerlove (1986), have noted that forecast errors in foreign exchange markets are notoriously heteroskedastic. Giovannini and Jorion (1987a) offer some 'back-of-the-envelope' calculations that suggest that the degree of variability over time in Ω_t is large enough to account for the empirical failure of CAPM (however, see Giovannini and Jorion (1987b) and Frankel (1987)).

The international finance literature does not offer a very good guide to the determinants of the variance of the forecast error. It seems plausible that forecasts should have higher variance in times of economic turbulence. One approach we take in this paper is to let Ω_t vary over time as a function of macroeconomic data such as the US money supply and oil prices. If, in fact, shocks to the US money supply and dollar oil prices increased the difficulty of forecasting foreign exchange rates, then the constant variance models of Frankel are misspecified. They base their measurement of the forecast variance at any time on the average past squared forecast errors. However, if the money supply or oil prices have been behaving erratically in the recent past, it is likely that the exchange rate forecast variance will increase.

A method of modelling time-varying variances that does not rely on macroeconomic data has been suggested by Engle (1982). In essence, he postulates that the variance in this period is likely to be large following a large error (positive or negative) in the previous period. In a univariate context, for example, we might see

$$\sigma_t^2 = \alpha + \beta \varepsilon_t^2,$$

where σ_t^2 is the variance for the forecast error made for the time t forecast of time $t + 1$ returns, and ε_t is the forecast error made at t for the time $t - 1$ forecast. This modelling of the variance is labelled autoregressive conditional heteroskedasticity (ARCH) by Engle. ARCH models have been used in the foreign exchange literature by Domowitz and Hakkio (1985), Hsieh (1988), Kaminsky and Peruga (1987), McCurdy and Morgan (1987, 1988), Diebold and Nerlove (1986), Baillie and Bollerslev (1987a,b), Lastrapes (1987), Hodrick (1987), Bodurtha and Mark (1987), Mark (1987), Attanasio and Edey (1987), Giovannini and Jorion (1987c), and Engle and Bollerslev (1986).

In this paper we estimate and test a six-country international CAPM, allowing for time-varying variances following both ARCH specifications and models relating the variance to macroeconomic data. We use aggregate asset data representing the nominal obligations of six governments—France, Germany, Italy, Japan, the UK and the US—and the rates of return from Eurocurrency markets from April 1973 to December 1984. These CAPM tests can be viewed as a direct extension of Frankel's tests by allowing for heteroskedasticity. This work is also quite similar to that of Bollerslev, Engle and and Wooldridge (1988), who estimate—but do not test the restrictions imposed by—a CAPM for domestic US assets, using nominal rates of return.

We also allow for a generalization of the Frankel-type CAPM by introducing the possibility that the empirical CAPM equation (1) does not hold exactly. We embed the model in the traditional measurement error framework and test the CAPM restriction under the assumption that the variance of the forecast error depends on observable data. We are able to identify the elements of the variance matrix of the measurement error because they are assumed to be time-invariant, as opposed to the variance of the forecast errors.

In section 2 we introduce the time-varying variance CAPM and test CAPM under the assumption that the variance is a function of macroeconomic variables. In the next section some multivariate ARCH models are presented and estimated. The CAPM restrictions are tested. Section 4 presents the measurement error model, and section 5 concludes.

2. CAPM WITH TIME-VARYING VARIANCES

A general model of the real rates of return on $N+1$ assets might be that the rates of return are related to the value of these assets as a share of total wealth. Thus one might expect that, for any given asset, its expected rate of return in equilibrium would be higher if the share of that asset in the market portfolio increased. The expected return on this asset may also be influenced by an increase in the values of other assets.

Choosing one asset as numeraire, the expected rates of return on the N-vector for the other assets relative to the numeraire are a function of the values of the supplies of these assets as shares of the total value of the $N+1$ assets. Thus, we could write:

$$E_t z_{t+1} = c + B_t \lambda_t, \tag{2}$$

where B is an $N \times N$ matrix of coefficients, and the other variables are defined as in section 1. In this general form of the equation, B could vary over time.

Equation (1) is clearly a restriction on equation (2)—it forces the matrix of coefficients in the N equations to be proportional to the covariance matrix of forecast errors of z_{t+1}. In the earlier papers that have tested international CAPM, B and Ω_t were treated as constants; here, we allow both to vary over time. As in the previous literature, we can test the restriction imposed by CAPM that B is proportional to Ω_t.

In this paper there are six aggregate assets. Each is essentially the outstanding debt at the end of period t of each of the six governments. The debt is calculated in such a way as to include only the value of debt in the hands of the public. For example, corrections are made for foreign exchange intervention by central banks that may remove some of the obligations of one government from public hands and replace it with another. The calculation of the data is described in detail in Frankel (1982b). The data set used here is an updated data set kindly provided to us by Alberto Giovannini. The asset share data are measured at end of the month, and run from April 1973 to December 1984. See the Appendix for a description of the data.

To produce an empirical model it is assumed that expectations are formed rationally. Equations (1) and (2) can be transformed into regression equations by replacing the expected value of z_{t+1} with the *ex-post* realized value of this variable, and appending an error term equal to the forecast error at the end of each equation. The *ex-post* real rates of return are used to calculate z_{t+1}, with the dollar as the numeraire asset. The rate of return on the asset of each country is calculated in dollar terms as $(1 + i_{t+1})S_{t+1}/S_t$, where i is constructed for each country from spot and 1-month forward exchange rates, and the Eurodollar 1-month rate assuming covered interest parity, and S is the end-of-period exchange rate in dollars per unit of each country's currency (e.g. the dollars per mark exchange rate). Each nominal rate is then deflated by the common deflator P_{t+1}/P_t, which is a dollar inflation index. P_t is a geometrically weighted average of price indices of the six countries (converted into dollar terms by multiplying by S_t). The *ex-post* rates of return are measured from July 1973 to January 1985. These data are described in more detail in the Appendix.

We assume that errors are distributed normally. The log likelihood for observation t is given by

$$\ln L = -2\cdot5 \ln 2\pi - 0\cdot5 \ln|\Omega_t| - 0\cdot5(z_{t+1} - c - B_t\lambda_t)'\Omega_t^{-1}(z_{t+1} - c - B_t\lambda_t). \tag{3}$$

When the CAPM constraint is imposed, $B_t = \rho\Omega_t$. We estimate the likelihood under alternative assumptions about the behaviour of Ω_t over time.

Table I reports the maximum-likelihood estimates of the model under the CAPM constraint and under the additional constraint that the variance matrix be constant over time. These estimates correspond to those reported by Frankel (1982b) and Frankel and Engel (1984).

Although the point estimate of ρ in this model is negative, its standard error is very large, so that essentially no economically reasonable value of ρ can be ruled out. In particular, $\rho = 0$ is not rejected, which would say that there is no evidence of a risk premium.

Table I also reports the Lagrange multiplier (LM) statistic for a test of CAPM. The

Table I. CAPM estimation, constant Ω

$$z_{t+1} = c + \rho(P'P)\lambda_t + \varepsilon_{t+1}$$
$$\mathrm{Var}(\varepsilon_{t+1}) = P'P$$
Log-likelihood: 1639·789088

FRA	GER	ITA	JAP	UK
The estimate of the vector c':				
0·0039629	0·0028871	0·0044543	0·0052062	0·0020641
(0·0168931)	(0·0172629)	(0·0140137)	(0·0173972)	(0·0133013)
The estimate of the coefficient ρ:				
−19·287650				
(42·676856)				
The estimate of the upper triangular matrix P:				
0·0322951	0·0288285	0·0223217	0·0179637	0·0161121
(0·0019034)	(0·0018396)	(0·0024820)	(0·0024482)	(0·0025755)
0	0·0165643	0·0018479	0·0034280	0·0046026
	(0·0010655)	(0·0017589)	(0·0030412)	(0·0023601)
0	0	0·0181945	0·0009552	0·0030412
		(0·0006800)	(0·0040832)	(0·0025898)
0	0	0	0·0270647	0·0051627
			(0·0015644)	(0·0018011)
0	0	0	0	0·0237012
				(0·0014565)

Standard errors in parentheses.
Test of CAPM restrictions
Lagrange multiplier test statistic = 33·221925
Marginal significance level = 0·002676 (14 d.f.)

alternative hypothesis is actually a somewhat restricted version of the general unrestricted version of the model in equation (2). In the test reported here, under the alternative hypothesis, B is still restricted to be symmetric, although it is no longer proportional to the covariance matrix. This means that a one-unit increase in the supply of asset j has the same effect on the relative return of asset i that a one-unit increase in the supply of asset i has on the return of asset j. CAPM is rejected at the 1 per cent level even against this restricted form of the alternative hypothesis. The test statistic reported is χ^2 with 14 degrees of freedom. (We are testing 15 proportionality restrictions while estimating the constant of proportionality, implying 14 unique restrictions.)

In this constant variance version there is not much encouraging news for the CAPM hypothesis. However, CAPM does not require that Ω_t be constant over time. Ω_t represents the variance conditional on information available at time t. It is a measure of dispersion of forecast errors for market participants. Frankel's formulation of CAPM is a significant advance on those empirical specifications that require the 'beta' of each asset to be constant over time or else vary in a deterministic way. The 'beta' for each asset will, in fact, vary as the supplies of the assets change over time. But there is another possible source of variation in the beta that Frankel does not allow for, and that is the change over time in Ω_t.

The CAPM hypothesis does not provide any particular clue to the source of change in the conditional variance of forecast errors over time. A more complete general equilibrium model of the economy would investigate the source of shocks to the economy, and indicate what might cause fluctuations in the forecast variance.[3] In the absence of such a model, we will test some plausible macroeconomic sources of variation.

Each element of the covariance matrix Ω_t could vary independently with the macroeconomic data (subject to the symmetry constraint on Ω_t). Thus, in our five-equation model, each of the 15 elements of Ω_t might be a linear function of some variable or variables x. As a practical matter, this five-equation system with the constraint imposed between coefficients and the variance matrix is very difficult to estimate. We find it desirable to parameterize the process parsimoniously, at least in our first attempt at estimating the time-varying variance model. Thus, we let the variance depend on only one variable at a time, and initially we model the variance according to

$$\Omega_t = P'P + hh'x_t. \tag{4}$$

In this equation, P is an upper triangular matrix of parameters to be estimated and h is column vector of parameters. The macroeconomic variable is given by x_t. In all cases the values of the data are positive numbers, so the form of equation (4) guarantees that the estimated Ω_t matrix is positive semi-definite. In practice, imposing this positive-semi-definiteness constraint is useful in achieving convergence of the maximum-likelihood estimates. Note that we are restricting the variance to depend only on one x_t (at a time) and that only 20 parameters are used to describe the relation between the variance of the forecast made at time t of $t+1$ variables, and that this variance is a function only of variables known at time t.

We have chosen macroeconomic variables that seem most likely to have influenced the variance of forecast errors of the exchange rate. We allow the variance to depend on stationary representations of the US money supply and of the dollar price of oil. Both variables had very large economic effects during our sample period both in the US and abroad. Furthermore, it

[3] We do not know the true model of the variance but, following Pagan (1984), Pagan and Sabau (1987) and Pagan and Ullah (1988), the estimates of ρ are almost certain to be inconsistent if the conditional covariance matrix is misspecified.

is likely that the size and unpredictability of these variables over this span of time added to the variability of forecasts of many macroeconomic variables, including exchange rates.

The assumption of rational expectations might lead one to conclude that only unexpected changes in oil prices or the money supply would increase the difficulty of making forecasts. In this view, consumers are able to form expectations of the money supply and oil prices given the past behaviour of these variables. A large, but expected, change in one of them will not add any uncertainty, or make forecasts of exchange rates more difficult. To allow for this possibility we fit ARIMA models to the logs of US M1 and dollar oil prices. We then squared the residuals, and allowed for the variance of the exchange rate forecast to increase if there were a large innovation in one of these variables in the previous month. The Appendix contains a description of the ARIMA models.

Table II reports the estimates of the model when the variance is allowed to depend on the square of the 1-month change in the log of the US money supply. Three of the coefficients in the h vector are individually significantly different from zero, and the χ^2 test indicates that they

Table II. CAPM estimation, variance as a function of square of change in log of US M1

$$z_{t+1} = c + \rho\Omega_t\lambda + \varepsilon_{t+1}$$
$$\mathrm{Var}_t(\varepsilon_{t+1}) = \Omega_t = P'P + hh'x_t$$
x_t = square of change in log US M1

Log-likelihood: 1647·625183

FRA	GER	ITA	JAP	UK
The estimate of the vector c':				
0·005956	0·004988	0·006123	0·006953	0·003338
(0·013681)	(0·013695)	(0·011231)	(0·014585)	(0·011425)
The estimate of the coefficient ρ:				
−23·934543				
(36·290540)				
The estimate of the upper triangular matrix P:				
0·030219	0·025191	0·020613	0·018436	0·019729
(0·002054)	(0·002419)	(0·002908)	(0·003298)	(0·003370)
0	0·015772	0·001692	0·005086	0·009071
	(0·001358)	(0·002119)	(0·003297)	(0·002637)
0	0	0·018193	0·001027	0·003226
		(0·000716)	(0·004228)	(0·002686)
0	0	0	0·026389	0·001813
			(0·001625)	(0·002341)
0	0	0	0	0·018658
				(0·002559)
The estimate of the vector h':				
1·590317	2·077879	1·201510	0·287016	−0·917194
(0·713113)	(0·645719)	(0·622900)	(0·607644)	(0·559770)

Standard errors in parentheses.

Test of CAPM restrictions

Lagrange multiplier test statistic = 56·225719

Marginal significance level = 0·000015 (14 d.f.)

are jointly significant at the 5 per cent level. Thus we can reject the constant Ω_t version of CAPM, in favour of a model in which the forecast variances are greater when there are large percentage changes in the money supply.

Given that this model is a significant improvement on the model whose estimates are reported in Table I, we now want to test whether the restrictions imposed by CAPM are binding. In Table II the coefficients were constrained to be proportional to the variance matrix Ω_t. The unconstrained model in this case would let the coefficient matrix B (from equation (2)) vary over time as a function of the square of the change in the log of the money supply, and not be constrained to be related in any way to the Ω_t matrix. At the bottom of Table II is reported the LM statistic for the test of the CAPM restriction against the alternative that B is not constrained to be proportional to the variance matrix of the residuals. The χ^2 statistic has 19 degrees of freedom and, as indicated in the table, shows that the CAPM restrictions are strongly rejected. Actually, visual inspection of Table II is enough to cast serious doubt on the CAPM hypothesis, since the point estimate of ρ is negative (but not significantly different from zero).

A note of caution is in order here. Suppose the LM test had failed to reject the CAPM restrictions in Table II. How should we interpret such a failure to reject? We estimate ρ quite imprecisely, and in fact cannot reject that ρ is zero. But this means that we cannot reject the hypothesis that our explanatory variables have no ability to explain our dependent variables, since all of the explanatory variables are multiplied by ρ. Thus, were we not to reject CAPM, probably the correct interpretation would be that we are unable to explain relative rates of return using asset shares—but we seem to do about as badly whether or not the restrictions of CAPM are imposed.

Table III reports the results of the estimation in which the variance of the forecast errors is a linear function of the square of the change in the dollar price of oil. Here we do not quite reject the hypothesis that the macro data do not help explain changes in the variances and that Ω_t is constant at the 5 per cent level. The LM test indicates that the CAPM restrictions are strongly rejected.

The hypothesis that the variance in the forecast errors is related to the size of unanticipated changes in money and oil prices are also tested in Table III. We in fact find that the squared ARIMA residuals for oil prices do a good job of explaining changes in the variance under the CAPM restrictions (in the sense that the likelihood is significantly improved over the constant Ω_t model), but the ARIMA residuals for money are not as successful. However, in both cases the LM test rejects the CAPM restrictions in favour of the more general model of equation (2).

Equation (4) restricts the matrix of coefficients that multiplies the macroeconomic variable to have only five independent parameters. We also estimated the general form of equation (4) in which the matrix multiplying the macro variable has 15 parameters. Specifically, we take the variance to be given by

$$\Omega_t = P'P + Q'Qx_t, \tag{5}$$

where Q is an upper triangular matrix. This formulation imposes the constraint that Ω_t be positive semi-definite. The five-parameter version of this model given by equation (4) is a restriction on equation (5), which forces all but the top row of Q to equal zero—providing ten zero restrictions.

In no case is the 15-parameter model a significant improvement on the five-parameter model of equation (4). Table III reports the log likelihoods, the χ^2 statistics and the p-values for the null hypothesis that all but the top row of Q is zero. In no case is the null hypothesis rejected at the 5 per cent level.

Table III. CAPM estimation, variance as a function of macroeconomic data

Square of change in log of price of oil (parsimonious model, eq. (4))
Log of likelihood = 1644·831445
LM test of CAPM restrictions:
  Chi-square statistic (10 d.f.) = 41·783589
    Marginal significance level = 0·001895

Square of residual from US. M1 ARIMA (parsimonious model, eq. (4))
Log of likelihood = 1642·882930
LM test of CAPM restrictions:
  Chi-square statistic (10 d.f.) = 43·015021
    Marginal significance level = 0·001290

Square of residual from oil price ARIMA (parsimonious model, eq. (4))
Log of likelihood = 1645·886909
LM test of CAPM restrictions:
  Chi-square statistic (10 d.f.) = 53·858668
    Marginal significance level = 0·000035

Square of change in log of US M1 (general model, eq. (5))
Log of likelihood = 1649·718806
Test of general model against parsimonious model:
  Chi-square statistic (10 d.f.) = 4·18724
    Marginal significance level = 0·93850486

Square of change in log of price of oil (general model, eq. (5))
Log of likelihood = 1645·836216
Test of general model against parsimonious model:
  Chi-square statistic (10 d.f.) = 2·009542
    Marginal significance level = 0·99626650

Square of residual from US M1 ARIMA (general model, eq. (5))
Log of likelihood = 1643·892504
Test of general model against parsimonious model:
  Chi-square statistic (10 d.f.) = 2·019542
    Marginal significance level = 0·99618817

Square of residual from oil price ARIMA (general model, eq. (5))
Log of likelihood = 1646·620739
Test of general model against parsimonious model:
  Chi-square statistic (10 d.f.) = 1·467660
    Marginal significance level = 0·99903254

In this section we have found some evidence that the variance of forecast error does change over time. In particular, we find that the square of the unanticipated monthly growth rate of dollar oil prices and of the monthly growth rate of US M1 are significant explainers of the variance of the residuals. Thus, the constant Ω_t version of CAPM can be rejected. However, these models offer little consolation for the more general CAPM, since the CAPM restrictions are in every case strongly rejected. The next section considers a more time-series-oriented model of the variance process—the ARCH model.

3. CAPM WITH ARCH

Often the economist does not know the true model of the variance. Without a full general equilibrium model of the economy, we do not know exactly which macroeconomic variables the variance should be related to. In such a case Engle's (1982) ARCH model is well-suited.

Engle's model does not require knowledge of the structure of the economy. Instead, it makes the reasonable postulation that, for example, if the absolute size of errors in $t-1$ are large, the conditional variance at time t would be larger than average. This is the essence of the ARCH hypothesis.

In this section we apply the general idea of ARCH to our five-equation CAPM system. We test two versions of ARCH. They take the general form:

$$\Omega_t = P'P + G\varepsilon_t\varepsilon_t'G, \tag{6}$$

where P is a constant upper triangular matrix, G is a constant symmetric matrix and ε_t represents the lagged forecast error (the error made in predicting the returns between $t-1$ and t). This formulation ensures that the estimated variance matrices are positive semi-definite. (This property is not necessarily satisfied in the ARCH formulation used by Bollerslev, Engle and Wooldridge.)[4]

Equation (6) represents a particular form of a first-order ARCH. In our applications we will take G first to be diagonal (so that it has five independent non-zero elements) and then we will consider the general symmetric case for G (with 15 independent elements). Even our 'general' case is a quite parsimonious form of a first-order ARCH. There are 15 elements in the time t conditional covariance matrix. In its most general formulation each of those 15 elements could be related linearly to each of the 15 elements in the moment matrix for the lagged residual. Thus, for general first-order ARCH we could postulate a model with 225 parameters relating the current conditional variance to the lagged errors. Moreover, in generalized ARCH models (see Bollerslev, 1986), the variance matrix is essentially related to a distributed lag of error moment matrices. In a multiple-equation model such as the one estimated here, the number of ARCH parameters increases quite quickly with the number of equations.

Two considerations motivate our model of equation (6). First, estimation of a five-equation ARCH system that imposes constraints between coefficients and elements of the variance matrix is very difficult. It is made much more managable by choosing specifications with a small number of parameters. The less parsimoniously parameterized versions are not only more difficult to estimate, but given the limited data set from the floating rate period, they also leave too few degrees of freedom for meaningful estimation. The second advantage of our formulation is that it constrains the estimates of the variance to be positive definite. This turns out to be of great practical importance in estimating a large multi-equation ARCH system.

In Table IV we present the result of an ARCH estimation in which the matrix G is diagonal. Thus, there are five ARCH parameters to estimate. The constant variance model could be thought of as a constrained version of this time-varying variance formulation, in which the five ARCH parameters are forced to be zero. It is easy to perform a likelihood ratio test of this constraint. Doubling the difference between the log likelihood for the model reported in Table IV and the one reported in Table I gives a χ^2 statistic with degrees of freedom equal to 37·78. This is easily significant at the 1 per cent level—so we can reject the constraint that the variance is constant over time.

The ARCH parameters range in size from about 0·23 to about 0·64, so the variance process appears stationary over time. (All the parameters are significantly different from one.) Four of the five parameters are significantly different from zero at the 1 per cent level in a two-sided t test.

[4] Baba *et al.* (1987) and Engle (1987) present formulations that impose positive-semi-definiteness.

Table IV. CAPM estimation, five parameter ARCH

$$z_{t+1} = c + \rho\Omega_t\lambda_t + \varepsilon_{t+1}$$
$$\mathrm{Var}_t(\varepsilon_{t+1}) = \Omega_t = P'P + G\varepsilon_t\varepsilon_t' G$$
Log-likelihood: 1658·682319

FRA	GER	ITA	JAP	UK
The estimate of the vector c':				
−0·006027	−0·007428	−0·004122	−0·004009	−0·006882
(0·006596)	(0·006869)	(0·005514)	(0·006892)	(0·005812)
The estimate of the coefficient ρ:				
13·270001				
(15·036330)				
The estimate of the upper triangular matrix P:				
0·030631	0·027416	0·021628	0·015429	0·015644
(0·002278)	(0·002153)	(0·002339)	(0·003147)	(0·002867)
0	0·015277	0·002311	0·002357	0·004313
	(0·001201)	(0·001541)	(0·003246)	(0·002660)
0	0	0·015184	−0·000316	0·002262
		(0·000578)	(0·003692)	(0·003157)
0	0	0	0·022775	0·005693
			(0·001720)	(0·002368)
0	0	0	0	0·023374
				(0·001620)
The estimates of the diagonal elements of G:				
0·425252	0·384488	0·478617	0·642844	0·227064
(0·103266)	(0·106483)	(0·082283)	(0·103571)	(0·116554)

Standard errors in parentheses.

Test of CAPM restrictions

Lagrange multiplier test statistic = 45·846083

Marginal significance level = 0·000521 (19 d.f.)

Allowing the variance to change over time greatly reduces the standard error on the estimated coefficient of relative risk aversion. It drops from 42·7, as reported in Table I, to 15·0 in the ARCH model presented in Table IV. The point estimate of ρ is extremely high—around 13·6. In a one-sided t test it is not significantly different from zero at the 5 per cent level.

This model with the CAPM constraint is tested against the alternative that the B matrix has the same form as Ω_t but is not constrained to be proportional to the covariance matrix. As the LM statistic in Table IV indicates, CAPM is still strongly rejected at the 1 per cent level.

Figure 1 is a graph of the estimated conditional variances from this model for the error in forecasting the return on German mark securities relative to the dollar. This graph is representative for all currencies. Also drawn on the graph is an estimate of the unconditional variance. In most periods the conditional variances are much smaller than the unconditional one. In the constant variance version of CAPM, the unconditional variance is an upper bound on the size of the conditional variance. Frankel (1986) argued, using the estimates of the unconditional variance as upper bounds on the conditional variances, that a change in the supply of outside assets is unlikely to generate much change in the risk premium, since the change in the risk premium is the product of the change in the asset supply and the conditional

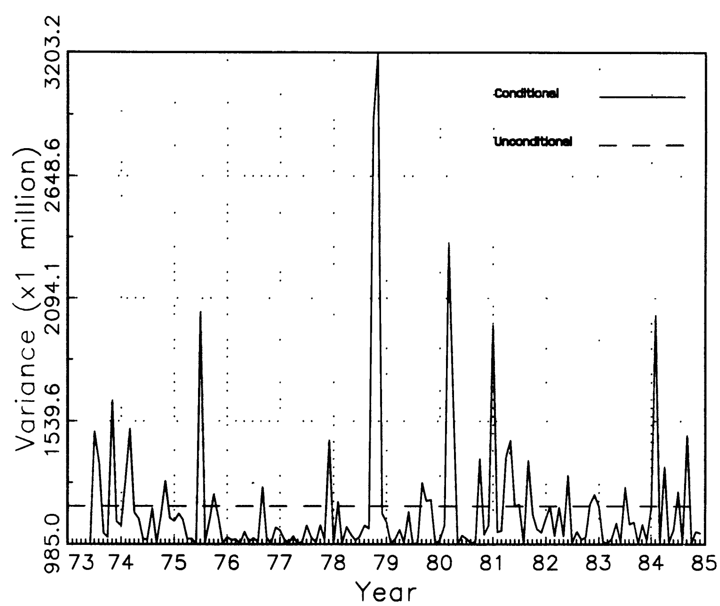

Figure 1. Variances—Germany

variance of exchange rate forecasts (times ρ). Pagan (1986) has pointed out that, in fact, the conditional variance need not be smaller than the unconditional variance if the variances change over time. Figure 1 confirms that for some periods the conditional variances are much larger than the unconditional variance. On average they should be smaller, however, as Frankel (1987) indicates.

If the variance does vary over time, the variability of the variances themselves might explain the observed size of risk premia within the framework of CAPM. This point is brought out in the exchange between Giovannini and Jorion (1987a,b) and Frankel (1987). The results from our ARCH estimation and CAPM tests, though, make this speculation moot. Our estimation does allow the variance to change, but CAPM is still strongly rejected.

Because the five-parameter ARCH model of CAPM is such a large improvement on the CAPM constrained to have a constant variance, we proceed to test a less sparsely parameterized ARCH model. Table V reports the results of the estimation in which the G matrix is allowed to be a general symmetric matrix. There are 15 independent elements in G.

The five-parameter ARCH is a restricted version of the ARCH in this model in which all the off-diagonal elements are forced to be zero. That imposes 10 restrictions on this richer specification. The likelihood ratio test for these restrictions yields a χ^2 statistic with 10 degrees of freedom of 43·34, which is easily significant at the 1 per cent level. Thus, we seem to gain a lot by going to this more general specification.

There is one troubling aspect to this particular form of the model. The characteristic roots of the estimated G matrix are $-0{\cdot}646$, $0{\cdot}198$, $0{\cdot}305$, $0{\cdot}421$ and $1{\cdot}144$. This last root indicates the possibility of non-stationarity in the variance process.[5]

This ARCH specification once again reduces the standard error of the estimate of ρ—from 15·0 in the five-parameter ARCH to 9·4 here. The estimate of ρ is negative and significantly

[5] Hong (1987) and Engle (1987) discuss integrated ARCH.

different from zero. This would be very troubling were it not for the fact that the restrictions of CAPM relative to a formulation of asset demands with a symmetric B are very strongly rejected. The rejection is much stronger than in any of our earlier specifications of CAPM. So, the model reported in Table V is not capturing the true behaviour of investors in international financial markets.

It is not clear why CAPM fares so badly in this case. Perhaps the non-stationarity in the variance is distorting the test statistics, but more likely the CAPM restrictions are just very strong relative to a model that allows returns to depend in a relatively unrestricted way on asset shares and lagged covariances.

It is possible that we have not considered a general enough ARCH specification. Perhaps the variance is related to the moment matrix of forecast errors at lags greater than 1. We tested for this possibility using the squared deviations from the means for each of the relative rates of return. Lagrange multiplier tests for ARCH with one lag, computed following the suggestion of Engle (1982), provided strong evidence in favour of that hypothesis (relative to homoskedasticity) for all but UK. Comparison of ARCH with 12 lags to ARCH with one lag gave no support to the more complex model.

In sections 2 and 3 we have estimated a variety of forms of CAPM with time-varying variances. There seems to be abundant evidence that the explanatory power of the model is increased by letting the conditional covariances of the asset returns change over time. Yet these more sophisticated models still provide no support for the CAPM restrictions between coefficients and variances when tested against the more general unconstrained asset pricing models. We perform six LM tests of the capital asset pricing model in sections 2 and 3, and they all reject the restrictions. In the next section we take up the possibility of some specific cases of misspecification in equation (1).

4. MEASUREMENT ERROR

The model presented in equation (1) and estimated in sections 2 and 3 is a model in which it is assumed that the CAPM equation holds exactly and all variables are measured without error, so that the residual in the estimating equation can be identified with the forecast error. There is a large class of empirical macroeconomic models that make the same assumptions in order to exploit 'orthogonality conditions'. These assumptions are particularly convenient because they rule out any simultaneous-equations problem. The right-hand side variables in equation (1) are uncorrelated with the error term because under rational expectations all currently known variables are orthogonal to forecast errors.

Suppose that equation (1) did not exactly describe investors' behaviour, but instead it was only correct on average. Preference shocks, for example, would be represented by a disturbance appended to the true model of equation (1). Then when we replace the expectation of z_{t+1} with its realization, the disturbance term in the estimating equation will be the sum of the forecast error and the preference shocks. In this case the explanatory variables—the asset shares—would be correlated with the error, since we would expect that as risk preferences change the values of the outstanding stocks of the assets would also change. Thus, the estimation techniques undertaken here would be rendered invalid. It should be understood, then, as Frankel and Engel discuss, that the CAPM tests discussed previously in this paper are tests that the CAPM equation (1) holds exactly.

There is another problem that the addition of demand errors cause that is specific to this model and does not necessarily appear in some other test of 'orthogonality conditions'. It is also present if there is measurement error in either the dependent or explanatory variables.

When some source of error other than the forecast error is present in the residual, it is no longer the case that under CAPM the coefficients on the asset shares should be proportional to the variance of the residual. They should only be proportional to the variance of the forecast error.

Although we offer no general solution to the problems of misspecification, we can generalize the tests in one direction when forecast errors have a time-varying variance. We can identify and estimate equation (1) even if there is measurement error in the rates of return. This identification is possible when the variance of the forecast errors is a function of observable data, and the variance of the measurement error is constant.

Our economic model is still given by equation (1), but the model to estimate no longer simply replaces $E_t z_{t+1}$ with $z_{t+1} - \varepsilon_{t+1}$. We now have:

$$z_{t+1} = c + \rho\Omega_t\lambda_t + \varepsilon_{t+1} + u_{t+1}, \tag{7}$$

where u_{t+1} is the measurement error of z_{t+1}. The variance of ε_{t+1} is given by Ω_{t+1} while the variance of u_{t+1} is equal to Σ. We assume that u_{t+1} is uncorrelated with ε_{t+1} and λ_t, and that Σ is constant.

The likelihood for the model in this case is given by:

$$\ln L = -2\cdot5 \ln 2\pi - 0\cdot5 \ln|\Phi_t| - 0\cdot5(z_{t+1} - c - B_t\lambda_t)'\Phi_t^{-1}(z_{t+1} - c - B_t\lambda_t), \tag{8}$$

where $\Phi_t = \mathrm{Var}_t(\varepsilon_{+1} + u_{t+1}) = \Omega_t + \Sigma$. When the CAPM constraint is imposed, the matrix of coefficients is proportional to the variance of the forecast errors: $B_t = \rho\Omega_t$.

It is useful to consider for a moment why Σ is identified in this model. Intuitively, identification comes because the part of the variance that also appears in the asset pricing equations (Ω_t) is time-varying, while the measurement error variance (Σ) is not. Suppose the forecast error variance were constant as in the Frankel empirical models. Then it would be impossible to distinguish between the forecast error variance and the measurement error variance. Suppose we had some estimate of this hypothetically constant Ω as well as estimates of ρ, Σ, and Φ that maximized the likelihood. We could obtain the same value of the likelihood with, for example, a $\Omega^* = 2\Omega$, a value of $\rho^* = 0\cdot5\rho$, a value of $\Sigma^* = \Phi - \Omega^*$, and the same estimate of Φ. However, when Ω_t is time-varying we can no longer perturb its value without changing the value of the likelihood, since the forecast error variance is the only part of the total error variance that changes with time.

We only consider models of the forecast error variance in which the variance depends on macroeconomic data, as in equation (4). While we are able to separate out the variance of the forecast errors from the variance of the measurement error, we cannot distinguish between the forecast error and the measurement error themselves. The ARCH model, however, requires that the forecast errors be identifiable, if we hypothesize that the variance of the forecast error depends on the lagged moment matrix of realized forecast errors. So, we must abandon ARCH as our model of time-varying variance of forecast errors, and instead rely on equation (4).

The models of section 2 are restricted forms of equation (7). They force the matrix Σ to be zero. As in the previous sections, we choose a form for the variance that ensures it is positive semi-definite. Equation (4) defines our model for Ω_t. For the variance of the measurement error we impose

$$\Sigma = Q'Q \tag{9}$$

where Q is an upper triangular matrix. The models of section 2 force Q to equal zero—a total of 15 restrictions.

We calculate maximum-likelihood estimates for the model using the two macroeconomic variables that seemed to do the best job in explaining the variances in section 2 of this paper—

Table V. CAPM estimation, 15-parameter ARCH

$$z_{t+1} = c + \rho\Omega_t\lambda_t + \varepsilon_{t+1}$$
$$\mathrm{Var}_t(\varepsilon_{t+1}) = \Omega_t = P'P + G\varepsilon_t\varepsilon_t'G$$
Log-likelihood: 1680·349931

FRA	GER	ITA	JAP	UK
The estimate of the vector c':				
0·005302	0·004158	0·002732	0·005903	0·001906
(0·005211)	(0·004958)	(0·004468)	(0·004900)	(0·003912)
The estimate of the coefficient ρ:				
−18·815029				
(9·366542)				
The estimate of the upper triangular matrix P:				
0·030750	0·028908	0·023892	0·018148	0·016024
(0·002431)	(0·002272)	(0·002072)	(0·003315)	(0·003051)
0	0·015238	−0·001725	0·003238	0·004367
	(0·001211)	(0·001288)	(0·003848)	(0·002422)
0	0	0·007587	0·004764	0·007590
		(0·001389)	(0·005438)	(0·004048)
0	0	0	0·025977	0·003559
			(0·001905)	(0·002234)
0	0	0	0	0·020771
				(0·001922)
The estimate of the symmetric matrix G:				
0·551903	0·018319	−0·334667	0·092724	−0·106092
(0·133893)	(0·081518)	(0·083272)	(0·073811)	(0·086655)
0·018319	0·276057	−0·180710	0·053321	0·033104
	(0·103955)	(0·084837)	(0·075722)	(0·080986)
−0·334667	−0·180710	0·689377	−0·322378	0·349194
		(0·122514)	(0·083084)	(0·095240)
0·092724	0·053321	−0·322378	0·251767	0·321065
			(0·154097)	(0·130203)
−0·106092	0·033104	0·349194	0·321065	−0·347991
				(0·155485)

Standard errors in parentheses.
Test of CAPM restrictions
Lagrange multiplier test statistic = 88·6136637
Marginal significance level = 0·000000 (29 d.f.)

the square of the change in the log of money and the squared residual from the oil price ARIMA.

These measurement error models do not significantly improve the explanatory power of the models estimated in section 2, where the asset pricing equation was assumed to hold identically. The model in which money helps explain forecast error variances yields a log-likelihood of 1652·55. The restricted form of this model, reported in Table II, has a log-likelihood of 1647·63. While the log-likelihood of the restricted model is of necessity lower than that of the unrestricted, the χ^2 test of the 15 restrictions yields a statistic of 9·84, which is far less than the 5 per cent critical level of 25·0.

The same conclusions can be drawn from the model in which the square of the innovation of the oil price is a determinant of the conditional variance of the forecast errors. The log of the likelihood for this model is 1651·08, compared to the log-likelihood for its corresponding restricted model reported in Table V of 1645·89. The χ^2 statistic with 15 degrees of freedom is 10·38, which again is insignificant.

While the evidence of sections 2 and 3 has provided little support for CAPM, it is clear that the measurement error model is not the solution to the misspecification.

5. CONCLUSION

CAPM is in many respects a very attractive model for pricing of international assets. It can be derived from a (restrictive) utility-maximizing framework, and the asset demand equations that it produces are a special case of those studied in the portfolio balance literature. Unfortunately, the restrictions that CAPM places on those more general asset demand equations—restrictions that imply that the returns on assets are functions not only of the supplies of assets, but also of the variances and covariances of the asset returns—have universally been rejected in previous literature. Although this paper allows for substantial generalization of CAPM, its restrictions are still not accepted.

One might think of several reasons why previous papers have rejected CAPM—but some of the most important problems with past models are taken care of here. One of the most obvious problems, as pointed out by Giovannini and Jorion (1987a) among others, is that in practice those who have implemented the Frankel test have assumed the forecast error variances and covariances to be constant over time. Given the abundant evidence of heteroskedasticity in these errors, this seemed like a natural point of misspecification in the empirical CAPM. However, we test several versions of CAPM with conditionally heteroskedastic errors, but still find little support for the model.

It is sometimes argued that a deficiency in the Frankel approach is that the only way it can provide a measure of the variance of expectational errors is by extrapolating from past expectational errors. It is argued that if there is a policy change or some other economic shock that increases uncertainty, then there is no way to pick this up reliably from the data. This argument carries much less weight when we allow the variance of the forecast error to be a function of economic variables whose changes are apt to be important generators of uncertainty—such as money growth or oil prices. Our models of sections 2 and 3 allow for just such a possibility, yet again CAPM gets no support.

Another potential problem with the Frankel set-up is that it requires the CAPM equation to hold exactly. We have relaxed this constraint in section 4 by allowing for an error that is uncorrelated with the forecast error and the right-hand-side variables. However, this model is no improvement empirically.

This paper extends the frontiers of estimation of international CAPM to consider some of the most important possibilities that have been suggested for the empirical failure of that model, yet the model still does not get a shred of support.

DATA APPENDIX

Our analysis relies on rate of return and aggregate asset data for the six countries in the study: France, Germany, Italy, Japan, UK and US. The assets studied are publicly held outstanding government debt denominated in the six currencies.

The calculation method, designed to measure debt at the end of each month, is described in detail in Frankel (1982b). We used an updated data set provided by Alberto Giovannini. Our other major source of data was the Data Resources Inc. DRIFACS data base. (We are indebted to Ken Froot and Susan Collins for providing us access to this data.) We also obtained data from the Citibank Citibase tape and the International Monetary Fund's IFS tape.

Asset shares

A complete description of how the values of the assets are calculated is included in Frankel (1982b). Briefly, for each country the asset data starts with the value of outstanding debt reported by each government. To this figure is added the cumulative value of foreign exchange purchases by the central bank of that country (which has the effect of exchanging foreign denominated assets for domestic denominated assets in the hands of the public). Subtracted from this total is the value of assets held in that currency by central banks (a figure which is obtainable through numbers available in the IMF Annual Report). Because no correction is made for debt held by the central bank of its own government, these figures for the values of the outstanding assets are the values of debt and monetary base held by the public for each country.

The analysis in the paper utilizes asset shares rather than the level of assets. The asset shares were computed from the asset levels, measured in dollars. The asset share data covered the period from June 1973 to December 1984.

Rates of return

The nominal rate of return for dollar assets was taken as the average of the bid and ask Eurodollar rates on 1-month securities, measured on the last day of the month that was not a holiday.

The nominal rate of return for the other five currencies was calculated assuming covered interest parity. The interest rate calculated on the last day of the month gives the return (known with certainty) for assets held for the forthcoming month. The exchange rates are averages of bid and ask rates, as were the forward rates.

The nominal rates of return are converted to *ex-post* real rates by a common price index.

The common price index, P_t, is computed as a geometric mean of the consumer price indices, IFS item 64, for the six countries at time t after they are converted to dollars by multiplying by the end of period exchange rate.

Economic variables

In section 2 we allowed the variance of the forecast errors to be a function of an oil price index and the US money supply. These data were obtained from the Citibase data tape for the period January 1973 to December 1984. The money supply measure used was Citibase variable FM1, US M1 measured in current dollars as a seasonally adjusted monthly average of daily figures. The oil price index used was the Citibase variable PW561, the producer price index for crude petroleum products (1967 = 100), not seasonally adjusted.

To measure the surprise component of the money supply and oil prices, we estimated ARIMA models for the natural logarithm of each series over the period January 1973 to December 1984. The autocorrelations of each series indicated that both the money supply and the oil price index were nonstationary in levels and stationary in first differences. Identification indicated that the money supply followed a moving-average process with one and four lags. The estimated model for the differences in natural logs, DM_t, is shown below along with the

Box–Pierce statistic calculated from the residuals (standard errors of the coefficients are shown in parentheses):

$$\underset{}{\mathrm{DM}_t} = \underset{(0\cdot0003996)}{0\cdot0055398} + \underset{(0\cdot0843936)}{0\cdot2759634}\ a_t - \underset{(0\cdot0862326)}{0\cdot2574734}\ a_{t-4}$$

$R^2 = 0\cdot115483$ Q-statistic (24 lags) $= 22\cdot24084$ SE of $a_t = 0\cdot004795$ SD of dependent var. $= 0\cdot005063$ 144 Observations (January 1973 to December 1984).

The first difference of the logarithm of the oil price series, DP_t, was identified as an AR(1). The estimated model is presented below along with the Box–Pierce Q statistic calculated from the residuals (standard errors of the coefficients are shown in parentheses):

$$\mathrm{DP}_t = \underset{(0\cdot0043962)}{0\cdot0119926} + \underset{(0\cdot0766518)}{0\cdot4095051}\ \mathrm{DP}_{t-1} + a_t$$

$R^2 = 0\cdot167357$ Q-statistic (24 lags) $= 21\cdot89200$ SE of $a_t = 0\cdot031151$ SD of dependent var. $= 0\cdot034019$ 144 Observations (January 1973 to December 1984).

Both ARIMA models appear to fit the series well.

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